



# Permanent Capacitor Single-Phase Induction Motor, D-Q Axis Modelling and Non-Linear Mathematical Analysis & Simulations

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**Abstract:** This paper presents a non-linear dynamic model of a permanent capacitor single-phase induction motor. The D-Q axis model of the permanent capacitor induction motor, based on the state vector analysis of the system is conducted; revealing the periodic and chaotic phenomenon under different system parameters. Accordingly, a chaotic-speed fan can be achieved by appropriately varying the motors[4]' internal parameters or the operation condition. Mathematical analysis and computer simulations are attached to testify the proposed non-linear model of the permanent capacitor single-phase induction motor.

**Keywords:** chaos, non-linear, permanent-capacitor, periodicity, phase-plot

## I. INTRODUCTION

The Permanent capacitor single-phase induction motors (SPCIMs) are commonly found in the drive systems of fans, compressors and pumps etc. Here in these particular modelling performance characteristics of the induction motors circuit of lumped parameters are used; offering simplicity and fast computation. Assumptions of the mathematical linearity of magnetic circuits and negligible slotting of the stator and rotor are considered.

Chaos can be defined as an a-periodic long term behaviour in a deterministic system that exhibits sensitive dependence on initial condition. Power systems, power converters, motor drives, telecommunications and medical electronics are few of the electrical systems that have already been identified to exhibit chaotic behaviours; characterized by a noise-like spectrum which has a continuous broad-band nature. As the initial state of a practical system can only be specified with some tolerance, the long-term behaviour of a chaotic system can never be predicted accurately.

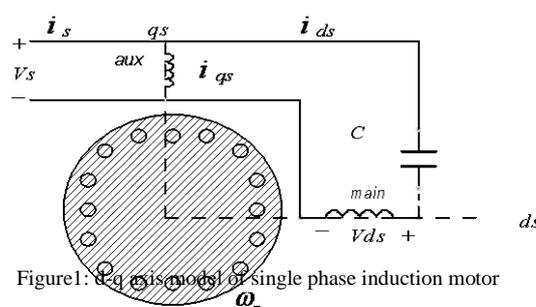


Figure1: D-q axis model of single phase induction motor



The paper here is sub-divided conveniently into parts. Part I. gives us the Introduction, Part II. has the D-Q axis modelling of the single-phase permanent-capacitor induction motor. The MATLAB/SIMULINK modelling of the system has been described in Part III. The results and MATLAB/SIMULINK simulations have been discussed in Part IV. Finally the conclusion is discussed in Part V. It is followed by references and bibliography.

## II. D-Q AXIS MODELING OF THE MOTOR

The D-Q model a single phase induction machine can be considered to be an unsymmetrical two phase induction machine. Three-phase machines' modelling can be easily be defined by the D-Q axis modelling. The equivalent circuit thus obtained can be represented for the SPIM is shown in alongside figure, and the machine is modelled by the following equations:

$$\begin{aligned}
 V_{ds}^s &= V_s \\
 V_{qs}^s &= V_s - \frac{1}{C} \int i_{ds} dt \\
 V_{ds} &= i_{ds} R_{ds} + p \lambda_{ds} - \omega_r \lambda_{qs} \\
 V_{qs} &= i_{qs} R_{qs} + p \lambda_{qs} + \omega_r \lambda_{ds} \\
 V_{dr} &= i_{dr} R_{dr} + p \lambda_{dr} \\
 V_{qr} &= i_{qr} R_{qr} + p \lambda_{qr}
 \end{aligned}$$

$$T_e = T_L + J \frac{d\omega_r}{dt} + B_m \omega_r \Rightarrow \frac{d\omega_r}{dt} = \frac{T_e}{J} - \frac{T_L}{J} - \frac{B_m}{J} \omega_r$$

