Quadtree Segmented Double Predictor DPCM Image Compression

Jia-Chyi Wu
Assistant Professor, Dept. of Communications, Navigation and Control Engr., National Taiwan Ocean University, Keelung, Taiwan

ABSTRACT: This study is to improve image data compression performance based on variable block-size quadtree image segmentation applied to double predictor differential pulse code modulation (DP–DPCM) image compressive algorithm. The quadtree segmentation method is applied to better allocate image characteristics. A variable block-size double predictor DPCM (VBDP–DPCM) image coding system works on an image been preprocessed into segments of variable size, square blocks, and each block is separately encoded by a DP–DPCM algorithm. Quadtree segmentation method is utilized to divide a given real-world image into variable size image blocks. The detail regions comprise more image features of a given image is segmented into blocks with smaller block size, and the background regions of the image will be assigned larger block size to the image blocks. After quadtree segmentation process, the average dissimilar values between the nearby pixels within an image block are abridged. Therefore, we can decrease the distribution range of the prediction error and diminish the quantization levels as well as the coding bit rate. We then adopt the double predictor DPCM technique to moderate the effect from the fed-back quantization error and not to augment the system complexity. The compression performance of this proposed method is about 5dB (or greater) coding gain in Signal-to-Noise Ratio (SNR) than that of a conventional DPCM system.

KEYWORDS: variable block-size; quadtree segmentation; differential pulse code modulation (DPCM); double predictor estimation

I. INTRODUCTION

The proposed quadtree segmented variable block-size double predictor differential pulse code modulation (VBDP–DPCM) image coding system operates on an image that has been preprocessed into segments of variable size, square blocks. A double predictor DPCM (DP–DPCM) system is then applied to encode each image sub-block separately. When transmitted over a noiseless channel, the distortion between the original signal and reconstructed signal is primarily due to quantization error provided that an optimal predictor is employed. Reconstructed signal degradation because of quantization error becomes severe at low bit rates due to the fact that large quantization errors are directly fed back into the predictor and to be used in subsequent estimation of future input signal [1]. The proposed VBDP–DPCM algorithm has been established to moderate the feedback loop quantization error effects in the vicinity at the expense of predictor optimality from a minimum mean-squared-error (MMSE) point of view. This VBDP–DPCM scheme divides a given image into variable size image sub-blocks and attempts to balance between sub-optimal predictor designs. The proposed scheme may expressively reduce feedback effects due to quantization errors locally with the objective of maximizing reconstructed image quality for each test image entirely.

When processing a given image frame, we initially apply the quadtree segmentation algorithm [2]–[4], [11]–[14] to divide a given test image into image sub-blocks having widely differing perceptual importance. The detail regions of a given image with more features activity will be segmented into blocks with smaller block size, and the background inactive regions of the image should be assigned larger block size for the areas. The block-sizes can vary from 128x128 to 8x8 in this study. The variable block-size image segmentation process is to better meet the non-stationary mathematical description of real-world images. Having comprehensive characterization of the underlying stochastic nature of a source signal generally results in a more efficient source encoder for that signal. After the quadtree image segmentation, the predicted differential values between the nearby pixels of an image block are reduced. Therefore, we can decrease the distribution range of prediction error and reduce the bit rate as well as quantization levels. We then
applied the DP–DPCM [6]–[8] scheme to each image sub-block. As in the case of conventional DPCM image coding system the emphasis of system design is focused on the prediction process which should reduce MMSE estimates of the input signals. Similarly, quantizer design is based on a match to the probability density function of a differential signal. The quadtree image segmentation method applied is to divide a given image according to the activity details within the image into variable size image sub-blocks. Each image sub-block will be more stationary than the original image frame and the predicted differential values between the nearby pixels of an image sub-block are condensed. The quantizer design employed in the encoder is locally optimal in the MMSE sense. Experimental results have been obtained by processing two test images named “LENA” and “WALT”. Objective measurement of system performance shows that the SNR obtained using the VBDP–DPCM scheme is about 5 dB or greater compared to that of conventional DPCM employing a predictor of the same order for low bit rate (R≤ 3) image compression.

In this paper, Section II describes the basic concepts regarding a conventional 2-D DPCM image encoder scheme as well as a double predictor DPCM encoder algorithm proposed earlier. Section III focuses on the proposed quadtree variable block-size image segmentation applied to a double predictor DPCM image compression applied in this study. The overall VBDP–DPCM image encoding system configuration and performance results are shown in Section IV. Finally, a summary and conclusions are presented in Section V.

