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Relativity, Energy & Complex Power by Using the Charge

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ABSTRACT

In this research, we investigate energy of particles, by using the charge Q, rather than using the mass M of particles. That is, since the mass M of electrons can decay faster than the charge Q of the same electrons. So in order to be as prompt as we can in technology, we have to be able to speak of relativistic power (electric, electronic, chemical, and nuclear) in the quantum theory. It then becomes evident that one has to make sure to fulfill the requirements of the 1st law of thermodynamics and the laws of conservation of energy, in space, in any chemical process, electric process. That is done and presented in this research by analyzing and further investigating with sufficient analysis the relationship between energy and charge in the 4-dimensional space-time. The Albert Einstein relativistic forces and mechanics have been considered here in. So, is the case for using the Relativistic Maxwell's equations, the Lorentz Transformation and further the rules and regulations of tensor analysis and differential geometry in the Continuum 4-D space-time. Moreover, I have used the same coordinate systems and frames or similar frames which were used by A. Einstein. The invariance of the charge has been employed to show the proof of the validity of this analysis for any value of the charge Q.

INTRODUCTION

In this analysis we derive the energy-charge equation by using tensor analysis and Albert Einstein Theory of Relativity, which characterizes the relationship between the ordinary energy ε of any particle and the ordinary charge Q of the same particle. This equation is also called the Moaqat Energy-Charge Equation which is $\varepsilon = \frac{QVc^2}{[1-(v^2/c^2)]^{1/2}}$

V is the magnitude ordinary velocity of that same particle under consideration, in Space-Time, and c the speed of light.

ε=8.85×10-12 Farad/meter=permittivity of free space in a vacuum, π=3.14 (approximately)

μ=1.260 nano H/meter=permeability of free space in a vacuum

c=3.0×108 meter/second=speed of the light in a vacuum

$$c^2 = 1/\mu\varepsilon$$

Axiom: In accords with the postulates of the Theory of Relativity and the Albert Einstein Relativistic Equations System in relativistic mechanics, it is known by the famous Newtonian Relativistic 1st Law of motion that one can arrive to the axiomatic statement that an object at rest stays at rest with a constant relativistic velocity unless active upon by a relativistic force. Based on the fact that, we have obtained the Moaqat energy-charge equation,

$$\varepsilon = \frac{QVc^2}{[1-(v^2/c^2)]^{1/2}}$$

It is seen that we can employ relativistic velocity U, of any particle where, $U = v/[1-v^2/c^2]^{1/2}$

V is the ordinary magnitude velocity of the same considered particle. That is, whence we can transfer electric, magnetic, heat, mechanical power in terms of the charge Q of the said particle. Again, we must note, here, in our conclusion, that , the charge Q resulting from the velocity U, of any particle under consideration is the charge resulting from applying any of the electric or magnetic fields on the same particle. Thus, in a similar discussion, it is obvious that we can use the charge Q of any considered particle in order to transform energy ε, from any prospective possible form to another possible form where we must have by tensor rules, the Maxwell's equations, differential geometry, in quantum mechanics that, the energy transferred ε or transformed is expressed in terms of that particle's charge Q by the Moaqat-energy charge equation which is: $\varepsilon = QUc^2$

That is concluded and explained in brief as follows by the use of Maxwell's equations and Albert Einstein's principle of Relativity. In studying the electrodynamics behaviors of any particle upon introducing the 4 potential Ω_α in space, we can form the 4 potential vector Ω_α such that

$$\Omega_\alpha = (A_i, \Omega_4) = (A_i, i\phi)$$

It is seen that the associated 4 electric vector E, and the 4 charge vector Qis regarded as the fields which affect any particle in motion, in space time, having the 4-relativistic velocity vector U, where V is the ordinary velocity and U is the relativistic velocity, of the same considered particle, while, in motion, in the 4-dimensional continuum space-time.

SOME PRE-RELATIVITY PHYSICS IN THE CONTINUUM 4-D SPACE-TIME

According to Albert Einstein in his "The Meaning of Relativity", A. Einstein, published by Princeton University Press, in 1922, we recall the following concepts and principles, which are considered by vector and tensor analysis and the theory of relativity in the continuum 4-D space-time. Now, by A. Einstein's principles in Relativity, if an event takes place anywhere, in space, we can assign to it three co-ordinates, x_i, i=1,2,3, and a time t, in a 4-coordinate system, as soon as we have specified the time of the clock at the origin O which is simultaneous with the event. Therefore, we give an objective significance to the statement of the simultaneity of distant events. It may be seen, that, the time so specified is at all events independent of the position of the system of coordinates in our space of reference, and is therefore an invariant with respect to the Lorentz group transformation [7].

Besides reciting the statement of the simultaneity of distant events and being concerned with the simultaneity of two experiences of an individual, it seen that we can use the covariance of the Lorentz Transformation, we can express with a sufficient high degree of efficiency in translation of coordinates the position of any particle in another system S', while taking, isotropy of the medium into account and homogeneity in cartesian coordinate systems by the geometrical line equation

$$x'_\alpha = y_\alpha = a_\alpha + LOR_{\alpha\beta} x_\beta$$

For simplicity by considering above that a_α to be zero, i.e. the origin 0 of the coordinate system S, we arrive at obtaining the Lorentz Transformation equations; characterized by the equations,

$$x'_\alpha = y_\alpha LOR_{\alpha\beta} x_\beta;$$

Similar arguments apply if using 3 components (2 real components, and a 3rd which may be selected as imaginary) in space-time [3,12,14].

THE GALILEAN TRANSFORMATION EQUATIONS

Consider two inertial frames S and S' That is, where frame S' moves with a constant 3 velocity, v along the common *x* and *x'* axes, respectively, where v is measured as relative to frame S. We assume the origins of S and S′ coincide at *t* = 0 and an event occurs at point *P* in space at some instant of time. An observer in frame S describes the event with space–time coordinates (*x*, *y*, *z*, *t*), whereas if we have an observer in frame S', then such an observer uses the coordinates (*x*', *y'*, *z*', *t*') to describe the same event. Now, from the geometry and by using the Galilean Transformation equations that the relationship among these various coordinates can be written

$$x' = x - vt,\ y' = y,\ z' = z,\ t' = t$$

The equations above are the Galilean space-time transformation equations. It must be noted, in the said Galilean equations, that time is assumed to be the same in both inertial frames [6].

So that by using Galilean Transformation Equations that means, within the framework of classical mechanics, all clocks run at the same rate, regardless of their velocity, so the time at which an event occurs for an observer in S is the same as the time for the same event in S'. By using the said Galilean Equations the time interval between two successive events should be the same for both observers. Although one sees that, this assumption may seem to be obvious, yet, it turns out to be incorrect in situations where the velocity v is comparable to the speed of light.

THE LORENTZ TRANSFORMATION EQUATIONS

Suppose that, two events occur at the points P and Q in space and are reported by two observers, one at rest in a frame S and another in a frame S' that is moving to the right with speed v. The observer in frame S reports the events with space-time coordinates (x, y, z, i, c, t), while the observer in frame S' reports the same events using the coordinates (x', y', z', i, c, t'). Then from above equation this predicts that the distance between the two points in space at which the events occur does not depend on motion of the observer: $\Delta x = \Delta x'$.

