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Optimal Scheduling Techniques for Charging the Batteries of Electric Vehicles under Time-Of-Use Pricing Using Binary Integer Programming

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ABSTRACT:This paper presents new optimal scheduling technique for charging the batteries of Electrical Vehicles (EVs) where the electric supply companies offer differential pricing depending on the Time-of-Use (TOU). The optimal schedule minimizes the total chargingcost of the battery of a specific EV. At the charging station, the arrival and departure time of the EV is taken into consideration along with the required charging power sequence.

KEYWORDS: Arrival time slot, Binary Integer programming, departure time slot, Kronecker product, Linear Sum Assignment Programming (LSAP), Plug-in Electric Vehicle (PEV), sort index, Time-of-Use pricing,charging stints,vec-operator.

I. INTRODUCTION

Battery driven Electric Vehicles (EVs) are ecologically superior to conventional fossil fuel driven vehicles. Periodic charging of the batteries is an important requirement in the operation and maintenance of EVs. Easily accessible and efficient Charging Stations will boost the usage of EVs. In a smart grid, the Time-of-Use (TOU) pricing of electricity scheme [1], [2] is a Demand Side Management (DSM) technique.Here, the electrical energy is differentially priced depending on the time of use in a day. The energy cost is priced at a higher rate during peak demand hours so that users are encouraged to consume more during lean periods. This will, to some extent, make way for over all uniform load distribution.In those places where the electricity market is highly regulated and the TOU pricing is adopted, it is possible to schedule the charging task to minimize the total cost of electricity used to charge the battery of the EV in the specified duration. The objective of this paper is to calculate the optimum charging schedules to minimize cost of electricity used under different charging constraints.Several solutions have already been proposed for optimum charging of EV batteries based on different techniques [3], [4], [5]. A game theoretic approach is used in [6].

II. TIME-OF-USE PRICING, ASSUMPTIONS AND NOTATIONS

A typical 24 hour TOU price information is shown in the bar graph of Fig.1. Normally the TOU tariff will be different for weekdays and holidays/weekends. In general, the TOU price varies on hourly basis during the 24 hour day. For convenience, the 24 hours of the day are divided into 24 standard slots of one hour each. The TOU time slots are sequentially numbered as j = 1, 2,…, 24. Slot 1 starts just after midnight(00 hrs) and occupies the time interval from 00 to 01 hrs. Slot i occupies the time interval (i–1) to i hrs.

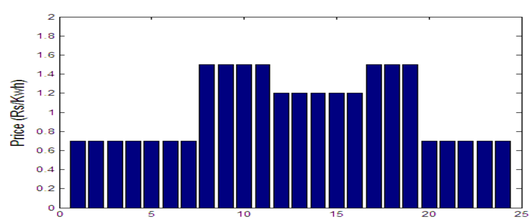

Fig.1. TOU Price information

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For a given TOU scheme, the unit price (rate) information is stored in the array R(j) for j = 1, 2,…,24. Here, R(j) is the price of the electrical energy in time slot j. Variable j represents the time slot index for the Ncontiguous hours of the day under consideration. Since TOU pricing is generally cyclic with a 24 hour period, it repeats itself after every 24 hours except for holidays and weekends. Therefore R(j) is periodic as,

$$R(j+24) = R(j) \quad (1)$$

Thus the price matrix is R = [R(1) R(2) … R(24)]. The size of R is (1, 24). It is a row vector. In some situations, all the 24 hours may not be available for charging. Therefore we may take the range of the price vector R fromR(1) to R(N). That is, R = [R(1) R(2) …. R(N)]. Invariably N=24. Therefore in our discussion we take N=24.

A. Charging Sequence

In this section, we consider the charging of one EV battery. To take advantage of the TOU pricing, we would like to charge the battery during the low price time slots as far as possible and to skip charging during high price time slots. Thereforethe charging of the battery will be intermittent (in burst). Thus we use ON-OFF charging. The battery charging ON-OFF time periods are also expressed in terms of the standard time slots adopted to describe TOU pricing. Thus an ON or OFF charging intervals will be an integers representing the hours of the day. The ON charging operation for aninterval of one hour is calleda*charging stint*. The charging stints are synchronous with the time slots of the day. That is, the starting and ending of a charging stint coincides with that of a time slot. A typical charging sequence is shown in Fig.2.

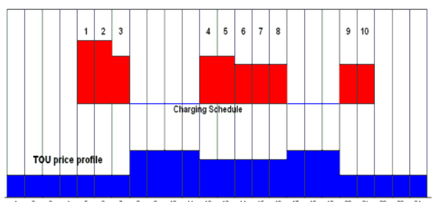

Fig.2. Charging Schedule with 3-ON periods along with TOU price profile

In Fig. 2, the charging operation has 10 charging stints. The charging stints are sequentially marked as,1, 2, 3, (four-gaps)4, 5, 6, 7, 8, (three-gaps) 9, 10. In general, let the total number of charging stints in a charging cycle be M. The individual stints are designated as i = 1, 2, …, M.Normally M is less than or equal to 24 so that a charging cycle has a duration less than or equal to 24 hours.