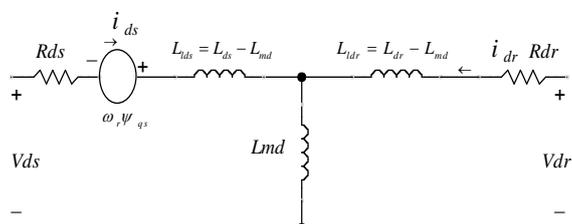


Figure 2: Equivalent Q-axis model of the induction motor

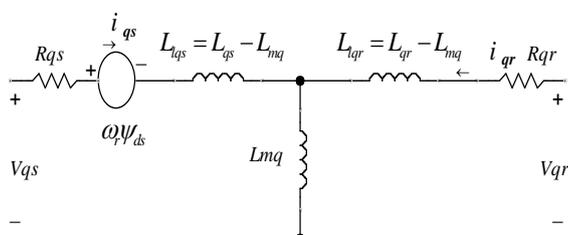


Figure 3: Equivalent D-axis model of the induction motor

where,



$R_{dr}$  = Direct axis rotor resistance,

$R_{qr}$  = Q-axis rotor resistance,

$L_{lds}$  = Direct axis stator leakage inductance,

$L_{lqs}$  = Q-axis stator leakage inductance,

$L_{md}$  = Direct axis mutual inductance,

$L_{ldr}$  = Direct axis rotor leakage inductance,

$L_{lqr}$  = Q-axis rotor leakage inductance,

$L_{mq}$  = Q-axis mutual inductance,

$\omega_r$  = Rotor angular speed and

$\omega$  = Speed of the reference frame,

### III. MATLAB/SIMULINK MODELLING

For a system possessing more than one unique behavior, as a parameter is varied, an abrupt change in the steady-state behavior of the system is called a bifurcation. Here we vary the amplitude of applied voltage. The voltage is varied from zero to 160 volt (r.m.s). The Two (main and auxiliary) stator windings are displaced 90 degree in space. Normally the capacitor is connected with auxiliary winding and the value is selected in such a way that the auxiliary current leads the main winding current by 90 degree for balanced operation. The modeling of the single phase permanent capacitor induction motor projects the non-linear model of the system. Here to inject non linearity the capacitor is connected to the main winding instead of auxiliary winding and the inductance value of the auxiliary winding is chosen in such a way that the phase difference of currents in the two winding is very less and also different in magnitude. It can be found that the chaotic speed waveforms offer the well-known chaotic properties, namely random-like but bounded oscillations. Also, these waveforms are a-periodic and very sensitive to the initial condition. Physically, these chaotic motions reveal the unbalanced status of the interaction between the magnetic fields by the main winding and the auxiliary winding.

The non-linear phenomenon in permanent capacitor single-phase induction motors (SPCIMs) are observed as the input voltage is varied from 0 to 160 Volts. Figures have shown above represents the speed waveforms and the corresponding phase-portraits, at various periodic-speed operations, namely the period-1, period-2 and period-5 operations. These waveforms consist with the well-known phenomenon of inevitable torque pulsation. It should be noted that once the operating condition are known, the motor parameter whose variation bringing chaos may be more than one. Possible candidates are the ratio of  $B_m$  to  $J$ , main winding resistance, split capacitor and so on.