Due to the fact that this prediction is contradictory to the notion of length contraction, then it is found that the Galilean transformation is not valid when v approaches the speed of light. In the following, we present the correct transformation equations that apply for all speeds in the range 0<v<c. These equations which are valid for all speeds and that enable us to transform coordinates from frame S to frame S' are called the Lorentz transformation equations:

$$x' = \gamma[x - Vt],$$
$$y' = y,$$
$$z' = z,$$
$$t' = \gamma[t - (vx / c^2)]$$

Further it must be noted that these transformation equations were developed by Hendrik A. Lorentz (1853-1928) in 1890 in connection with electromagnetism. However, it was for Albert Einstein who recognized the physical significance of these equations. That is where he took the bold step of interpreting them within the framework of the special theory of relativity. One important aspect to notice here is the difference between the Galilean and Lorentz time equations. In the Galilean case, the time t=t'. However, in the Lorentz case, the value for t' assigned to an event by an observer O' in frame S' depends both on the time t and on the coordinate x as measured by an observer O in the S frame, which is consistent with the notion that an event is characterized by four space-time coordinates (x, y, z, i c t). So, in other words, in relativity, space and time are not separate concepts. However, they rather are closely interrelated with each other. If one wishes to transform coordinates in the S' frame to coordinates in the S frame, then one simply replaces v by -v, and then interchange the primed and unprimed coordinates in the Lorentz transformation equations

$$x = \gamma[x' - Vt'],$$
$$y = y',$$
$$z = z',$$
$$t = \gamma[t' + (vx' / c^2)]$$

When v<<c, the Lorentz transformation equations should reduce to the Galilean equations [6]. However, as v approaches zero, v/c<<1, therefore γ>1 and the Lorentz transformation equations, stated above, indeed reduce to the Galilean space-time transformation equations in the Galilean transformation equations, stated earlier, above. In many situations, we would like to know the difference in coordinates between two events or the time interval between two events as seen by observers O, and O'. From the Lorentz equations stated above we can express the differences between the four variables x, x', t, and t' in the following form

$$\Delta x' = \gamma[\Delta x - v\Delta t],$$
$$\Delta y' = \Delta y,$$
$$\Delta z' = \Delta z,$$

For transforming from frame S to S'

$$\Delta t' = \gamma[\Delta t - (v\Delta x / c^2)]$$

(6), p 1130 That is, while we use the equations

$$\Delta x = \gamma[\Delta x' - v\Delta t'],$$
$$\Delta y = \Delta y',$$
$$\Delta z = \Delta z',$$
$$\Delta t = \gamma[\Delta t' + (v\Delta x' / c^2)]$$

For transforming from frame S' to S. Further discussions are presented about the Lorentz transformation equations stated above. Meanwhile, we present, here below, rotation of the coordinates from one frame to another, for additional explanation about the said equations, in the continuum 4-Dimensional space- time.

OBTAINING TENSORS IN THE THEORY OF RELATIVITY

There are many ways to obtain tensors in space-time by using the theory of relativity. We shall discuss few. Readers are advised to refer to textbooks in Relativity about more ways for derivation of tensors. By covariance According to Albert Einstein and Tensor rules of transformations, it must state that:

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If a figure is defined with respect to every system of Cartesian co-ordinates in a space of reference of n dimensions by the n^r numbers

$A_{\mu\nu\_\lambda\ldots}$ (r= number of indices), then these numbers are the components of a tensor of rank r if the transformation law is

$$A'_{\mu'\nu'\_\lambda'\ldots} = b_{\mu'}$$
$$_{\mu}b_{\nu'},\ _{\nu}b_{\lambda'\lambda}\ldots.A_{\mu\nu\lambda}\cdots$$

It follows, by means of the known law of transformation for the arbitrary components εμ for linear orthogonal transformations, we can indeed easily find the law of transformation for the aμν, as follows such that

$$a'_{\sigma\tau} = b_{\mu\sigma}b_{\tau\nu}a_{\mu\nu}$$

Now by the rules of tensors and also those of the theory of relativity, we can present the notion of multiplication of tensors. We may obtain a tensor of rank (α+β) from a tensor of rank α and a tensor of rank β, by multiplying all the components of the first tensor by all the components of the second tensor as follows, for example:

$$T_{\mu\nu\rho}\cdots_{\alpha\beta} = A_{\mu\nu\rho}\ldots.B_{\alpha\beta\lambda}\ldots.$$

Tensors can also be obtained by differentiation [7].

ROTATION OF COORDINATES AND THE LORENTZ EQUATIONS OF TRANSFORMATION

Continuing with our discussion, we have to take into consideration the phenomena of rotation of the coordinates from any system S to any other system S'. We present as done earlier by Albert Einstein the apparatus shown below for the rotation of the coordinates from any arbitrary coordinate system S, to any other coordinate system S', in the 4-Dimenional space M^4 (also called Makowski's Space–Time, or just Space-Time, in many occasions that is by the general concept of relativity). So it is seen that in the process of rotation of the coordinates from a system S to a system S', the Lorentz matrix of transformation may be used as shown in **Figure 1**.

By employing the magnitude v of the 3 velocity of any particle moving relative to any frame S, we can express the general Lorentz transformation equations associated with such a particle. Considering a rotation of coordinates, by an angle ψ, from the system S to the system S', we must obtain by the Albert Einstein theory of relativity, the equations

$$x' = x\cos\psi + ict\sin\psi$$

$$ict' = -ix\sin\psi + ct\cos\psi$$

(7), p.35 and (18), p.22. Alternatively, we may write the above 2 equations, as follows

$$x' = x\cos\psi + il\sin\psi$$

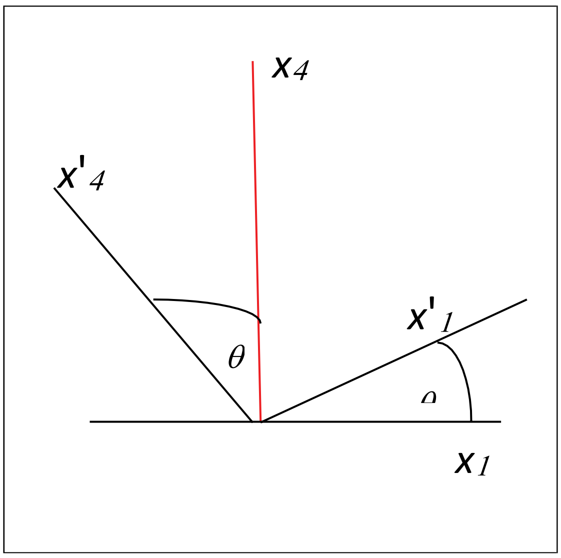

Figure 1. Rotation of Coordinates.

$$il' = -ix\sin\psi + l\cos\psi$$

$$i = \sqrt{-1}$$

Where, t is the ordinary real time of any such considered particle,

l is the ordinary real light time of the same particle.

$$\cos\psi = 1/[1-V^2]^{1/2},$$

$$\sin\psi = -iV/[1-V^2]^{1/2}$$

$$V = -i\tan\psi$$

$$l = ct$$

Where, above the angle ψ of rotation taken to be as purely imaginary that is, by using the magnitude v of the 3 velocity of any particle, then, the general arbitrary Lorentz transformation group equations may be expresses as follows, taking the following form.

$$x'_\alpha = \begin{cases} x' = \dfrac{x}{[1-\beta^2]^{1/2}} - \dfrac{Vt}{[1-\beta^2]^{1/2}} \\ y' = y \\ z' = z \\ it' = \dfrac{it}{[1-\beta^2]^{1/2}} - \dfrac{iVx/c^2}{[1-\beta^2]^{1/2}} \end{cases}$$

Where, by busing the magnitude velocity v of any considered particle, we have $\beta = V/c$