B. Power and Energy Drawn by the battery.

Let the charging power drawn by the concerned EV battery in its successive charging stints (time periods)be represented by P(i)'s for i = 1, 2,…, M. The unit of P(i) is in Kilowatts. Here, P(i)'s represent the battery charging power sequence. The total charging energy E, in Kilowatt hours, drawn by the battery in one charging cycle would be,

$$E = P(1) + P(2) + \cdots + P(M) = \sum_{i=1}^{M} P(i) \qquad (2)$$

The charging power sequence is represented by the row matrix P = [P(1) P(2) … P(M)]. The size of P is (1, M).

C. Arrival and departure time of the EV

We assume that the battery of the EV is available for charging as soon as the EV arrives at the charging station. Let the EV arrive at the charging station at T_1 hrs. IfT_1isat the beginning of the time slot J, then the arrival slot,designated as a,is taken as J. If t_1 is somewhere in the middle of the time slot J, then a is taken as (J+1). That is the arrival time slot a is given by,

$$a = \lceil T_1 \rceil + 1 \qquad (3)$$

Thus if T_1 = 01.30 hrs, it has arrived in the middle of time slot 2. Therefore a = 3. The departure slot b corresponding to the departure time T_2hrs, is given by,

$$b = \lceil T_2 \rceil - 1 \qquad (4)$$

Thus if T_2 = 12.30 hrs, then b = 12. The charging operation can take place between a andb as shown in Fig. 3. Here, the available slots for charging are from a tob.That is from 3 to 12. In Fig. 3, the time slots are marked in red numbers. If the departure time belongs to the next day, add 24 to T_2 to get the effective T_2.Since the first slot available for charging is a, the first charging stint x(1) can be a or greater than a. Similarly, the last charging stint can occur on or before b. Thusx(1) ≥ aand x(M) ≤ b. That is,

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$$a \leq x(i) \leq b \quad (5)$$

Since we have M charging stints, they should occur within the range a tob. Therefore the total number of available time slots should be greater than or equal to M. That is,

$$(b–a+1) \geq M \qquad (6)$$

In oursolution, we assume that the condition specified by Eq.(6) is satisfied.

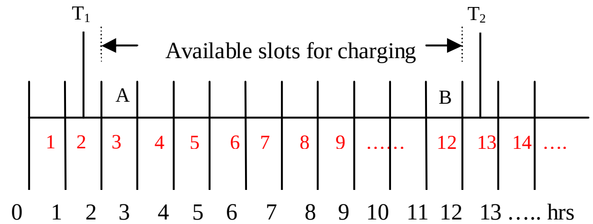

Fig.3. Daily time slots and available slots for charging

D. Charging Schedule

Let the first battery charging stint occur at time slot designated as x(1) where x(1) is one of thetime slots. (In Fig.1, x(1) = 5.) Let the charging power in stint 1 beP(1). Therefore the power P(1) is drawn in the time slot x(1). Let the second charging stint be at time slot x(2). (In Fig. 2, x(2) = 6.) Naturally x(2) >x(1) because, the second stint occurs after the first one. There may be gaps between x(2) and x(1).Power drawn during x(2) is P(2). Similarly the i th charging stint occupies the time slot x(i). The time slots occupied by the consecutive charging stints are represented by x(1), x(2),…,x(M). The charging powers drawn are,

P(1) during day time slot x(1)
P(2) during day time slot x(2)
…………………………………
P(i) during day time slot x(i)
…………………………………
P(M) during day time slot x(M)

Thus the power drawn for charging in slot x(i) is P(i) for i =1,2, 3,…, M. The sequence x(i) for i = 1 to M forms the charging schedule. Thus the charging schedule denoted by symbol X is,

$$X = [\ x(1),\ x(2), \dots x(i),\dots, x(M)] \qquad (7)$$

In Fig.2, the charging schedule X is,

$$X=[\ 5, 6, 7, 12, 13, 14, 15, 16, 20, 21]$$

The charging energy during charging stint i of 1 hour duration is P(i)*1=P(i).This P(i) occurs in the time slot x(i). The energy price in slot x(i) is given by R(x(i)). Therefore the corresponding charging cost would be R(x(i))*P(i). Hence the total cost for all the stints is,

$$C = R(x(1))*P(1)+R(x(2))*P(2)+,\dots+R(x(M))*P(M)$$

That is,

$$C = \sum_{i=1}^{M} R(x(i)) * P(i) \qquad (8)$$

Here, our objective is to minimize C by properly choosing the sequence ofx(i)'s.