II. RELATED WORK

In a conventional 2-D DPCM image encoder having a feedback loop around the quantizer, the prediction sequence values \( \hat{x}_i \) depend on the previously quantized error values \( \tilde{x}_i \) rather than on past unquantized input data values \( x_i \). This ensures that the transmitter and receiver components of the DPCM system can track and reconstruct the input signal waveform in a stable fashion. For a given \( N \times N \) image with pixel intensity values \( x_{ij}, j = 0, 1, \ldots, N–1 \), a typical 2-D causal image predictor having third-order support used to determine the estimation sequence is described by

\[
\hat{x}_i = b_i \tilde{x}_{i,j-1} + c_i \tilde{x}_{i-1,j} + d_i \tilde{x}_{i-1,j-1},
\]

which can be compactly express as

\[
\hat{x}_i = b_0 \tilde{x}_{i} + c_0 \tilde{x}_d + d_0 \tilde{x}_j ,
\]

where \( b_0, c_0, \) and \( d_0 \) are the predictor coefficients of the DPCM predictor and \( \tilde{x}_b, \tilde{x}_c, \) and \( \tilde{x}_d \) are the data values input to the predictor filter corresponding to locations \( b, c, \) and \( d \) in the image field, as shown in Figure 1.

Figure 1. Corresponding location of \( b, c, \) and \( d \) with the current pixel \( a \) in the image field.

Let \( \rho \) denote the one-lag covariance \( E(x_{i,j}x_{i,j})/E(x_{i,j}^2) \) and let \( \rho = \rho_{aa} = \rho_{bb} = \rho_{cc} = \rho \), \( \rho' = \rho_{aa} = \rho_{bc} = \rho_{dd} = \rho_{ab} = \rho_{bd} = \rho_{cd} = \rho' \), where \( J = 2 \) for an image model with separable auto-correlation function and \( J = \sqrt{2} \) for an isotropic image model [5]. The optimal third-order predictor is defined by the prediction coefficients:

\[
b_i = c_i = \frac{\rho(1-\rho')}{1+\rho^2-2\rho\rho'},
\]

\[
d_i = \rho' = \frac{2\rho^2(1-\rho')}{{1+\rho^2-2\rho\rho'}},
\]
Performance for the conventional DPCM systems is shown that the quantization in the encoder loop is not adequately fine if the encoding rate is 3 bits/pixel or less, the quantization error is not small enough so as to be ignored in the design of the optimal linear predictor. In order to balance between sub-optimal predictor designs and significantly reduce feedback effects due to quantization errors with the objective of maximizing reconstructed image quality, a double predictor DPCM algorithm has been proposed [6].

Figure 2 illustrates the block diagram of a DP–DPCM algorithm. The corresponding DPCM decoder consists of two cascaded filtering stages. The DP–DPCM scheme has an overall structure similar to that of a double differentiator-integrator system[9]. Filter coefficient values are chosen under constraints that guarantee system stability. There are two predictor filters, H and F in this scheme, where Q is a PDF-optimal quantizer of either a uniform or non-uniform structure. The output of the first subtractor, e(n), is input to a second subtractor whose other input is a local estimate of the differential signal itself. The output signal ŝ(n) is the difference sequence e(n)–ê(n). The quantizer output sequence α(n) is fed back into the adder along with the estimation ê(n) of e(n) producing ê(н)which is the input to the filter F and to the primary prediction loop. Conceptually, ê(н)should be statistically closer to the prediction error e(n) rather than the quantizer output signal α(n).

\[ x(n) \rightarrow e(n) \rightarrow ŝ(n) \rightarrow Q \rightarrow \alpha(n) \rightarrow Channel \]

(a) Encoding system

\[ Channel \rightarrow \delta(n) \rightarrow \tilde{e}(n) \rightarrow F \rightarrow ŝ(n) \rightarrow y(n) \]

(b) Decoding system

Figure 2. Block diagram ofDP–DPCM system.