The above is further explained in below. If we select any coordinate system S of the 4 coordinates x_α, we wish to consider the rotation of the original coordinates to the new coordinates x'_α in another system (frame) S'. By introducing new coordinates upon the rotation of the coordinates, by an angle θ, from the system S to the system S', we must obtain by the Albert Einstein theory of relativity, the equations

$$x' = x\cos\theta + ict\sin\theta$$

$$ict' = -ix\sin\theta + l\cos\theta$$

Where, the angle θ of rotation from the frame S to frame S' is taken to be as purely imaginary. Thus, upon the rotation of the coordinates by an angle of θ, from the system S to the system S', we must arrive at the Lorentz transformation equations, which are characterized by the following set of equations, and the transformed 1st rank covariant infinitesimal positional 4- vector x'_α (with lower indices), in the system S', by using vector indices α=1, 2, 3, 4, such that:

$$x'_\alpha = \begin{cases} x'_1 = \gamma[x_1 - Vt_1] \\ x'_4 = \gamma[x_4 - (\mathrm{V}/\mathrm{c}^2)\,\mathrm{t}_4] \\ x'_2 = x_2 \\ x'_3 = x_3 \end{cases}$$

Where,

$$\sin\theta = -iV/c[1-V^2/c^2]^{1/2}$$

$$V/c = -i\tan\theta$$

$$x_1 = x, \quad x_4/c = it,$$

$$x_1 = x, \quad x_4/c = it,$$

$$x'_1 = x', \quad x'_4/c = it',$$

$$\mathrm{t}_1 = \mathrm{t}, \quad t_4 = ix,$$

$$\mathrm{t}'_1 = \mathrm{t}', \quad t'_4 = ix',$$

We have the angle θ of rotation being as purely imaginary. So, in frame S', we obtain the small differential changes 4- vector

$\Delta x'\alpha$ are expressed as follows relative to frame S where,

$$\Delta x'_{\alpha} = \begin{cases} \Delta x' = \gamma[\Delta x - V\Delta t \\ \Delta y' = \Delta y \\ \Delta z' = \Delta z \end{cases}$$

$$i\Delta t' = \gamma[i\Delta t - i(V/c^2)\Delta x$$

THE POSITIONAL 4-DIFFERENTIAL ELEMENTS dx_α IN SPACE-TIME

As it was discussed in 1900s' by Albert Einstein it is seen that anyone can form the 4-differentials, dx_α in any frame S that has the origin O with the coordinates.

$$x_1 = x,\ \ x_2 = y,\ \ x_3 = z,\ \ x_4 = T = il = ict,\ \ i = \sqrt{-1}$$

Where, t is the proper (ordinary) real time of any particle and l is the real light time of the particle. T=il is the imaginary light time of the same particle. The relativistic real time is denoted by t, where, relativistic differential element dt of any particle under consideration is related to the proper real time differential dt, by using the coefficient γ as follows:

$$dt = \gamma dt = [1-(V^2/c^2)]^{-1/2}dt$$

$$dt = [1-\beta^2]^{-1/2}dt$$

$$\beta = V/c$$

So, it is obvious in a similar way to the above, by using the proper (ordinary) imaginary time differential dT that we arrive at obtaining the relativistic imaginary time dT by using the coefficient such that

$$dT = [1-\beta^2]^{-1/2}dT$$ [3,5,16]

That is, where, the differential elements dx_α are expressed, as follows,

$$\begin{aligned} dx_\alpha &= (dx_1, dx_2, dx_3, dx_4) \\ &= (dx, dy, dz, dT) \\ &= (dx, dy, dz, d[ict]) \\ &= (dx, dy, dz, d[il]) \end{aligned}$$

By co-variance, we have above, in any reference frame S, that

$$dx_1 = dx,\ dx_2 = dy,\ \ dx_3 = dz,\ dx_4 = dT = icdt = d[il]$$

Further where we have above relative to any frame S, that

$$dx_1 = dx,\ \ dx_4/c = idt,$$

$$dt_1 = dt,\ \ dt_4 = idx$$

Consequently in a similar fashion, while relative to any frame S', we have that,

$$x'_1 = dx',\ \ dx'_4/c = idt',\ \ dt'_1 = dt',\ \ dt'_4 = idx'$$

Consequently, the 1st rank co-variant infinitesimal differential element position 4–vector dx'_α, having lower indices as subscripts, in the coordinate system S', in terms of the coordinates dx_α of the system S, such that by the Lorentz transformation matrix, we obtain

$$dx'_{\alpha} = \begin{cases} dx' = \gamma[dx - Vdt] \\ dy' = dy \\ dz' = dz \end{cases}$$

$$idt' = \gamma[idt - i(V/c^2)dx]$$

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Then in positional vector form, using indices, we get the 4- differential elements stated above by using lower indices and the Lorentz transformation matrix, we get

$$dx'_\alpha = \begin{cases} dx_1' = \gamma[dx_1 - Vdt_1] \\ dx_2' = dx_2 \\ dx_3' = dx_3 \end{cases}$$

$$dx_4' = \gamma[dx_4 - (V / c^2)dt_4]$$

Replacing v by -v in the previous positional 4-vector x'_α, of the system S', upon the rotationof the coordinates from the system S' to the system S, then by the Albert Einstein theory of relativity as well, we then, obtain the 1st rank covariant infinitesimal positional coordinates x_α, with lower indices in the system S, in terms of the coordinates x'_α, such that

$$x_\alpha = \begin{cases} x_1 = \gamma[x'_1 + Vt'_1] \\ x_4 = \gamma[x'_4 + (V / c^2)t'_4] \\ x_2 = x'_2 \\ x_3 = x'_3 \end{cases}$$

THE LORENTZ INVERSE EQUATIONS OF TRANSFORMATIONS

We wish to consider the inverse of the Lorentz equations of transformations stated above. That may be done by replacing the velocity v by -v, and changing the signs of orientation. Thus, we arrive at obtaining the said inverse equations of transformations, as in the following. These are

$$x_\alpha = \begin{cases} x = \dfrac{x'}{[1-\beta^2]^{1/2}} + \dfrac{Vt'}{[1-\beta^2]^{1/2}} \\ y = y' \\ z = z' \end{cases}$$

$$it = \frac{it'}{[1-\beta^2]^{1/2}} - \frac{ivx'/c^2}{[1-\beta^2]^{1/2}}$$

THE INFINITESIMAL DISPLACEMENT LINE ELEMENT VECTOR dL_α

In the theory of relativity, that of electromagnetism, electricity, thermodynamic, etc it is needed that we find a differential line element vector, so that we can operate with the said theories, transform coordinates from one set to another in space. Considering the formation of small line element changes ΔL_α, at any event, in the system S, then it is seen that the small changes differential line element vector ΔL_α, in 4-Space, and specifically, in space-time, is such that

$$\Delta L_\alpha = (\Delta L_1, \Delta L_2, \Delta L_3, \Delta L_4)$$

Consequently, we obtain the infinitesimal line element 4-vector dL_α, for the flow of charges, objects, etc in the system such that,

$$dL_\alpha = (dL_1, dL_2, dL_3, dL_4)$$

α=1,2,3,4

Then by applying transformations of the 4-line element vector dL_α from the system S, to another system S', at any corresponding event, we can form

$$\Delta L'_\alpha = (\Delta L'_1, \Delta L'_2, \Delta L'_3, \Delta L'_4)$$

As a consequence, the transformed infinitesimal line element 4-vector dL_α, for the flow of charges, objects upon transformation from one state in one coordinate system to another state in another, such that

$$dL'_{\alpha} = (dL'_1, dL'_2, dL'_3, dL'_4)$$

Thus, we can use the small displacement changes ΔL_{α} and $\Delta L'_{\alpha,}$ stated above for the purpose of applying limits of integration and the applications of 4- line integrals Green's and Stokes' Theorem, etc in the 4-Dimensional continuum space-time. Similar to the above, it may be seen that we can form the infinitesimal arbitrary line 4-vector L'_{α} as functions of the coordinates x', y', z' and T', in the coordinate system S' by using 1st rank covariant components, with lower indices as subscripts, in the coordinate system S', where

$$L'_1 = L'_1(x', y', z', T')$$
$$L'_2 = L'_2(x', y', z', T')$$
$$L'_{\alpha} = L'_3 = L'_3(x', y', z', T')$$
$$L'_4 = L'_4(x', y', z', T')$$

We note above, that in the system S' we have t' as the ordinary real time, l' as such that: $l' = ict'$

and T' as the imaginary light time $T' = il' = ict'$ [14].