III. CONSTRAINED OPTIMIZATION

Optimal battery charging schedule is to determine the sequence x(i) for i = 1 to M to minimize C as given by Eq. (8). The following data are given.

1. Number of TOU time slots N.
2. Number of charging stints M.
3. TOU pricing scheme represented by R(j) for j = 1 to N.
4. Battery charging power sequence P(i) for i = 1 to M.
5. Arrival and departure timesT_1 and $T_2$of the EV. Thus a and b are known.

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While determining the sequence x(i) for i = 1 to M, the following constraints are to be satisfied.

1. $x(i)\in\{1 \text{ to } N\}$ for i = 1, 2,..., M (9)
2. $x(i+1) > x(i)$ for i=1, 2,..., (M–1) (10)
3. $a \leq x(i) \leq b$ (from Eq. (5)) (11)

A. Solution

Case 1): Consider the case where the charging power P(i) is constant for all i's. That is,
$P(1) = P(2) = ... = P(i) =...= P(M) = P$
In this case, minimization of C as given by Eq. (8), is same as minimization of F given by,

$$D = \sum_{i=1}^{M} R(x(i)) \qquad (12)$$

From Eqs.(5),$a \leq x(i) \leq b$ for i = 1 to M. Hence the available range for R(x(i)) in Eq. (12) is, from R(a) to R(b). Let us designated this sequence as G given by,

$$G = [\ R(a), R(a+1), R(a+2),\ldots, R(b)\ ] \qquad (13)$$

Let L represent the number of elements in G. Then,

$$L = (b–a+1) \qquad (14)$$

From Eqs. (6) and (14),

$$L \geq M \qquad (15)$$

The elements of sequence G are,

$$\begin{gathered} G(1) = R(a) \\ G(2) = R(a+1) \\ \ldots\ldots\ldots\ldots \\ G(k) = R(a+k–1) \qquad (16) \\ \ldots\ldots\ldots\ldots \\ G(L) = R(a+L–1) = R(b) \end{gathered}$$

Thus the terms available for summation in Eq. (12) are from the sequence G. From this sequence, we have to choose M terms to minimize F.

The solution is simple. We have to pick thelowest M terms from G. Then, their sum will be minimum compared to any other combinations. To pick the lowest M terms, we sort G in the ascending orderand take the first M terms. The corresponding sort indices give the locations of these M terms in G. This process is carried out as follows.

$$[H, sort\_index\ ] = sort(G) \qquad (17)$$

H is the sorted array and sort_index is the array of sort indices which specify the locations H(1), H(2), *etc.*In array G. Thus,

$$\begin{gathered} H(1) = G(sort\_index(1)) \\ H(2) = G(sort\_index(2)) \\ \ldots\ldots\ldots\ldots \\ H(k) = G(sort\_index(k)) \qquad (18) \\ \ldots\ldots\ldots\ldots \\ H(L) = G(sort\_index(L)) \end{gathered}$$

Here, L = size of array G = (b–a+1).

Let the first M terms of sort_index be stored in array Q as,

$$Q = sort\_index(1{:}M) = [\ Q(1), Q(2),\ldots, Q(M)\ ] \qquad (19)$$

Thus, H(1) = G(Q(1)), H(2) = G(Q(2)), and so on. That is,

$$H(k) = G(Q(k)) \quad \text{for } k = 1, 2,\ldots,M \qquad (20)$$

From Eqs. (16) and (20),

$$H(k) = G(Q(k)) = R(a+Q(k)–1) \qquad (21)$$

Let us use the symbol V(k) to represent a+Q(k)–1. That is,

$$V(k) = a+Q(k)–1 \quad \text{for } k = 1 \text{ to } M \qquad (22)$$

From Eqs. (21) and (22), H(k) = R(V(k)) for k = 1 to M. Since H(k)'s are the M lowest terms of G, sum of H(k)'s minimize D as given by Eq. (12).

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Therefore, the minimum of D is,

$$D_{min} = \sum_{i=1}^{M} H(k) = \sum_{i=1}^{M} R(V(k)) \qquad (23)$$

Hence, the indices of R's which minimize F are given by V(k) = a+Q(k)–1 for k = 1 to M. Thus x(i)'s belong to the set {V(k) for k = 1 to M}. Since x(i)'s have to be a sequence in the ascending order, x(i)'s are obtained by sorting V(k)'s in the ascending order. Thus the optimal sequence S is given by,

$$S = [\, x(1),\ x(2), \ldots x(i), \ldots, x(M)] = \mathrm{sort}(V) \qquad (24)$$

The above process of determining the optimal sequence S is described in Algorithm 1.

Algorithm 1. The inputs are: TOU price sequence R(j)'s. The lowerand upper limits a and b on x(i)'s.

Output: Optimal sequence S =[x(1), x(2), … x(i),…, x(M)].