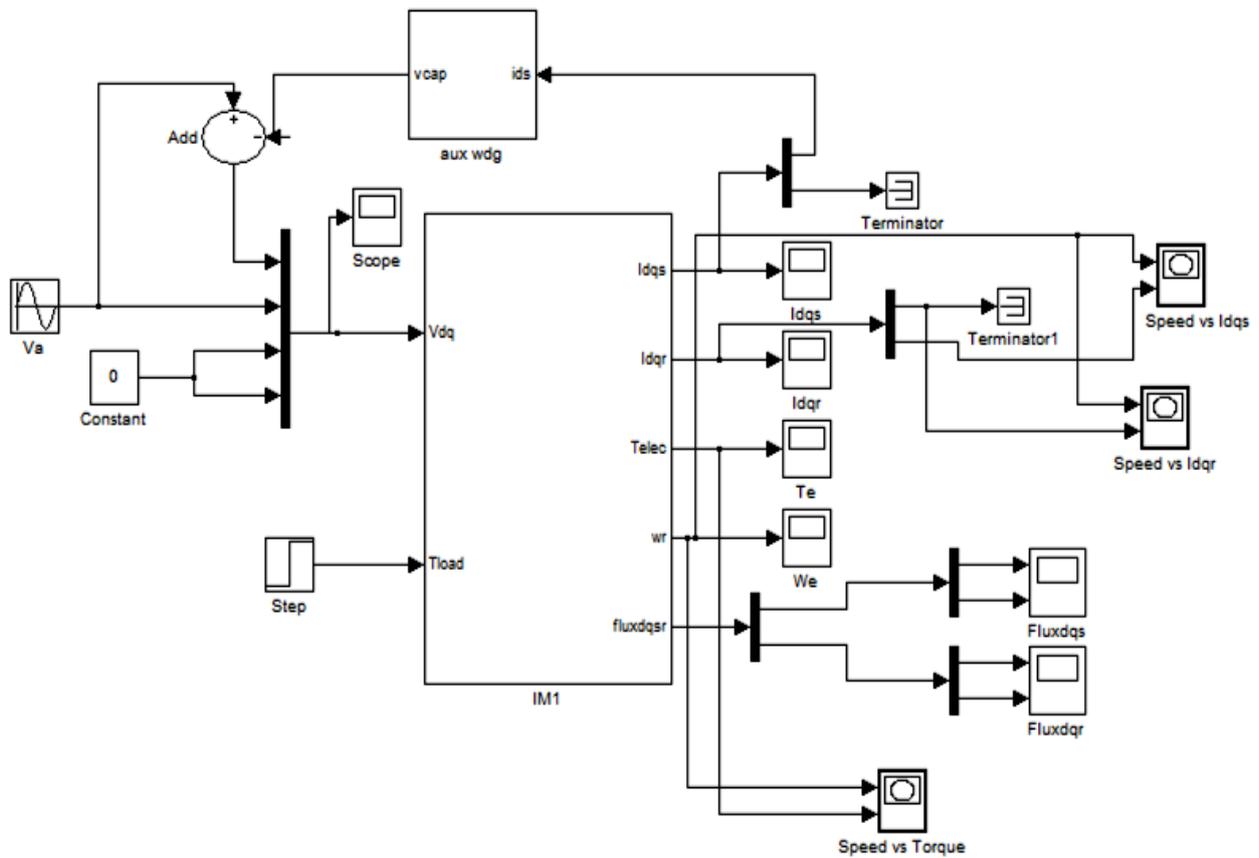


Figure 4: MATLAB/SIMULINK model of dynamic model of a permanent capacitor single-phase induction motor.

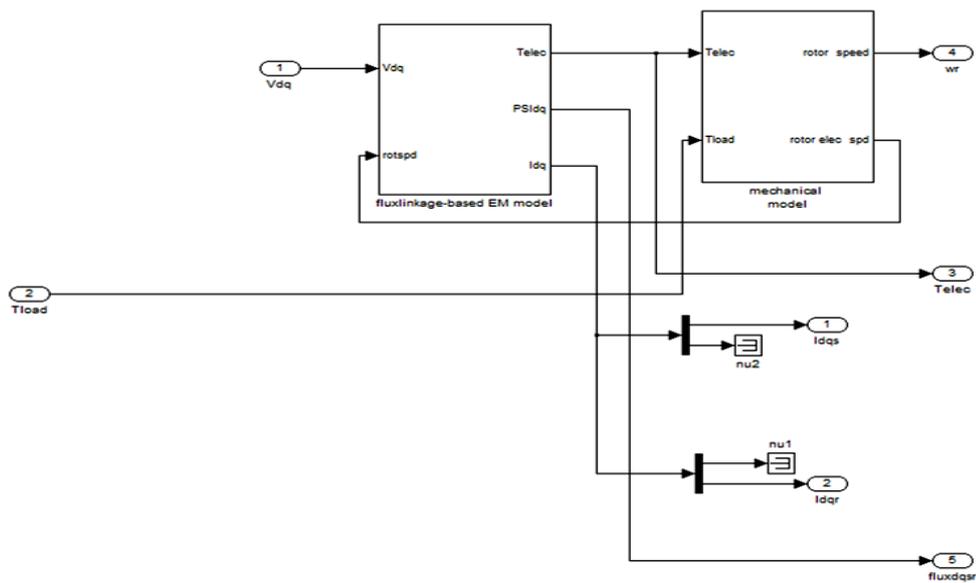


Figure 5: MATALB/SIMULINK sub-systems for the induction motor



Parameters	Value	Parameters	Value
Pair Of Poles	1	$L_{lqr}$	0.000482 H
$R_{dr}$	20 $\Omega$	$L_{mq}$	0.12 H
$R_{qr}$	20 $\Omega$	J	0.00007Kg-m <sup>2</sup>
$L_{lds}$	0.0211 H	$B_m$	0.001e-2 N.m.s
$L_{lqs}$	0.0117 H	$C_{run}$	211.5 $\mu$ F
$L_{md}$	0.2356 H	$R_{sd}$	9.5 $\Omega$
$L_{ldr}$	0.000482 H	$R_{qs}$	4.5 $\Omega$
Frequency (f)	40Hz		

