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Recurrence of Space-time Events

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ABSTRACT

A causal-directed graphical space-time model has been suggested in which the recurrence phenomena that happen in history and science can be naturally explained. In this Ramsey theorem inspired model, the regular and repeated patterns are interpreted as identical or semi-identical space-time causal chains. The same colored paths and sub graphs' in the classical Ramsey theorem are interpreted as identical or semi-identical causal chains. In the framework of the model, Poincare recurrence and the cosmological recurrence arise naturally. We use Ramsey theorem to prove that there's always a possibility of predictability whatever how chaotic the system.

INTRODUCTION

Historical Recurrence

History is a record of past events. In the literature, historical recurrence is an hypothetical concept refers to the repetition of similar events in history. This concept, sometimes expressed by the quote history repeats itself, has attracted the attention of many thinkers and authors through the history with the absence of any rigorous scientific base to explain it [1-5]. It might be best expressed in Mark Twain's words that no occurrence is sole and solitary, but is merely a repetition of a thing which has happened before, and perhaps often [6]. For some historic examples see [1] where Trompf traced historically recurring patterns of political thought and behavior in the west since antiquity. The recurrence idea also exists in science and has been discussed in different contexts. In dynamical systems, Poincare's recurrence theorem [7] states that in general, all systems will return not once but in nitely many times to a con guration very close to its initial one. The time elapsed until the recurrence is the Poincare recurrence time and depends on the exact initial state and required degree of closeness [8]. A quantum recurrence theorem which is a quantum analog of Paincare's recurrence theorem was proved in 1975 [9]. Birkho's recurrence theorem (1972) states: if X is a compact metric space and T is a continuous map of X into itself, there exists some point x_o 2 X and some sequence n_k ! 1 with T^{nk} x_o ! x_o. In geophysics, the study of the recurrence of the earthquake events is very important. It has been shown in [10] that the recurrence intervals between earthquake events strongly depend on history.

Distribution of the recurrence times and the mean residual time until the next earthquake strongly depend on the previous recurrence time. In cosmology, several recurrence models have been introduced in which the evolution of the universe is cyclic (Ekpyrotic and cyclic cosmology). The name Ekpyrotic represents the contractive phase of eternally-recurring destruction and recreation [11,12]. In [13], a cosmological model has been proposed in which the universe undergoes an endless sequence of cosmic epochs each beginning with a bang and ending in a crunch.

Ramsey theory: Ramsey theory is the study of unavoidable regularity in large structures. It is a foundational result in combinatory named after Frank Ramsey who did seminal work in this area before his death in 1930 [14]. The theory was developed extensively by Erdos [15]. The original theorem proved by Ramsey could be written as: In any colouring of the edges of a sufficiently large complete graph, one will find monochromatic complete sub graphs. So, no matter how chaotic we try to arrange certain objects, we will find ourselves creating a very highly organized and structured object within it. The quote "complete disorder is impossible" describing Ramsey theory is attributed to Theodore Motzkin [16]. The In nite version of Ramsey's theorem states that: If X is a countably in nite set and for any n 2 N the subsets of size n, X^n, are colored in in nitely many colors, then there is an in nite set M X where all subsets of M of size n are colored in the same color. For the graphs G_1; G_2 ; : : :; G_t, the graph Ramsey number r(G_1; G_2; :::; G_t) is the smallest integer R with the property that any complete graph of at least R vertices whose edges are partitioned into t color classes contains a monochromatic sub graph isomorphic to G_i in the i-th color for some i, 1 i t. These are classical Ramsey numbers or simply Ramsey numbers when all graphs are complete graphs which correspond to the original definition, later extended to any graph [17]. Ramsey theorem has been a useful tool in many areas of mathematics. There have been also some attempts to use graph theory in social sciences (see [18] as an example). Some attempts to use Ramsey theory in the study of history has been mentioned by Gasarch in [19]. The classical Ramsey Theorem has since been generalized in many ways; most of these generalizations are concerned with finding monochromatic substructures in various colored structures [20-22]. Monochromatic paths and cycles have been investigated by many authors, for example see [22,23] it's useful to quickly recall some basic definitions: The complete graph K_n of order n is a simple graph with n vertices in which an edge connects every pair of vertices. A clique in a graph is a set of pairwise adjacent vertices. Since any sub graph induced by a clique is a complete sub graph, the two terms and their notations are usually used interchangeably. A path from vertex V_0 to vertex V_k is a sequence V_0; E_0; V_1; E_1; V_2; E_2; :::; $V_{k\ 1}$; $E_{k\ 1}$; V_k of vertices V_k and edges $E_{k\ 1}$. A cycle in a graph is a path from some vertex V_0 back to V_0 (a closed path) where no edge appears more than once in the path sequence. A directed graph is one in which the direction of any given edge is defined. Conversely, in an undirected graph we can move in both directions between vertices. Two graphs are isomorphic when the vertices of one can be re labeled to match the vertices of the other in a way that preserves adjacency.