1. Get the sequence G using Eq. (13).
2. Sort G in the ascending order to get H and the sort_index using Eq. (17).
3. Get Q, the first M terms of G using Eq. (19).
4. Add offset (a–1) to the elements of Q to get V as, V(k) = Q(k)+ a–1 for k = 1 to M
5. Get X by sorting array V in the ascending order as, X = sort(V).
6. Get D_{min} using Eq. (23).

B. solution when P(i)'s are not uniform.

When P(i)'s are not same, the objective function to be minimized is given by Eq. (8), reproduced here.

$$C = \sum_{i=1}^{M} P(i) * R(x(i)) \qquad (8)$$

Our aim is to determine x(i)'s to minimize C, subjected to the constraints given by Eqs. (9), (10) and (11). Here x(i)'s take integer values in the range from a to b. This problem can be reformulated as a*Linear Sum Assignment Problem* (LSAP) [7], [8]as follows.

C. *Linear Sum Assignment Problem*

R(x(i)) which is the x(i) th element of R can be rewritten as,

$$R(x(i)) = \sum_{j=1}^{N} \delta(j - x(i)) * R(j) \qquad (25)$$

where $\delta(n) = 1$ for $n = 0$ and $\delta(n) = 0$ for $n \neq 0$. Thus $\delta(n)$ is a binary variable whose value is either 0 or 1.

From Eqs. (8) and (25),

$$C = \sum_{i=1}^{M} P(i) * \sum_{j=1}^{N} \delta(j - x(i)) * R(j)$$

This is rewritten as,

$$C = \sum_{i=1}^{M} \sum_{j=1}^{N} P(i) * \delta(j - x(i)) * R(j) \qquad (26)$$

$$\text{Let} \quad Y(i, j) = \delta(j - x(i)) \qquad (27)$$

for i = 1 to M and j = 1 to N.

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In the light of Eqs. (27), Eq. (26) becomes,

$$C = \sum_{i=1}^{M}\sum_{j=1}^{N} P(i) * Y(i,j) * R(j) \qquad (28)$$

Here Y(i, j) is the element at row i and column j of the binary matrix Y. The size of Y is(M, N).When Y(i, j) =1, charge stint i is assigned to the time slot j. Then P(i)*Y(i, j)*R(j) =P(i)*R(j) represents the cost of charging with stint I assigned to slot j. To choose Y(i, j) to Minimize C given by Eq. (26) is a *Linear Sum Assignment Problem*(LSAP) [4]. We have to assign appropriate charging stint i to the TOU time slot j so that C is minimum.
Eq.(28) can be written in the matrix form as,

$$C = P*Y*R^T \qquad (29)$$

where P =[P(1) P(2) … P(M)] is a row matrix of size (1, M) and R = [R(1) R(2) … R(N)] is the price matrix of size (1, N).

D. Constraints of the LSAP

In our assignment problem, charging stint i should be assigned to a single time slot. Therefore,

$$\sum_{j=1}^{N} Y(i,j) = 1 \text{ for } i = 1 \text{ to } M \qquad (30)$$

When the arrival and the departure time slots a andb are given, the summation range of Eq. (28) for j would be from j = a to b. No assignments are made outside this range. Therefore, the summation terms are zeros outside this range. Hence Y(i, j) = 0 outside this range. Therefore Eq. (30) is modified as,

$$\sum_{j=a}^{b} Y(i,j) = 1 \text{ for } i = 1 \text{ to } M \qquad (31)$$

This means only one element of each array Y(i, j) for j =a to b has to be 1 and the remaining elements have to be zeros.From Eq. (28), the time slot assigned for charge stint i is, j = x(i). Therefore,
Y(i, j) = 1 when j = x(i)
and Y(i, j) = 0 when $j \neq x(i)$
for i =1 to M.
The above condition is expressed as,

$$\sum_{j=1}^{N} Y(i,j) * j = x(i) \qquad (32)$$

Now for row (i+1), the above equation becomes,

$$\sum_{j=1}^{N} Y(i+1,j) * j = x(i+1) \qquad (33)$$

We know from Eq. (10), that x(i+1) > x(i). This means, the column locations of 1's should progressively increase as we go from the present row to the next row. Therefore, from Eqs. (10), (32) and (33),

$$\sum_{j=1}^{N} Y(i+1,j) * j > \sum_{j=1}^{N} Y(i,j) * j \qquad (34)$$

Eq. (34) should hold good for i =1 to (M–1).
Thus the LSAP is to determine Y to minimize C given by Eq. (30), subjected to the conditions of Eqs. (31) and (34). Another important constraint is, a TOU time slot cannot accommodate more than 1 battery charging stints. We assume that a single battery is gettng. It can be one or none.
This constraint can be formulated as,

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$$\sum_{i=1}^{M} Y(i,j) \le 1 \text{ for } j = 1 \text{ to } N \qquad (35)$$

E. Column Major format of Y or vec(Y)

The optimization problem is to determine the binary matrix Y to minimize C given by Eq. (29). Determination of Y is easy if Y is in the column major or single column format. The column major format of Y is obtained by arranging the columns of Y one beneath the other. This result is obtained using the *vec*-operator [7] as follows.