The predictor coefficients can be written as

\[ b_j = e_j = \frac{\rho_e \left(1 - \rho_e' \right)}{1 + \rho_e' - 2 \rho_e}, \]

\[ d_j = \rho_e' \frac{2 \rho_e' \left(1 - \rho_e' \right)}{1 + \rho_e' - 2 \rho_e}. \]

We have assumed that the average value \( E[e(n)]=0 \), the one-lag covariance of \( e(n) \), \( \rho_e \) is then defined as

\[ \rho_e = \frac{E[(x_n - \tilde{x}_n)(x_{n-1} - \tilde{x}_{n-1})]}{\sigma_e^2} = \frac{E[e_ne_{n-1}]}{\sigma_e^2} \]

where \( \tilde{x}_n \) and \( \tilde{x}_n \) are defined as in eq. (2). \( \sigma_e^2 \) is the variance of the error sequence \( e(n) \) and \( E[e_ne_{n-1}] \) is the error correlation. For \( \sigma_e^2 \) is of the form as following [10]

\[ \sigma_e^2 = \sigma_x^2 [1 + 2 b_j^2 + d_j^2 + 4 b_j \rho_x \left(d_j - 1\right) - \rho_e' \left(2d_j - 2b_j^2\right)]. \]

we can obtain the covariance \( \rho_e \) to be of the following equation
In a variable block-size image compression system, we segment the input image into variable size blocks, for which a “detail” region comprising more image features activity is partitioned into a smaller block size, each sub-image block can be more sensibly described the appearances in the region, and a “background” region with less image variation assigned a larger block-size. We have espoused a top-down quadtree approach [2] to obtain the tractability of a variable block-size segmentation procedure while eluding the excessive overhead needed to characterize more sophisticated image segmentation techniques. To segment a 256x256 image by using the quadtree procedure, we begin with dividing a given 256x256 input image into four 128x128 equal size blocks. Each 128x128 image block is set to be a root(initial node), from which the algorithm passes through the tree from level to level. At each node, a test is executed to determine whether the image block represented by that node is a “background” region or a “detail” region. If the test agrees that the block is a “background” region, then the node becomes a leaf. If not, the segmentation proceeds to the next level and the four branches of the node are each examined consecutively. The final quadtree structure is represented by one bit of side information for each node, which indicates whether that node is a nonleaf or a leaf node. By adapting the predictor and quantizer structures for each variable size image block the statistical non-stationary inherent in real-world images is considerably revealed. Encoding efficiencies are realized by: (i) achieving better edge renovation in busy areas of an image; (ii) considerable rate reductions in areas of near-constant gray level which is characteristic of the larger block size regions within an image.

The segmentation test used in this study is implemented as follows: For the image block $x_{mn}^{k}$, we first estimate the mean value, $E[x^{k}]$, and the variance, $Var(k)$. Then, we compare the value of $Var(k)$ with $\theta E[x^{k}]$, where $\theta$ is a preset threshold. The quadtree segmentation threshold value $\theta$ was first picked randomly large and should be reduced to deal with the anticipated distortion until a tolerable reconstructed image quality achieved. If $Var(k) \geq \theta E[x^{k}]$ for the examined image block, then the block is marks as a “detail” region and the segmentation course proceeds to the next level. The test procedure of the quadtree algorithm is illustrated in Figure 3.

$$\rho_{e} = \frac{2(1-b_{e}) \rho_{e} - 2(c_{e} + d_{e}) \rho'_{e}}{1 + 2b_{e}^{2} + d_{e}^{2} + 4b_{e} \rho_{e}(d_{e} - 1) - \rho'_{e}(2d_{e} - 2b_{e}^{2})}$$

(9)

We now are able to calculate the optimal filter coefficients of predictor F by substituting $\rho_{e}$ and $\rho'_{e}$ into eq. (5) and eq. (6). The variance of the difference sequence appearing at the output of the second subtractor is given by

$$\sigma_{e}^{2} = \sigma_{e}^{2} \left[ 1 + 2b_{e}^{2} (1 + \rho'_{e}) + d_{e}^{2} + 4b_{e} \rho_{e} (d_{e} - 1) - 2d_{e} \rho_{e} \right],$$

(10)

where $\rho'_{e} = E[e_{e}d_{e}] / \sigma_{e}^{2}$. The variance of $\xi(n) = e(n) - \xi(n)$, denoted as $\sigma_{e}^{2}$, and the square root, $\sigma_{e}$ (standard deviation of $\xi(n)$) is the parameter adjusting the dynamic range of the locally optimal quantizer $Q$.

III. PROPOSED ALGORITHM

To improve image data compression performance further, we have proposed a combined system based on variable block-size quadtree image segmentation applied to double predictor differential pulse code modulation (DP–DPCM) image compressive algorithm.
Figure 4 presents the proposed variable block-size double predictor DPCM image coding system, which improves the performance of DPCM by adapting the predictor and quantizer structures for each variable size image block. For the VBDP–DPCM image coding, an $N \times N$ image with pixel intensity values $x_{i,j}, j = 0, 1, \ldots, N-1$, is segmented into blocks of size $M \times M_k$, where the block size can vary from 128x128 to 8x8. The pixel values of the $k^{th}$ sub-image block is denoted by $x_{mn}, m, n = 0, 1, \ldots, M_k-1$, and is expected to possess statistical stationarity over the spatial domain.