SPACE-TIME CAUSAL CHAINS

Starting with the fact that every space-time event can be represented as a point in the four dimensional space-time manifold. We consider the space-time macroscopic' events as a random distribution of countably in nite space-time points. In this system causality cannot be violated and hence the arrow of time must be respected. The points are connected to each other's by edges represent the causal relations and we are allowed to move from one point to another only in the future direction, i.e. edges are future directed time-like vectors. So we actually have a partially connected network or a graph with sub graphs and paths. Since some events are related to each other's and some are not, it cannot be a complete graph or fully connected network. In this space-time event graph, No self-loops allowed. Loops are edges connected at both ends to the same event which implies the existence of closed time-like loops in the space-time manifold allowing traveling to the past [24]. The kinds of graphs which guarantee the fulfillment of the arrow of time condition with no loops are the Directed acyclic graph (DAG). That kind of graphs sometimes referred to as causal graphs or path diagrams. Causal-directed graphical models were developed in the philosophy of science and statistical literature, they are also used in epidemiology, genetics and related disciplines (see [25] and references therein). Denoting edges by E and vertices by V we can set the following definition.

Definition: space-time directed acyclic graph G is a pair (V; E) where V is a set of events and E is a set of causal relations E f (V_1; V_2) j V_1; V_2 2 V g.

Where an ordered pair of two events (V_1; V_2) defines the space-time event arc' directed from V_1 to V_2. Every directed path defines a space-time causal chain. A space-time causal chain is defined as an ordered sequence of events in which any one event in the chain causes the next.

Ramsey numbers for directed graphs were introduced by Erdos and Moser [26,27], The directed analogue of the two possible arc colour is the two directions of the arcs and the analogue of monochromatic' is that all arc-arrows point the same way i.e. acyclic. Applying Ramsey theorem to this countably in nite chaotic distribution of space-time events implies the existence of regular and repeated patterns in the form of identical and semi-identical paths and sub graphs, i.e. identical space-time causal chains **(Figure 1).** Having a countable in nite number of edges connecting randomly distributed countably in nite number of points makes the existence of identical paths and cliques guaranteed. Identical directed paths correspond to identical space-time causal chains. We could state the following.

Conjecture: For a countable in nite number of space-time events, there is a countably in nite number of identical or semi-identical causal chains.

The identical and semi-identical causal chains are explained by the striking similarities among events and sequences of events at different space-time locations. This is just what is called historic recurrence which is clearly observable in the study of human history. According to that, the repetition of similar events in history is unavoidable. Similar analysis is applicable for dynamical systems as we are going to see in the next sections.

PREDICTABILITY AND EVOLVING SYSTEMS

How possible is it to predict a future causal chain that is identical to a past causal chain before it happens? The predictability of macroscopic space-time events is very different from the predictability of point particles in dynamical systems where we easily predict the particle's future motion. Suppose we have Two causal chains A_1 ! A_2 ! A_3 ! A_4 and B_1 ! B_2 ! B_3 ! B_4. The predictability strength of a certain future event depends on the degree of similarity between the previous events in the two chains. Here, the more similar the two events A_3 and B_3 are, the more similar the two events A_4 and B_4 will be. Exceptions happen due to the nature of human beings as living creatures with a will. However, there is always a possibility to predict the future events and this is what we are going to prove in this section making use of Ramsey theorem.