Let Y = [Z(1) Z(2) … Z(j) … Z(N)]

where Z(1), Z(2),…, Z(j),… Z(N) are the columns of Y.

Then vec(Y) is defined as,

$$U = vec(Y) = \begin{bmatrix} Z(1) \\ Z(2) \\ \cdot \\ \cdot \\ Z(N) \end{bmatrix} \qquad (36)$$

The *vec*-operator vectorizes the matrix. The well known theorem [9] using *vec*-operator states that,

$$vec(A*Y*B) = (B^T \otimes A)*vec(Y) \qquad (37)$$

For compatible size matrices A, Y and B. In eq(37), $\otimes$ is the Kronecker product [9] operator.

From Eqs. (29), (36) and (37),

$$vec(C) = vec(P*Y*R^T) = (R \otimes P)*U \qquad (38)$$

Since C is a scalar, vec(C) = C itself. Therefore the cost of charging C is,

$$C = (R \otimes P)*U \qquad (39)$$

$$(R \otimes P) = [\ R(1)*P\ \ R(2)*P\ \ldots\ R(j)*P \ldots R(N)*P] \qquad (40)$$

The size of $(R \otimes P)$ is (1, N*M). The size of U is (N*M, 1).

U is the binary vector to be determined for minimum C. The cost matrix of the problem is $(R \otimes P)$. Now we can apply the Binary Integer Programming [10] to optimize C given by Eq. (39).

G. Binary Integer Programming

.The objective function to be minimized is given by Eq. (39). The constraints specified by Eqs. (31) is reformulated for U as follows. In Eq. (31), j varies from j = a to j = b for each i. Eq. (31) can be expressed as a matrix product as,

$$Y*S^T = [ones(M, 1)] \qquad (41)$$

Where [ones(M, 1)] is a column vector of all ones with size (M ,1).

H is given by,

$$S=[zeros(1, a-1)\ \ ones(1, b-a+1)\ \ zeros(1, b-1)] \qquad (42)$$

S is a binary row vector with elements from location a through b are 1's while the remaining elements are zeros.

In Eq. (37), put A = I= Identity matrix of size M. then Eq. (37) gives,

$$vec(I*Y*B) = (B^T \otimes I)*vec(Y) \qquad (43)$$

putting B = S^T, and knowing that vec(Y0) = U, Eq. (43) becomes,

$$vec(Y*S^T) = (S \otimes I)*U \qquad (44)$$

FomEqs.(44) and (41),

$$(S \otimes I)*U = vec([ones(M, 1)]) = [ones(M, 1)] \qquad (45)$$

Thus the equality constraint specified by Eq. (31) is equivalent to,

$$(S \otimes I)*U = [\ ones(M, 1)] \qquad (46)$$

Now, consider the inequality constraint Eq. (35) which can be re written as,

$$[ones(1, M)]*Y \le [ones(1, N)] \qquad (47)$$

Putting A = [ones(1, M)] and B = I= Identity matrix of size N in Eq. (37), we get,

$$vec([ones(1, M)]*Y*I) = (I \otimes [ones(1, M)])*U \qquad (48)$$

Taking vec-operation on both sides of Eq. (47), we get,

$$vec([ones(1, M)]*Y) \le vec([ones(1, N)]) \qquad (49)$$

From Eqs. (49) and (48),

$$(I \otimes [ones(1, M)])*U \le [ones(N, 1)]) \qquad (50)$$

Here I = Identity matrix of size N. Note that,

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vec([ones(1, N)]) = [ones(N, 1)])

Thus, Eq. (50) represents the inequality constraint of Eq. (35) in terms of U.

Now, consider Eq. (32) which can be expressed in terms of the matrix product as,

$$Y*[\,1\;\;2\;\;3\ldots\;\;N]^T = [x(1)\;\;x(2)\;\;\ldots x(M)\,]^T \quad (51)$$

$$\text{Let } L = [\,1\;\;2\;\;3\ldots\;\;N] \quad (52)$$

Then Eq. (51) becomes,

$$Y*L^T = [x(1)\;\;x(2)\;\;\ldots x(M)\,]^T \quad (53)$$

From Eq. (37), putting A = I of size(M) and B = L^T, we get

$$vec(I*Y*L^T) = (L\otimes I)*vec(Y) = (L\otimes I)*U \quad (54)$$

$$\text{That is,}\quad vec(Y*L^T) = (L\otimes I)*U \quad (55)$$

From Eqs. (53) and (55),

$$(L\otimes I)*U = [x(1)\;\;x(2)\;\;\ldots x(M)\,]^T \quad (56)$$

$$\text{Let }\; X = [x(1)\;\;x(2)\;\;\ldots x(M)\,]^T \quad (57)$$

Then Eq. (56) can be written as,

$$(L\otimes I)*U = X \quad (58)$$

Consider the (M–1)*M circulant matrix W as,

$$W = \begin{bmatrix} 1 & -1 & 0 & 0 \ldots 0 & 0 \\ 0 & 1 & -1 & 0 \ldots 0 & 0 \\ \ldots & & & & \\ 0 & 0 & 0 \ldots & 1 & -1 \end{bmatrix}$$