For each segmented image block, the first-order and second-order statistics are estimated over an entire sub-block of image data. Hence, the complete image frame retains globally statistical non-stationary over the spatial domain. The local mean is determined and subtracted from the $M \times M_k$ image block to obtain a zero-mean image block. To be simplified, quantizers with uniformly spaced output levels were designed on the basis of a Laplacian probability density function (PDF) describing the differential input sequences for each image block. A third-order predictor structure is used both for the prediction of $\hat{x}(n)$ of the image signal based on $x(n)$ and for the prediction of the difference signal $\hat{e}(n)$ based on $\tilde{e}(n)$. Follow the procedures delivered in [8], the design process is modified and outlined in the following steps.

Step 1: For a given image, we apply the quadtree segmentation algorithm to divide the image into variable size image blocks having widely differing perceptual importance. Estimations of the local mean value, variance, and covariance coefficient $\rho$ for each image block are provided.

Step 2: The local mean is deducted from the pixels’ value in each image block to obtain a zero-mean image block.

Step 3: Determine the optimal predictor coefficients $b_{x,opt}, c_{x,opt}, d_{x,opt}$ from eq. (3) and eq. (4). For the predictor coefficients values $(b_{x,opt}, c_{x,opt}, d_{x,opt})$, compute the corresponding variance $\sigma_i^2$ of the error sequence $e(n)$ and $E[e(e)]$, the error correlation for the second-stage filter $F$.

Step 4: With the values of variance $\sigma_i^2$ and $E[e(e)]$, we can calculate $\rho_1^2 = E[e(e)]/\sigma_i^2$ and $\rho_2$. Then substitute $\rho_1^2$ and $\rho_2$ into eq. (5) and eq. (6) to solve the optimal prediction coefficients $(b_x, c_x, d_x)$ for the second-stage filter $F$.

Step 5: The coefficients $(b_x, c_x, d_x)$ are substituted into eq. (10) to calculate the variance $\sigma_i^2$ and then the standard deviation, $\sigma_i$ of $\hat{e}(n)$. $\sigma_i$ is the parameter to adjust the dynamic range of the locally optimal quantizer $Q$.

Figure 4. Block diagram for variable block-size DP-DPCM system.
Step 6: For each zero-mean image block, we can determine the predictor coefficients based on the previous steps. The difference sequences after the second-stage predictor can be quantized with the step-size determined in step 5 and then output to channel.

IV. SIMULATION RESULTS

The source coding performance of the proposed variable block-size DP–DPCM system presented in this study is generally about 5 dB or greater than that of a conventionally design DPCM system when operating at low bit rates (bit rate \( R \leq 3 \) bits/pixel). Noteworthy improvement in performance can be denoted for values of coding rate \( R \) greater than 3. Simulation results obtained in this study are summarized in Tables I and II. The output signal-to-noise ratio (SNR) is defined as \( \text{SNR} = 10 \log (\sigma_i^2 / \sigma_e^2) \) in dB, where \( \sigma_i^2 \) is the input image variance and \( \sigma_e^2 \) is the reconstructed error variance. Shown in Table I and II are the image coding results for the VBDP–DPCM system for 256x256 “LENA” and “WALT” images, respectively. The source coding performance of the VBDP-DPCM system has higher SNR compared to a conventional stationary-based DPCM system in general. The reconstructed images using the scheme in Figure 4 are mostly of higher quality both subjectively and objectively. Values of SNR equal to 12.68 dB, 17.57 dB, and 22.71 dB are obtained for the 256x256 “LENA” image at the rates of 1, 2 and 3 bits/pixel, respectively, for threshold \( \theta = 5 \). Values of SNR equal to 16.23 dB, 20.62 dB, and 24.81 dB are achieved for the “WALT” image at the rates of 1, 2 and 3 bits/pixel, respectively, for threshold \( \theta = 5 \). These results are better than those found using a conventional, stationary DPCM encoder/decoder system.

Table I
Comparison of source coding results for conventional DPCM and VBDP–DPCM systems.“LENA” image.