For a given space-time location, different events happen at different times. So there is a flow of events going through each space-time location. We might then consider a general case of a moving disorder; chaos. We need to investigate the degree of predictability for a system of an in nite number of points moving chaotically. We start by setting the following definition relating the degree of predictability of any dynamical system to its degree of disorder.

Definition: A dynamical system is said to be completely unpredictable if and only if it is in a complete disorder.

Theorem complete unpredictability is impossible.

Proof: Proposition the system is completely unpredictable. For this system, at any fixed time t, we get a static disordered distribution of points for which Ramsey theorem is valid, so this static snapshot of the system is not in a complete disorder. This is valid for any snapshot taken at any time t_i during the evolution of the system. Denoting the system's degree of disorder by S and the possibility of predictability' by P (S), we have: 8 t_i 9 P (S). But from the above Definition, this contradicts with the proposition that the system is completely unpredictable. So the proposition is false and the complete unpredictability of this chaotic system is impossible. The proof is valid for space-time future events and for dynamical systems too **(Figure 1).**

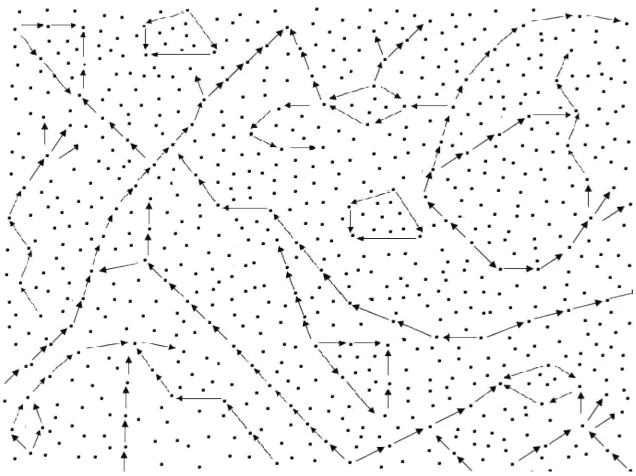

Figure 1. Macroscopic causal chains with the vertices represent the events and the edges are future directed time-like vectors. For example, three arrows directed toward a point means three reasons caused that event and so on. For a countably in nite number of events, the existence of identical causal chains is guaranteed. Identical directed paths and sub graphs are interpreted as identical causal chains at different space-time locations. Historic recurrence hypothesis and some recurrence theorems such as Poincare and cosmological recurrence have a natural interpretation here.

OTHER RECURRENCE THEOREMS REVISITED

Poincare's recurrence and the cosmological recurrence both can and a natural interpretation in the framework of the causal directed graphical space-time model. In Poincare's recurrence theorem, systems will return not once but in nitely many times to a conjuration very close to the initial one. This simply will happen as the system passes through a causal chain that is identical or semi-identical to its initial one. Since there is a countably in nite number of identical or semi-identical causal chains, the system will return in nitely many times to a con guration very close to its initial one. The same analysis applies for the cosmological recurrence. The recurrence will happens when the universe passes through causal chain identical to its initial one.

CONCLUSION

A directed acyclic space-time graph model has been suggested in which a natural explanation of the long standing hypothesis of historic recurrence could be found. Applying Ramsey theorem on a accountably in nite, or sufficiently large, number of space-time events emphasizes the existence of regular patterns in the causal structure of space-time events. The suggested analysis shows that historic recurrence might not be just a hypothetical concept but a necessity associated with any sufficiently large number of space-time events. Other recurrence theorems can also be naturally explained such as Poincare recurrence and cosmological reoccurrence. There is always a possibility of predictability depends on the system's degree of disorder.

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