Now,

$$W*X = \begin{bmatrix} x(1) - x(2) \\ x(2) - x(3) \\ \ldots \\ X(M-1) - x(M) \end{bmatrix} \quad (59)$$

The size of W*X is ((M–1), 1).

Then, the condition of Eq. (10) can be expressed as,

$$W*X \le [-1\;-1\;\;-1\ldots -1]^T \quad (60)$$

From Eqs. (58) and (60), we get,

$$W*(L\otimes I)*U \le [-1\;-1\;\;-1\ldots -1]^T$$

This can be rewritten as,

$$W*(L\otimes I)*U \le (-1)*[ones\,(M-1,1)] \quad (61)$$

Thus condition of Eq. (10), which is same as the condition represented by Eq.(34) is represented by Eq. (61).

Finally the binary integer programming is formulated as,

Minimize $C = (R\otimes P)*U$

The row vectors R of size (1, N) and P of size (1, M) are given. The equality constraint is given by

$$(S\otimes I)*U = [\,ones(M,1)] \quad (62)$$

The inequality constraint are,

$$(I\otimes[ones(1, M)])*U \le [ones(N, 1)])$$

$$W*(L\otimes I)*U \le (-1)*[ones(M-1, 1)]$$

The two inequalities can be combined into a single matrix inequality as,

$$Aineq * U \le Bineq \quad (63)$$

$$\text{Where } Aineq = \begin{bmatrix} I\otimes[ones(1,M)] \\ W*(L\otimes I)\,] \end{bmatrix} \quad (64)$$

$$\text{And } Bineq = \begin{bmatrix} I\otimes[ones(1,M)] \\ (-1)*[ones(M-1,1)] \end{bmatrix} \quad (65)$$

The cost to be minimized is reproduced here,

$$C = (R\otimes P)*U \quad (39)$$

Let us use symbolF to represent $(R\otimes P)^T$ as,

$$F = (R\otimes P)^T \quad (66)$$

Then, C is given by,

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$$C = F^T * U \quad (67)$$

In Eq. (62) call $(S \otimes I)$ by Aeq and [ones(M, 1)] by Beq.

That is let, $Aeq = (S \otimes I)$ (68)

and $Beq = [\text{ ones}(M, 1)]$ (69)

Then the equality constraint Eq. (62) becomes,

$$Aeq * U = Beq \quad (70)$$

Thus, the Binary integer programming problem is specified as,

$$\text{Minimise} \quad C = F^T * U \quad (67)$$

subjected to the following constraints.

$$Aeq * U = Beq \quad (70)$$

$$Aineq * U \leq Bineq \quad (63)$$

Our aim is to find U to minimize C.

This problem is solved using the Matlab*bintprog* [11] as,

$$U = \text{bintprog}(F, Aineq, Bineq, Aeq, Beq) \quad (71)$$

This function returns the optimal binary vector U that minimizes C.

F. Recovering Y from U

From the optimal U, the equivalent optimal binary matrix Y of size (M, N) is recovered using the *reshape* [9] function as,

$$Y = \text{reshape}(U, M, N) \quad (72)$$

The reshape function returns the M-by-N matrix Y whose elements are taken column-wise from U.

The locations of 1 in Y give the allocation of charging stints to the respective TOU time slots. If Y(I, J) =1, it means the charging stint I allocated to the time slot J. That is, the row index represents the charge stint and the column index represents the TOU time slot.

The optimal allocation time slots x(1) , x(2),...,x(M) are determined by finding the column locations of 1's in Y for the successive row locations. This is done using the Matlab*find* function as,

$$[\, I \;\; J] = \text{find}(Y) \quad (73)$$

Here the subscript set[IJ] gives the locations of 1's in Y. Column vector I gives the row index and the column vector J gives the corresponding column index of 1's in Y. The size of J is (M, 1). The contents of I will be from 1 to M. The optimum charging schedule is given by,

$$X = [\, x(1), \; x(2), \ldots x(i), \ldots, x(M)] = J^T \quad (74)$$

J^T is the transpose of J.

IV. OPTIMAL SCHEDULING ALGORITHM

Algorithm2. Inputs: TOU price array R of size(1,N) and the charging power sequence P of size (1, M).

Output: Optimal schedule X = [x(1) x(2) ... x(M)].