So that by taking the differential element dL'_{α}, we have that

$$dL'_{\alpha} = \begin{cases} dL'_1 = dL'_1(x', y', z', T') \\ dL'_2 = dL'_2(x', y', z', T') \\ dL'_3 = dL'_3(x', y', z', T') \\ dL'_4 = dL'_4(x', y', z', T') \end{cases}$$

ADDITION THEOREM FOR VELOCITIES

According to Albert Einstein, if we consider combining two special Lorentz transformations with the relative velocities v_1 and v_2, it follows, that, the resulting velocity of the single Lorentz transformation which takes the place of the two separate ones is given by the equation

$$V_{12} = i\tan(\psi_1 + \psi_2)$$

$$= i\frac{\tan\psi_1 + \tan\psi_2}{1 - \tan\psi 1 \tan\psi_2}$$

$$= i\frac{V_1 + V_2}{1 + V_1 V_2}$$

Then for the simple case of parallel velocities, as for example, the speeds U in any inertial frame S, then U' in any inertial frame S', and v the Albert Einstein addition law of velocities then takes the form,

$$U = [U' + V]/[1 + (U'V / C^2)]$$

The inverse of the above addition law may be obtained by interchanging the primed velocities by unprimed and vice versa then changing the velocity v sign [12].

SPACE-TIME AND USING THE A. EINSTEIN MAXWELL'S 4-D EQUATIONS IN RELATIVITY

In the following we study in general, the 4-vector properties of the 4-D electrodynamics of any particle under the effects of the ordinary electric and the ordinary magnetic field H. If we apply a 0-rank magnetic potential 4- vector A, then this will generate the magnetic flux density 4-vector B such that:

$$\mathrm{B} = curlA = \nabla xA$$
$$= curl_{\alpha} Ae^{\alpha}$$
$$= {}_{\varepsilon\alpha\beta\lambda}\partial_{\beta} A_{\lambda} e^{\alpha}$$

Where, above is the 4-permutation tensor, in space-time and,

$$curl_{\alpha} A = {}_{\varepsilon\alpha\beta\lambda}\partial_{\beta} A_{\lambda}$$

for any arbitrary 4-vector A.

αβγ =1,2,3,4. Then, if we introduce the scalar potential function ϕ such that

$$\begin{aligned} \phi &= \phi(x, y, z, T) \\ &= \phi(x, y, z, i\ c\ t) \\ &= \phi(x, y, z, i\ l) \end{aligned}$$

It follows by using 4 gradient vector operator, in the above then we can generate the 0-rank electric field strength 4-vector E, such that

$$\mathrm{E} = -\nabla\phi - {}_{\mathrm{i}}\partial A / \partial(i\ c\ t)$$
$$i = \sqrt{-1}$$

The previous equation of the electric field strength 4-vector (also called the electric field 4-vector) can be written as follows

$$\mathrm{E} = -\nabla\phi - \partial A / {}_{\mathrm{c}}\partial_{t}$$

Considering the covariance of the 4-electric field in any reference frame S, the above equation may be expressed as follows, in the continuum 4-Dimension space-time, $E_{\alpha} = -\partial_{\alpha}\phi - \partial A_{\alpha} / {}_{\mathrm{c}}\partial_{t}$

Where, again in the above equation the imaginary light time of any particle is

$$T = i\ c\ t = i\ l$$
$$\tau = i\ t,$$

relative to the basis, $e^{\alpha} = (i, j, k, -\tau)$

$$= (e^{1}, e^{2}, e^{3}, e^{4})$$

It must be noted above that t is the ordinary real proper time of the particle under consideration, and T is the ordinary imaginary proper time of the same particle.

It is important to mention that by permutation of the indices, the 1st rank covariant components curl $_{\alpha}$A of the magnetic potential vector A will generate the magnetic field strength 4-vector H also called the magnetic field 4-vector, in space-time. The curl of the vector A is expressed by using 2×2 determinants as stated in below where:

$$\mathrm{B} = \mathrm{curlA} = \nabla\ \mathrm{xA} = \mathrm{curl}_{\alpha}\ \mathrm{Ae}^{\alpha}$$
$$curl_{\alpha} A = {}_{\varepsilon\alpha\beta\lambda}\partial_{\beta} A_{\lambda}$$

We have for the magnetic field B in 0-rank vector form by using covariant indices that

B=Curl A

However, using alphabetical indices, we can express the magnetic flux density 4- vector B as follows, such that

$$\mathrm{B} = \mathrm{curlA} = \left\{ \begin{vmatrix} \partial z & \partial\tau \\ Az & A\tau \end{vmatrix} i + \begin{vmatrix} \partial\tau & \partial x \\ A\tau & Ax \end{vmatrix} j + \begin{vmatrix} \partial x & \partial y \\ Ax & Ay \end{vmatrix} k - \begin{vmatrix} \partial y & \partial z \\ Ay & Az \end{vmatrix} \tau \right\}$$

relative to the contra-variant basis

$$\begin{aligned} e^{\alpha} &= (i, j, k, -\tau) \\ &= (\mathrm{e}^{1}, \mathrm{e}^{2}, \mathrm{e}^{3}, \mathrm{e}^{4}) \end{aligned}$$

Taking into account the isotropy of the media and homogeneity, we obtain the magnetic field strength 4-vector, such that

$$B = \mu H$$

μ= permeability of free space in a vacuum

By permutation of the indices, using the signature +++-, and using 2×2 determinants, in the 4-Dimensional space-time, then we can express the curl of the electric field E by using 1st rank covariant components or the 1st rank contra-variant components of the curl of the vector E. It is necessary to state that, the 1st rank covariant components curl$_{\alpha}$ E of the curl of any vector E are expressed by using 2×2 determinants as in below where:

$$curlE = \nabla \times E = curl_{\alpha} Ee^{\alpha}$$

That's above equation which is similar to any arbitrary vector space,

$$curl_{\alpha} E = \varepsilon_{\alpha\beta\lambda} \partial_{\beta} E_{\lambda}$$

That is, where, we have, in 0-rank vector form, by using covariant indices curl E= invariant 0- rank [1,3,5]

$$= \left\{ \begin{vmatrix} \partial_3 & \partial_4 \\ E_3 & E_4 \end{vmatrix} i + \begin{vmatrix} \partial_4 & \partial_1 \\ E_4 & E_1 \end{vmatrix} j + \begin{vmatrix} \partial_1 & \partial_2 \\ E_1 & E_2 \end{vmatrix} k - \begin{vmatrix} \partial_2 & \partial_3 \\ E_2 & E_3 \end{vmatrix} \tau \right\}$$