Table 1: Motor parameters

#### IV. RESULTS AND SIMULATIONS

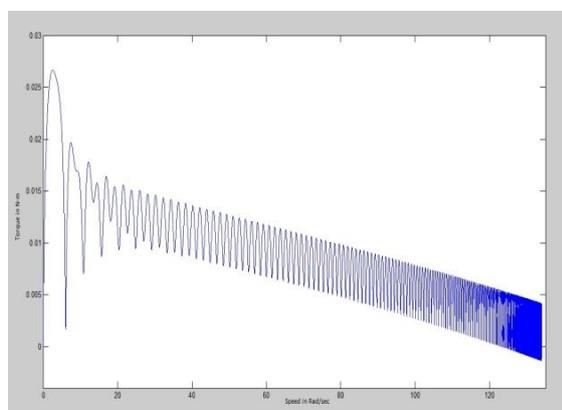


Figure 6: Speed-Torque Curve for 10 Volt (R.M.S)

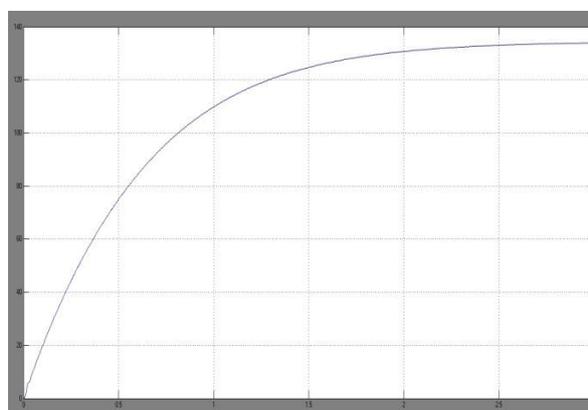


Figure 7: Speed Waveform for 10 Volt (R.M.S)

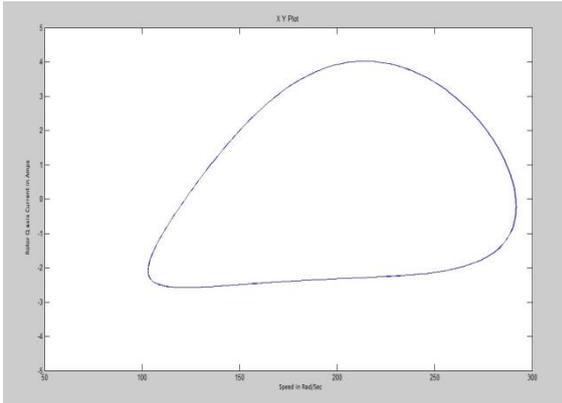


Figure 8: Iqr vs speed for 80 Volt (R.M.S)

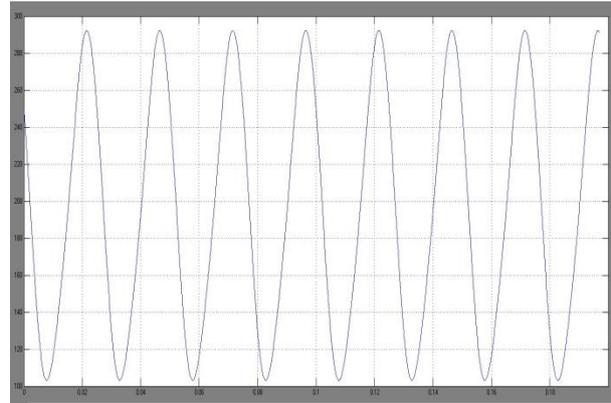


Figure 9: Speed waveform (Period One) for 80 Volt (R.M.S)

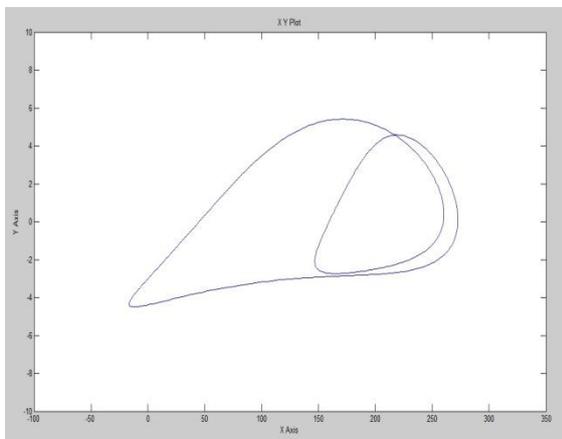


Figure 10: Iqr Vs Speed for 100 Volt (R.M.S)