<table>
<thead>
<tr>
<th>Quadtree Scheme</th>
<th>( R = 1 ) bit/pixel</th>
<th>( R = 2 ) bits/pixel</th>
<th>( R = 3 ) bits/pixel</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DPCM</td>
<td>DP-DPCM</td>
<td>DPCM</td>
</tr>
<tr>
<td>None</td>
<td>7.69</td>
<td>11.18</td>
<td>12.16</td>
</tr>
<tr>
<td>( \theta = 10.0 )</td>
<td>11.68</td>
<td>12.50</td>
<td>16.09</td>
</tr>
<tr>
<td>( \theta = 5.0 )</td>
<td>12.20</td>
<td>12.68</td>
<td>16.56</td>
</tr>
<tr>
<td>( \theta = 1.0 )</td>
<td>12.53</td>
<td>12.90</td>
<td>16.73</td>
</tr>
</tbody>
</table>

Table II
Comparison of source coding results for conventional DPCM and VBDP–DPCM systems.“WALT” image.

<table>
<thead>
<tr>
<th>Quadtree Scheme</th>
<th>( R = 1 ) bit/pixel</th>
<th>( R = 2 ) bits/pixel</th>
<th>( R = 3 ) bits/pixel</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DPCM</td>
<td>DP-DPCM</td>
<td>DPCM</td>
</tr>
<tr>
<td>None</td>
<td>8.09</td>
<td>13.39</td>
<td>12.58</td>
</tr>
<tr>
<td>( \theta = 10.0 )</td>
<td>13.38</td>
<td>15.79</td>
<td>17.91</td>
</tr>
<tr>
<td>( \theta = 5.0 )</td>
<td>14.87</td>
<td>16.23</td>
<td>18.94</td>
</tr>
<tr>
<td>( \theta = 1.0 )</td>
<td>15.03</td>
<td>16.58</td>
<td>19.27</td>
</tr>
</tbody>
</table>

Figure 5 compares the subjective quality of reconstructed “LENA” images obtained with the VBDP–DPCM system (\( \theta = 5 \)) to the quality obtained with a stationary, conventionally designed DPCM encoder at transmission bit rates \( R = 1 \) bit/pixel. The reconstructed image shown in Figure 5(d) faithfully reproduce gray-level variations in regions of high spatial activity, e.g., the feathers in the “LENA” image are sharper as compared to the blurring present in Figure 5(a), the result for the conventional DPCM encoder. Figure 6 and Figure 7 show the visual quality of the reconstructed/VBDP-DPCM “LENA” and “WALT” images at different coding rates of 1, 2, and 3 bits/pixel, respectively. The decoded images normally have slightly sharper image details, due to the AR predictions adapted to local statistics, as well as the filtering operation to moderate the venomenous effects of quantization errors being fed back into the prediction loop of the DPCM code as shown in Figure 2(a).
V. CONCLUSION AND FUTURE WORK

The new double predictor DPCM coding system with variable block-size investigated in this study results in improved source coding performance when compared to a conventionally designed DPCM image encoding scheme for bit rate $R \leq 3$ bits/pixel. Objective SNR results demonstrate that the new VBDP–DPCM encoding scheme yields better reconstructed image quality vis-a-vis conventional, stationary DPCM image encoding system. Performance improvements are the results summarized as:

- (a) Conventional DPCM
- (b) Double Predictor DPCM
- (c) Quad-Tree DPCM
- (d) VBDP-DPCM

Figure 5. Reconstructed image quality of “LENA” for different encoding schemes, $R = 1$ bit/pixel.

Figure 6. Reconstructed “LENA” image for different bit rates.

Figure 7. Reconstructed “WALT” image for different bit rates.
Image segmentation results in the statistical non-stationary inherent in real-world images being better exploited;

The DP–DPCM encoding scheme is applied to each segmented sub-image block. We are able to adapt the predictor and quantizer structures for each variable size image block;

The balance in design of sub-optimal difference signal predictors and a filtering operation to reduce the deleterious effects of large quantization errors being fed back into the prediction loop for small values of $\lambda$

The source coding performance of the variable block-size DP–DPCM algorithm is generally about 5 dB or greater than that of a conventionally designed DPCM coding system when operating a low bit-rates. The improved DPCM system provides better edge reproduction that is perceptually important to the viewer.

REFERENCES.


BIOGRAPHY

Jia-ChyiWu received the M.E.E. degree from the Catholic University of America at Washington, D. C., and the Ph.D. degree from Rutgers University, New Jersey in 1998. He currently is an assistant professor in the Department of Communications, Navigation and Control Engineering, National Taiwan Ocean University, Keelung, Taiwan. He is responsible for teaching and research in the areas of digital communications and information coding theory. His current research interests include digital communications, joint source-channel rate allocation, information and coding theory, and digital image transmission.