It follows, by using alphabetical indices that we have curl E= invariant 0- rank [3,5]

$$= \left\{ \begin{vmatrix} \partial_z & \partial_\tau \\ E_z & E_\tau \end{vmatrix} i + \begin{vmatrix} \partial_\tau & \partial_x \\ E_\tau & E_x \end{vmatrix} j + \begin{vmatrix} \partial_x & \partial_y \\ E_x & E_y \end{vmatrix} k - \begin{vmatrix} \partial_y & \partial_z \\ E_y & E_z \end{vmatrix} \tau \right\}$$

relative to the contra-variant basis $e^{\alpha} = (i, j, k, -\tau)$ we have to mention here that the 1st rank covariant components $curl_{\alpha}$ H of the curl of the vector H can be expressed, as in the following, where

$$curlH = \nabla \times \mathrm{H}$$

$$\nabla \times \mathrm{H} = curl_{\alpha} He^{\alpha}$$

Then, by using covariant indices, we have

curl H = invariant 0-rank

$$= \left\{ \begin{vmatrix} \partial_3 & \partial_4 \\ H_3 & H_4 \end{vmatrix} i + \begin{vmatrix} \partial_4 & \partial_1 \\ H_4 & H_1 \end{vmatrix} j + \begin{vmatrix} \partial_1 & \partial_2 \\ H_1 & H_2 \end{vmatrix} k - \begin{vmatrix} \partial_2 & \partial_3 \\ H_2 & H_3 \end{vmatrix} \tau \right\}$$

It follows, by using alphabetical indices that we have curl H=invariant 0-rank

$$\left\{ \begin{vmatrix} \partial_z & \partial_\tau \\ H_z & H_\tau \end{vmatrix} i + \begin{vmatrix} \partial_\tau & \partial_x \\ H_\tau & H_x \end{vmatrix} j + \begin{vmatrix} \partial_x & \partial_y \\ H_x & H_y \end{vmatrix} k - \begin{vmatrix} \partial_y & \partial_z \\ H_y & H_z \end{vmatrix} \tau \right\}$$

relative to the basis $e^{\alpha} = (i, j, k, -\tau)$

The 4 current density vector in any cartesian frame S of the coordinates, $x_{\alpha} = (x_1, x_2, x_3, x_4)$

$$T = i\ l = i\ c\ t$$

That is, above where in any frame S we have that the 4th dimension of the positional vector as representing the proper imaginary light time T of any particle,

$$T = i\ l = i\ c\ t$$

Further, where t is the real proper ordinary time, and denotes the real light time of the same particle, such that,

$$l = c\ t,$$

having above, c being the speed of light as measured in a vacuum.

By using covariance and considering the 4 electric field E_{α}, and the 4 magnetic field vector H_{α} in space-time, we recall the Albert Einstein-Maxwell's equations, which are stated in the following form by employing 4 vectors:

$$\frac{\partial H_{\alpha\beta}}{\partial x_{\beta}} = \frac{\partial E_{\alpha}}{{}_{c}\partial t} + i_{\alpha}$$

$$\frac{\partial E_{\beta}}{\partial x_{\alpha}} = \frac{\partial E_{\alpha}}{\partial x_{\beta}} - \frac{\partial H_{\alpha\beta}}{{}_{c}\partial t}$$

The divergence of the 4-electric field, characterizes the charge by using Albert Einstein- Maxwell's equation, in the continuum space-time. So, in employing 4 vectors in Space for the electric field, we obtain the relativistic charge density ρ which is a scalar, and is expressed as follows taking the following form, by invariance of the charge Q in frame S such that

$$\nabla_{\alpha} E^{\alpha} = \rho = scalar$$

In curvilinear coordinates, the divergence in the preceding is expressed as follows, by using the del operator, and, covariant differentiation [1,3,5]

$$\nabla_{\alpha} E^{\alpha} = \rho = scalar$$

Now, the 4 current density vector J_{α}, of any electrically charge signal, is such that,

$$J_{\alpha} = {}_{\rho} u_{\alpha}$$

where, $\rho = [1-\beta^2]^{-1/2} \quad \rho = \gamma\rho$

ρ being the ordinary (may be also called the proper) charge density. We have discussed the Electromagnetic Field Tensor of Albert Einstein in another study analysis. For more information on that, readers are encouraged to refer to more details and view more info on this issue as mentioned in textbooks of relativity. It follows upon the transformation of the Relativistic equations of the A. Einstein-Maxwell's equation, to any cartesian frame S' of the coordinates

$$x'_{\alpha} = x_{\alpha}$$

having above, that the transformed positional vector $x_{\alpha} = x'_{\alpha}$, as follows, in frame S',

$$\begin{aligned} x_{\alpha} &= x'_{\alpha} \\ &= (x'_1, x'_2, x'_3, x'_4) \\ &= (x', y', z', i\ c\ t') \\ &= (x', y', z', i\ l') \\ &= (x', y', z', T') \end{aligned}$$

It must be noted above, in frame S' that we have the 4th dimension of the positional vector x'α representing the transformed proper imaginary light time T' of any particle under consideration in frame S, where,

$$T' = i\ l' = i\ c\ t'$$

We further note above that, t' is the transformed real proper ordinary, and denotes the real light time of the same particle, in frame S'

$$l' = c\ t'$$

By covariance, using 4 vectors then, the Albert Einstein-Maxwell's equations take the following form, upon transformation from frame S to frame S', in space-time:

$$\frac{\partial H'_{\alpha\beta}}{\partial x'_{\beta}} = \frac{\partial E'_{\alpha}}{{}_{c}\partial t'} + i'_{\alpha}$$

$$\frac{\partial E'_{\beta}}{\partial x'_{\alpha}} - \frac{\partial E'_{\alpha}}{\partial x'_{\beta}} = \frac{\partial H'_{\alpha\beta}}{{}_{c}\partial t'}$$

Using 4 vectors in Space of the electric field, the ordinary transformed charge density ρ', which is a scalar, takes the following form, by invariance of the charge Q, in frame S', that the relativistic charge density ρ', such that,

$$\frac{\partial E'_{\alpha}}{\partial x'_{\alpha}} = \rho' = scalar$$

The classical Maxwell's equation in their basic form, in any transformed other frame S can be written as follows, taking the following form [3,5,13], such:

$$\nabla \times H = \frac{\partial E}{\partial T} + J$$

$$\nabla \times E = \frac{\partial H}{\partial T}$$

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$$divE = \rho$$

$$divH = 0$$

We note above, that in any curvilinear coordinates frame S=S', then the above equations are expressed as follows, for any considered particle,

$$\nabla_{\alpha} E^{\alpha} = \rho$$
$$\nabla_{\alpha} H^{\alpha} = 0$$

where above, T is the transformed light time of any considered particle [1,3,5,19],

$$T = T' = i\ l' = i\ c\ t'$$

Upon transformation of 4 of the field vectors from frame S to frame S we see that the transformed electric field strength 4-vector E as a 0-rank, the magnetic field strength 4 vector-H as a 0-rank as well. That's while ρ is the transformed charge density in magnitude scalar form in frame S=S', and J as the 0-rank electric current density 4-vector. Electromagnetic properties of any particle in terms of T the light time of any particle which is purely imaginary. Using the light time T of any particle, in space-time, we have upon transformation of the coordinates that T'= i c t'. The Relativistic Albert Einstein-Maxwell's Equations, which may will take the form as follows in cartesian coordinates:

$$\frac{\partial H_{\mu\nu}}{\partial x_{\nu}} = \frac{\partial E_{\mu}}{c\,\partial_{\tau}} + i_{\mu}$$

$$\frac{\partial E_{\nu}}{\partial x_{\mu}} - \frac{\partial E_{\mu}}{\partial x_{\nu}} = \frac{\partial H_{\mu\nu}}{c\,\partial_{\tau}}$$

$$\mu, \nu = 1,2,3,4$$

So that upon the transformation of the coordinates of the A. Einstein- Maxwell's equation, we get

$$\frac{\partial H'_{\mu\nu}}{\partial x'_{\nu}} = \frac{\partial E'_{\mu}}{c\,\partial_{t'}} + i'_{\mu}$$

$$\frac{\partial E'_{\nu}}{\partial x'_{\mu}} - \frac{\partial E'_{\mu}}{\partial x'_{\nu}} = \frac{\partial H'_{\mu\nu}}{c\,\partial_{t'}}$$