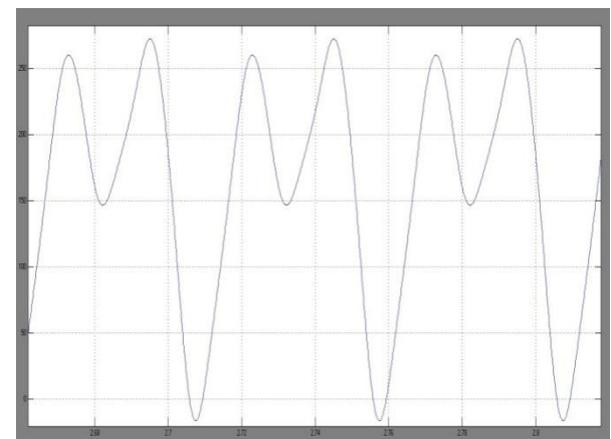


Figure 11: Speed waveform (Period Two) for 100 Volt (R.M.S)

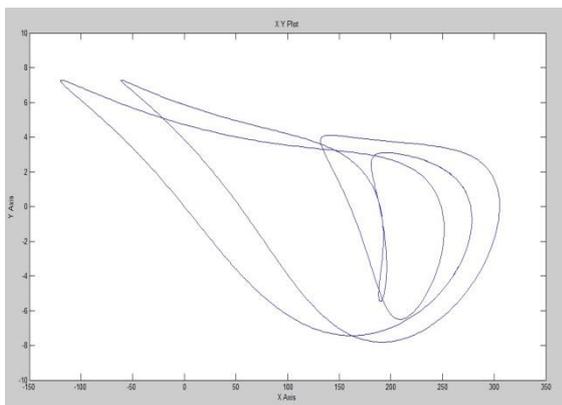


Figure 12: Iqr Vs Speed for 140 Volt (R.M.S)



Figure 13: Speed waveform (Period Five) for 140 Volt (R.M.S)