1. Get the Kronecker product,
   $F = (R \otimes P)^T$
2. Get matrices Aineq and Bineq from Eqs. (64) and (65).
3. Get matrices Aeq and Beq from Eqs. (68) and (69).
4. Get the optimal binary column vector U as,
   U = bintprog(F, Aineq, Bineq, Aeq, Beq)
5. Get Y as,
   Y = reshape(U, M, N)
6. Get J as,
   [I J] = find(Y)
7. Get the Optimal Time slot schedule X as,
   $X = J^T$
8. Over.

V. RESULTS & DISCUSSION WITH EXAMPLES

Example 1:TOU price sequence is given in TABLE 1. The price is in some appropriate units. Charging Power is constant.TABLE 1. TOU price information for 24 time slots

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Time-slot j	1	2	3	4	5	6
R(j)	14	10	12	9	8	10
Time-slot j	7	8	9	10	11	12
R(j)	21	9	15	6	12	21
Time-slot j	13	14	15	16	17	18
R(j)	12	18	13	23	12	21
Time-slot j	19	20	21	22	23	24
R(j)	23	9	25	9	20	12

The arrival and departure time slots are, a = 3 and b = 12. The number of charging stints are M = 7.
From Eq. (13),
G = [R(3) R(4) .... G(12)]
= [12 9 8 10 21 9 15 6 12 21].
Here, the size of G is L = 12 – 3+1 = 10.
On sorting we get,
H = [6 8 9 9 10 12 12 15 21 21].
Sort_index = [8 3 2 6 4 1 9 7 5 10].
First M elements of sort_index are,
Q = sort_index(1:7) = [8 3 2 6 4 1 9].
Adding offset (a–1) to Q we gwet,
V = Q+2 = [10 5 4 8 6 3 11].
Sorting V in the ascending order, we get the final schedule as,
X = [3 4 5 6 8 10 11].
Charging in these time slots minimizes the energy cost.
Dmin given by Eq. (12) is, D_{min} = 66. This will be the energy cost if P(1) = P(2) =...=P(M) = 1.

Example 2: The TOU and M values are same. But a and b are as follows. a = 18 and b = 7 next day. Effective b is 24+7=31. The available charging slots can be written as,
[a, a+1, ... 24, 25,...,31]=[18, 19,...,31].
The corresponding G is made up of R(a) to R(24) of this day and R(1) to R(b) of the next day. G is given by,
G = [R(a), R(a+1),...,R(24), R(1), R(2),...,R(7)]
= [R(18), R(19),...,R(24), R(1), R(2),...,R(7)].
= [21 23 9 25 9 20 12 : 14 10 12 9 8 10 21]
Number of elements of G is, L = b+24–a+1= 14. On sorting G we get,
H= [8 9 9 9 10 10 12 12 14 20 21 21 23 25]
Sort index = [12 3 5 11 9 13 7 10 8 6 1 14 2 4]
Q = sort index (1:7) = [12 3 5 11 9 13 7]
V=Q+a–1 = Q+17 = [29 20 22 28 26 30 24]
X = sort (V) = [20 22 24 26 28 29 30]
Indices 26, 28, 29 and 30 belong to the next day. The corresponding next day actual indices are obtained by subtracting 24 from them. Thus S can be rewritten as,
X = [20 22 24 next day: 24 56].
The corresponding D_{min} = 67.

Example 3. Here the price vector R is same as in Example 1. Arrival and departure time slots A andB are, A = 6 and B = 12. The charging profile P is, P = [8 7 6 5 4 3 2].
The result after running the binary integer program is found to be,

I = [1 2 3 4 5 6 7].&J = [6 8 10 11 13 17 20].

The minimum cost C = 341

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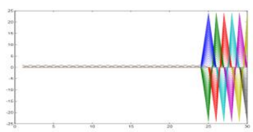
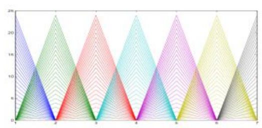
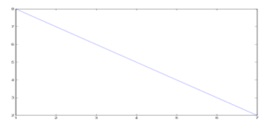
VI. SIMULATION RESULTS



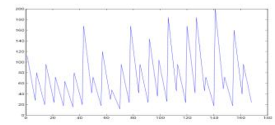
Graph1: Plot for [A].



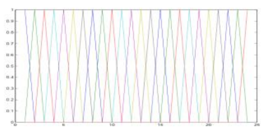
Graph 6: Plot for [F].



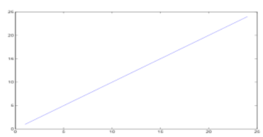
Graph 11: Plot for [P].



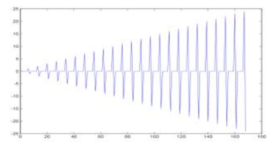
Graph 16: Plot for [F].