On the other hand, for the transformed charge density, we have:

$$\frac{\partial E'_{\mu}}{\partial x'_{\mu}} = \rho'$$

Moreover, where above, $T = i\ c\ t$ is the transformed imaginary light time in system S'. That is, while for the transformed charge density, we have in cartesian coordinates, that:

$$\frac{\partial E_{\mu}}{\partial x_{\mu}} = \rho$$

That is, in curvilinear coordinates, in frame S', we have $\nabla_{\alpha} E^{\alpha} = \rho'$

$$\nabla_{\alpha} H^{\alpha} = 0$$

and so forth for the divergence of the magnetic field. That is, we have above, that Iμ, being the transformed relative 4-current density vector, upon transformation. So that, we have, from the previous Albert Einstein- Maxwell's equations explicitly in the system S' of the coordinates that these equations for the curl of the magnetic field, will take the form, upon stated, in the preceding, upon the transformation of coordinates.

THE 4-POTENTIAL VECTOR Ω THE ELECTRIC SCALAR POTENTIAL ϕ

Again, it is important to mention that the physical meaning of obtaining an electric field in any region is emphasized by the preceding details. We state that, that is associated with generating electricity, which is mainly characterized by introducing the 4 magnetic potential vector ϕ_α, as detailed above. It follows that, upon presenting the arbitrary 4 magnetic potential vector ϕ_α, we generate the 4 potential vector Ω_α, without any incident. It is clearly evident, in space–time. By introducing any uniformed 4 potential vector Ω_α, such that $\Omega_\alpha = A_\alpha = (A_i, i\phi)$

It is seen above that, we have resolved the arbitrary vector potential into 3 real components A_i . Moreover, it must be noted that, in space-time, for any arbitrary electric scalar potential function ϕ (x, y, z, i l), for any considered moving, solely under the effects of any field and the charge Q, above we have that the real electric scalar potential

$$\phi = \phi(\mathrm{x}, \mathrm{y}, \mathrm{z}, \mathrm{T}) = \text{scalar invariant}$$

(4), p.179,180

$$T = i\ l = i\ c\ t$$

It follows that, the pure imaginary electric scalar potential, is such that

$$\phi = \phi(x, y, z, i\ l)$$
$$i = \sqrt{-1}$$

where above the ordinary real light time l of any considered particle, is such that $l = c\ t$; noting above that, t is the ordinary real proper time of the same considered particle. We have during the motion of any material particle under the effects of the electric field E, and the magnetic field H, relative to the inertial frame S, in space-time, that 4 electric field vector E_α and the 4 magnetic field vector H_α, are obtained from the application the 4 potential vector Ω_α where,

$$\begin{aligned}\Omega_\alpha &= A_\alpha \\ &= (\Omega_i, i\phi) \\ &= (\mathrm{A}_i, i\phi)\end{aligned}$$

Once again from above, we recall that, we obtain by using the 4 gradient ∇_α, for any particle in motion, relative to the inertial frame S in the 4 space-time, in a similar way to use the gradient in 3-space. Then, the 4 electric field vector E_α, which will take the form, as follows with respect to the real light time l, of any considered particle such that

$$\begin{aligned}E_\alpha &= -\nabla_\alpha \phi - \frac{\partial \Omega_\alpha}{\partial l} \\ &= (e_i, E_4) \\ &= (e_i, iH / c)\end{aligned}$$

where, above we have the 4^{th} coordinate E_4 being purely imaginary, where

$$E_4 = iH / c$$

Alternatively, we can express from the proceedings, the same equation of the previous 4 electric field vector, by using 4 partial derivatives, in inertial frame S, as follows such that

$$\begin{aligned}E_\alpha &= -\partial_\alpha \phi - \frac{\partial \Omega_\alpha}{\partial l} \\ &= (e_i, E_4)\end{aligned}$$

Now by using 0-rank vectors, we can express the same electric field equation, in space-time, above as follows, by using the 4-gradient ∇ (also called the 4 - del operator ∇), such that [5,19],

$$E = -\nabla \phi - \frac{\partial \Omega}{\partial l}$$

It follows again from above, in obtaining the 4-magnetic field vector H_α (which is also called the 4-magnetic field vector H_α), relative to the inertial frame S, by using the cross-product vector operation in the 4-dimensional space-time, where the 4 magnetic field vector H_α will take the form, as follow

$$\begin{aligned}H\alpha &= {}_{\varepsilon\alpha\beta\lambda} \partial_\beta \Omega_\lambda \\ &= (h_i, H_4) \\ &= (h_i, {}_i E / c)\end{aligned}$$

$$\alpha, \beta, \lambda = 1,2,3,4$$
$$H_4 = {}_i E / c$$

Where, above εαβλ is the permutation tensor in space-time, and where we have the 4th coordinate H_4 of the 4-magnetic field vector H_α being purely imaginary. εαβλ being as the permutation tensor in space-time. Then by using bold notation 0-rank vectors, we express the above magnetic field equation, by employing the 4- gradient ∇ (also called the 4-del operator ∇), in space-time, which will take the form, as follows such that,

$$H = \nabla \times \Omega$$
$$= curl\Omega$$

We pause for a moment here to mention that the 4-magnetic field induction vector B_α affecting any particle in motion under the effects of the 4-electric field vector E_α, is obtained from the cross product of the 4-velocity vector and the 4-electric field vector E_α, in space-time, in any arbitrary medium, as taking the form follows such that,

$$B_\alpha = \mu I_\alpha / 2\pi s = (b_i, B_4) = (b_i, iD/c)$$

where above, the electric 4 induction vector D_α, in any isotropic media, is such that, $D_\alpha = {}_\varepsilon E_\alpha$

That above, which is being measured at any distance s from any 4 electric current vector I_α. Now, by using the Biot Savart-Law, then the magnetic field, is denoted by B_α, where

$$B_\alpha = {}_{\varepsilon\alpha\beta\lambda} U_\beta E_\lambda / c^2 = (b_i, B_4)$$

That is, alternatively where from the above we have from the 4-magnetic field induction vector B_α [13,14] which affects any considered particle in motion due to the effects of the 4- electric field vector E_α, that

$$B_\alpha c = {}_{\varepsilon\alpha\beta\lambda} U_\beta E_\lambda / c = (b_i c, B_4 c)$$

It must be mentioned that, the 4-electric field intensity vector E_α as done by A. Einstein may take the following form, in the 4-D space-time, in the theory of relativity, such that

$E_\alpha = (e_i, E_4) = (e_i, {}_i H / c)$ and, as usual we have the indices α=1,2,3,4 and i=1,2,3.