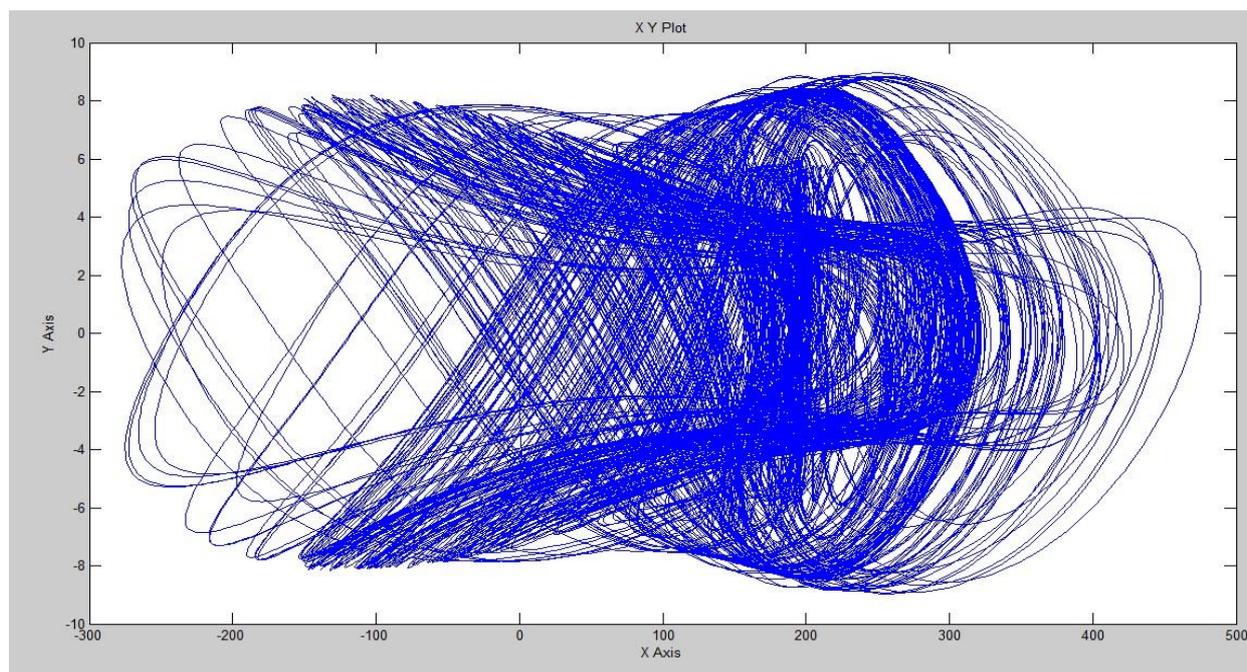


Figure 14: Iqr vs Speed for 160 Volt (R.M.S)

The figures displayed above from figure 6 to figure 15 exhibits the phase-plane portrait of the system, displaying the periodicity of the permanent-capacitor motor. The phase portraits of rotor current in the quadrature-axis direction versus speed for various input voltages are plotted and the resultants are observed, determining the periodicity of the system output. The same periodicity can be observed by plotting the speed versus time graph for the corresponding voltages. The output voltages are varied from the initial r.m.s. voltage of 10volts, 80 volts (here, period one is obtained) and to a final voltage value of 160 volts (the system here enters chaos). Rotor reference frame are chosen so that the rotor variables can be closely observed and due to the chosen reference frame the induced emf due to the rotor rotation circuit is absent ( $\omega = \omega_r$ ).

It is to be noted that number of turns in the windings of D-axis and Q-axis are same. Hence  $N_d/N_q=1$ . As since they have different cross-sectional area, they have different values of resistances and inductances and hence the resultant inductance; all the values being referred to the stator side.

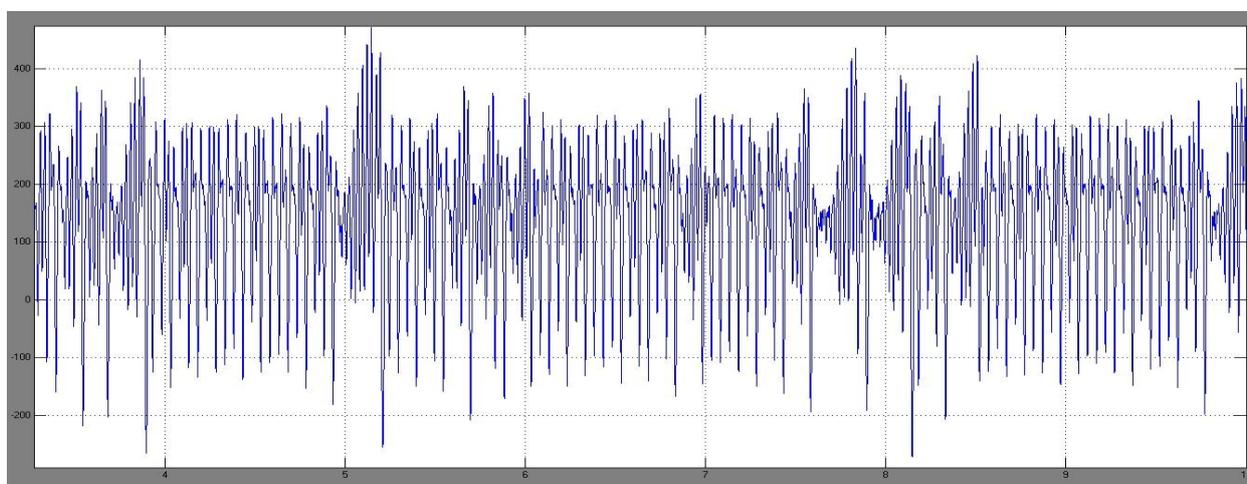


Figure 15: Chaotic Speed for 160 Volt (R.M.S)



## V. CONCLUSIONS

The non-linear phenomenon in permanent capacitor single-phase induction motors (SPCIMs) are observed as the input voltage is varied from 0 to 160 Volts. Figures have shown above represents the speed waveforms and the corresponding phase-portraits, at various periodic-speed operations, namely the period-1, period-2 and period-5 operations. These waveforms are consistent with the well-known phenomenon of inevitable torque pulsation. It should be noted that once the operating condition are known, the motor parameter whose variation bringing chaos may be more than one. Possible candidates are the ratio of  $B_m$  to  $J$ , main winding resistance, split capacitor and so on.

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