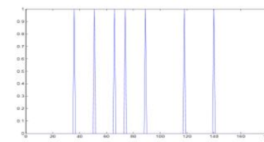
Graph2: Plot for [A1].



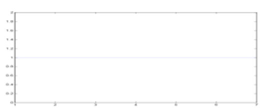
Graph 7:Plot for [G].



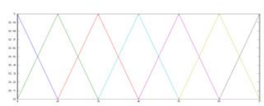
Graph 12: Plot for [Q].



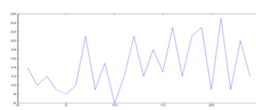
Graph 17:Plot for [X].



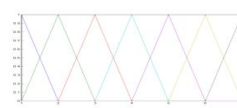
Graph3: Plot for [A2].



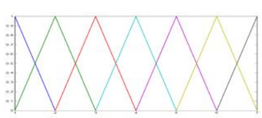
Graph 8:Plot for [H].



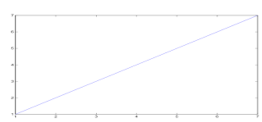
Graph 13:Plot for [R].



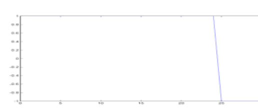
Graph 18: Plot for [XX].



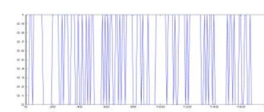
Graph 4:Plot for [Aeq].



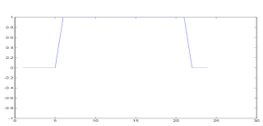
Graph 9:Plot for [I].



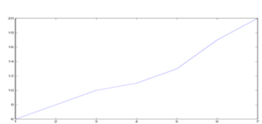
Graph 14: Plot for [B].



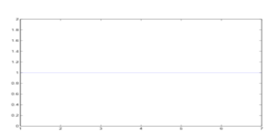
Graph 19:Plot for [I].



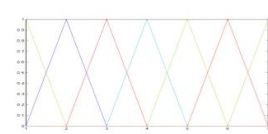
Graph5:Plot for [C].



Graph 10: Plot for [J].



Graph 15: Plot for [Beq].



Graph 20:Plot for [N].

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VI. CONCLUSION

A Binary Integer Programming approach is presented for minimizing the EV battery charging cost under the differential Time-of_Use (TOU) price scheme. In this paper the TOU time slot interval is takenas 1 hour. Our method holds good for different values of the time slot interval either higher or lower granularity.

REFERENCES

[1]C. W. Gellings, "The concept of demand-side management for Electricutilities,"*Proc. IEEE* , vol. 73, pp. 1468–1470, 1985.

[2] S. Rahman and SR. Rinaldy, "An efficient load model for analyzingdemand side management impacts,"*IEEE Trans. Power Syst*.vol. 8, pp. 1219–1226, 1993.

[3] S.Deilami, A. S. Masoum, P. S.Moses, and M. A. S. Masoum, "Realtimecoordination of plug-in electric vehicle charging in smart grids to minimize power losses and improve voltage profile," *IEEE Trans. Smart Grid*, vol. 2, no. 3, pp. 456–467, Sep. 2011.

[4] J. A. Peças Lopes, F. J. Soares, P. M. Almeida, M. Moreira da Silva"Smart Charging Strategies for Electric Vehicles: Enhancing Grid Performance and Maximizing the Use of Variable Renewable Energy Resources," 24th International Electric Vehicle Symposium and Exposition (EVS24) InProceedings 24th International Electric Vehicle Symposium and Exposition (EVS24) (May 2009), pp. 1-11.

[5]D. T. Phan, JinjunXiong, S. Ghosh "A distributed scheme for fair EV charging under transmission constraints,"*2012 American Control Conference (ACC) Proceedings* In 2012 American Control Conference (ACC) Proceedings (June 2012), pp. 1053-1058.

[6] W. Tushar, W. Saad, H. V. Poor, D. B. Smith, "Economics of Electric Vehicle Charging: A Game Theoretic Approach," *IEEE Transactions on Smart Grid* In IEEE Transactions on Smart Grid, Vol. 3, No. 4. December 2012, pp. 1767-1778.

[7] M. Akg¨ul, "The linear assignment problem, in Combinatorial Optimization," M. Akg¨uland S. Tufecki, eds., Springer Verlag, Berlin, 1992, pp. 85–122.

[8] Rainer E. Burkard, Eranda C¸ela, "Linear Assignment Problems and Extensions," Handbook of Combinatorial Optimization: Supplement, Volume 1. Ding-Zhu Du, Panos M. Pardalos. Springer, Oct 31, 1999.

[9] Jan R. Magnus and Heinz Neudecker (1999), *Matrix Differential Calculuswith Applications in Statistics and Econometrics*, 2nd Ed., Wiley. pp. 31-35.

BIOGRAPHY

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