In a similar way, the 4-magnetic field, it is necessary to state above that the 4-magnetic field intensity vector H_α may take the following form, in the 4-D space-time, as it was done by Albert Einstein in the theory of relativity such that,

$H_\alpha = (h_i, {}_i e / c) = (h_i, H_4)$ and as usual we have the indices α=1,2,3,4 and i=1,2,3.

The 4-magnetic field induction vector B_α (also called the magnetic flux density 4-vector) is then obtained in space-time, for any arbitrary medium as follows such that,

$$B_\alpha = \mu H_\alpha + M_\alpha = (b_i, B_4)$$

where, we have expanded above, the scalar magnetic induction into a 3-component vector b_i, in space-time. Further, the 4th coordinate of the magnetic flux density 4-vector B_4 is purely imaginary, such that $B_4 = iD/c$

It must be mentioned that 4-vector M_α is called the magnetization vector of the medium under consideration. In isotropic media, the above equation of the 4-magnetic induction vector B_α will become as follows, taking the following form, such that

$$B_\alpha = \mu H_\alpha = (b_i, B_4) = (b_i, iD/c)$$
$$B_4 = iD/c$$

In a similar fashion, the 4-electric field induction vector D_α (which is also called the electric flux density 4-vector) is obtained in space-time, for any arbitrary medium where,

$$D_\alpha = \varepsilon E_\alpha + P_\alpha = (D_i, D_4) = (D_i, iB/c)$$
$$D_4 = iB/c$$

(14), p.191

Then, similarly it must be mentioned above that, we have expanded the scalar electric induction into a 3 component vector Di, in space–time. We state again in the above that the 4th coordinate of the electric flux density 4- vector D_4 as being purely imaginary, such that

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$D_4 = iB/c$

We have to stress above that the 4-vector P_α is called the polarization vector of the medium under consideration. Now, that is where we have in isotropic media, that the above equation of the 4-electric induction vector D_α as taking the following form, where

$$D_\alpha = \varepsilon E_\alpha = (D_i, D_4)$$

THE ELECTROMAGNETIC FIELD TENSOR $F_{\alpha\beta}$

Once again we recall that, as it was done by Albert Einstein in the theory of relativity, in the early 1900's, who formed many electromagnetic field tensors, it is seen that the 2nd rank components $\phi_{\alpha\beta}$ are the components of a 2nd rank skew symmetric tensor. Although, we will talk in great details about the electromagnetic field tensor $\phi_{\alpha\beta}$, and the field tensor $F_{\alpha\beta}$, which plays an important role in the Electrodynamics of particle, yet a brief discussion about the said tensor is presented here. By resolving into 3 components e_i, the magnitude electric field intensity E which was obtained from the Lorentz electromagnetic (EM) force, we can start forming the electromagnetic field tensor $F_{\alpha\beta}$. Similarly, by resolving the magnitude of magnetic field B, introduced above, into 3 components, we then obtain those of the magnetic h_i (having 1 component as imaginary). Thus, the components $F_{\alpha\beta}$ may be made up of the electric field components e_i and those of the magnetic field h_i in space-time. Then, we recall from above that, by using 1st rank components of the electric field and also of the magnetic field, we arrive at obtaining the electromagnetic field tensor

$$F_{\alpha\beta} = \begin{bmatrix} 0 & h_z & -h_y & -{}_ie_x \\ -h_z & 0 & h_x & -{}_ie_y \\ h_y & -h_x & 0 & -{}_ie_z \\ {}_ie_x & {}_ie_y & {}_ie_z & 0 \end{bmatrix}$$

That is, we have formed above the electromagnetic field tensor $F_{\alpha\beta}$ noting that, from skew symmetry, we have the components $F_{\alpha\beta} = -F_{\beta\alpha}$ and also $F_{\alpha\alpha} = F_{\beta\beta} = 0$

Now, by using the Maxwell's Equations we have, div e =ρ= invariant upon any coordinate transformation

Then, it is convenient to recall the Albert Einstein Electromagnetic (EM) field tensor $\phi_{\alpha\beta}$, which plays an important role in the theory of relativity, in the 2nd rank components $\phi_{\alpha\beta}$ are in fact the components of a 2nd rank skew symmetric tensor. We have explicitly, in a closely similar fashion, that the components $\phi_{\alpha\beta}$ are characterized in space-time, from the components of the electric field e_i and those of the magnetic field h_i, where

$$\phi_{\alpha\beta} = \begin{bmatrix} 0 & h_{13} & h_{12} & -{}_ie_x \\ h_{31} & 0 & h_{23} & -{}_ie_y \\ h_{21} & h_{32} & 0 & -{}_ie_z \\ {}_ie_x & {}_ie_y & {}_ie_z & 0 \end{bmatrix}$$

The components h_{ij} may be considered as the 2nd rank components of the magnetic field h and so on. Further, the 2nd rank components e_{ij} of the field may be used in the consideration of other 2nd rank skew- symmetric electromagnetic tensors; as proposed by Albert Einstein, in his scientific book "The meaning of relativity" p.23-50, published in 1922, 1923–by Princeton University Press. Thus, as it was done by A. Einstein, we have formed above the electromagnetic (also denoted EM) field tensor $\phi_{\alpha\beta}$, noting that, from skew symmetry we have the components $\phi_{\alpha\beta} = -\phi_{\alpha\beta}$ and also $\phi_{\alpha\alpha} = \phi_{\beta\beta} = 0$,

α,β=1,2,3,4; i,j=1,2,3.

MOMENTUM, ENERGY, CHARGE & THE ALBERT EINSTEIN ENERGY TENSOR $T_{\alpha\beta}$

We recall once again the A. Einstein field intensities tensor $\phi_{\alpha\beta}$ which has a great importance in the theory of relativity. It must be noted, the field intensities potential $\phi_{\alpha\beta}$, which is a 2nd rank anti–symmetric tensor, such that

$$\phi_{\alpha\beta} = \begin{matrix} 0 & h_{13} & h_{12} & -{}_ie_1 \\ h_{31} & 0 & h_{23} & -{}_ie_2 \\ h_{21} & h_{32} & 0 & -{}_ie_3 \\ {}_ie_1 & {}_ie_2 & {}_ie_3 & 0 \end{matrix}$$

It must be mentioned that the components of the 2nd rank tensor $\phi_{\alpha\beta}$ are skew–symmetric. That is, we have above that $\phi_{\alpha\beta} = -\phi_{\alpha\beta}$ and $\phi_{\alpha\alpha} = \phi_{\beta\beta} = 0$

We shall talk more about the intensive tensors and electromagnetic field tensors. The A. Einstein energy tensor $T_{\alpha\beta}$ is expressed as follows taking the following form, such that

$$T_{\alpha\beta} = (-1/4)\delta_{\alpha\beta}\phi^2{}_{\lambda\theta} + \phi_{\alpha\theta}\phi_{\beta\theta}$$

α,β,λ,θ=1,2,3,4

That is, we must bear in mind that the skew-symmetric entries $T_{ij} = -T_{ji}$ are purely imaginary, while the $T_{ij} = -T_{ji}$ are real [7].

The Albert Einstein energy tensor $T_{\alpha\beta}$ takes also the following form $T_{\alpha\beta} = \sigma U_\alpha U_\beta$. We have the energy density η characterizing the energy per unit volume.

INVARIANCE OF ENERGY IN ANY VOLUME

Select any arbitrary volume $d\tau$ in any region in any entity in space-time, where, $d\tau = dx\ dy\ dz\ dt$. It follows immediately by invariance of the energy upon any rotation of the volume or transformation of the volume of any entity, that the invariant differential form quantity $T_{44}d\tau =$ invariant upon any transformation [14,19]. That is upon transformation of the volume from any frame S of the coordinates

$$\begin{aligned} x_\alpha &= (x_1, x_2, x_3, x_4) \\ &= (x, y, z, i\ l) \\ &= (x, y, z, T) \end{aligned}$$

where, $x_1 = x, x_2 = y, x_3 = z, x_4 = T = i\ c\ t = i\ l$

to any other frame S ' of the coordinates

$$\begin{aligned} x'_\beta &= (x'_1, x'_2, x'_3, x'_4) \\ &= (x', y', z', i\ l') \\ &= (x', y', z', T') \end{aligned}$$

where, $x'_1 = x', x'_2 = y, x'_3 = z, x'_4 = T' = i\ c\ t' = i\ l'$

We must have by invariance of differential forms that,

$$T'_{44}\, d\tau' = T_{44} d\tau = \text{invariant}$$

where, T_{44} is taken here to be as purely imaginary (unless otherwise specified). That is, in other words we must have from above by invariance of differential forms & differential geometry above

$d[\gamma QUc] = T'_{44}\, d\tau'/c = T_{44} d\tau / c =$ invariant upon any rotation or transformation

We have by taking the angle between the charge vector and the 4 velocity vector to be as imaginary, then, by the scalar product of 4 vectors in space-time that,

$Q.U = Q^\alpha U_\alpha$ =invariant upon rotation

Where, we have made notion of the invariance of scalar product upon any rotation in space-time. The 4 charge vector Q and the relativistic 4-velocity vector U, of any particle under consideration as detailed above, we obtain the transformed relativistic velocity U, upon the rotation of the relativistic velocity U. Further, we obtain the transformed relativistic charge Q, upon the rotation of the relativistic charge Q. To explain about the angle between the 4 charge vector Q and the 4 velocity vector U, we note above that as the real angle between the 4 charge vector and the 4-velocity vector, such that the invariance of differential forms becomes clearly obvious in the previous steps. So that we must obtain, by invariance of the scalar product of 4 vectors, in space-time M^4, that

$$d[\gamma\xi / c] = \text{invariant} = T'_{44}\, d\tau'/c = T_{44} d\tau / c = \text{invariant upon transformation}$$

$$d[\gamma QUc] = \text{invariant} = T'_{44}\, d\tau'/c = T_{44} d\tau / c = \text{invariant upon transformation}$$

On the other hand we have,

$$\mathrm{d}\{Q^{\alpha}U_{\alpha}C\} = \text{invariant}$$
$$= d\{[1-\beta^2]^{1/2} QUc\}$$
$$= d\{[1-\beta^2]^{-1/2} \xi / c\} = \text{invariant upon any transformation}$$

Further, at the existence of the ordinary proper mass m of any material particle in space we must obtain, by considering the relativistic mass M of the same material particle under consideration by invariance of 4- forms in space-time that [1,9,12,14,19],

$$d[Mc] = d[\gamma Mc] = T_{44} d\tau / c = d[\gamma \xi / c] = \text{invariant}$$

Hence, the energy ξ of any particle, under the effects of a field or more, is expressed in terms of the charge Q as follows [14,19],

$$\xi = QUc^2$$

We now consider the energy of any material particle having a proper ordinary mass m, under the effects of a field or more. That is, where, the relativistic mass M of the same considered material particle (if such a particle exits under any circumstances, as in the case above) and again T_{44} is taken here, to be as purely imaginary (unless otherwise specified). Now, by differentiating covariant in curvilinear coordinates systems, we have the force K_{α} per unit volume, is generated from the A. Einstein tensor $T_{\alpha\beta}$, where,

$$K_{\alpha} = T_{\alpha\beta;\beta}$$

However, by covariance in Cartesian orthonormal coordinates, the volume 4-force K_{α} will take the following form [1,4], by using partial derivatives

$$K_{\alpha} = \partial T_{\alpha\beta} / \partial x_{\beta}$$

So that, by using the 3×3 stress Maxwell's tensor p_{ij}, which is expressed explicitly below. Further, by using the 3 pointing vector Si, in Cartesian orthonormal coordinates, so that we get

$$\partial p_{11} \partial p_{12} \partial p_{13} \partial S_1$$
$$K_1 = \frac{-\partial x_1}{\partial p_{21}} \frac{-\partial x_2}{\partial p_{22}} \frac{-\partial x_3}{\partial p_{23}} \frac{-i\partial_{(i\,l)}}{\partial S_2}$$
$$K_2 = \frac{-\partial x_1}{\partial p_{31}} \frac{-\partial x_2}{\partial p_{32}} \frac{-\partial x_3}{\partial p_{33}} \frac{-i\partial_{(i\,l)}}{\partial S_3}$$
$$K_3 = \frac{-\partial x_1}{\partial S_1} \frac{-\partial x_2}{\partial S_2} \frac{-\partial x_3}{\partial S_3} \frac{-i\partial_{(i\,l)}}{\partial \eta}$$
$$l = c\ t$$

The importance of the physical meaning of the energy tensor $T_{\alpha\beta}$ becomes apparent and evident, when we realize by the Albert Einstein's theory of relativity that the energy tensor $T_{\alpha\beta}$ is expressed as follows in terms of the Maxwell's stress, such that

$$T_{ij} = p_{ij}$$
$$T_{ji} = p_{ji}$$
$$T_{\alpha i} = -T_{i\alpha} = -iS_i$$
$$T_{44} = \eta$$

Where, the Maxwell's stress tensor takes the form, as follows

$$p_{ij} = -e_i e_j - h_i h_j + (1/2)\delta_{ij}\{e^2 + h^2\}$$
$$\alpha, \beta = 1, 2, 3, 4$$
$$i, j = 1, 2, 3$$

Further, η is the energy density. Moreover, it must be noted that we have the energy density η expressed as follows in terms of the 3 fields, such that

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$$\eta=(1/2)\{(e_1^{\,2}+e_2^{\,2}+e_3^{\,2})+(h_1^{\,2}+h_2^{\,2}+h_3^{\,2})\}$$ (7), p.45-47

$$p_{11}=-e_1e_1-h_1h_1+\{(e_1^{\,2}+e_2^{\,2}+e_3^{\,2})/2\}+\{(h_1^{\,2}+h_2^{\,2}+h_3^{\,2})\}$$
$$p_{12}=-h_1h_2-e_1e_2$$
$$p_{13}=-h_1h_3-e_1e_3$$

By theory of relativity, the A. Einstein 4-force density K_α, as per unit volume, K_α is obtained in Cartesian coordinates from the Albert Einstein energy tensor $T_{\alpha\beta}$ as follows:

$$K_\alpha=\frac{\partial T_{\alpha\beta}}{\partial x_\beta}$$

However, in curvilinear coordinates the same A. Einstein force density K_α, as per unit volume may be obtained by covariant differentiation of the energy tensor such that $K_\alpha=T_{\alpha\beta;\beta}$ [7].

That is where, the imaginary mechanical energy density $i\lambda$, as taken per unit volume upon rotation is characterized by the 4th coordinate of the 4-force density vector K_α, as follows $K_4=i\lambda$

Then by considering any arbitrary volume, having the volume differential element dV, it follows by integration, that the relative 4 momentum vector J_α, is obtained as in the following such that,

$$J_\alpha=\int K_\alpha dx\;dy\;dz\;dl$$

$\alpha=1,2,3,4$, where, $T=i\;l=i\;c\;t$

We have above, in differentials, that the kinetic energy ξ, of any particle being expressed in imaginary differential form as follows such that [7,19]

$$id[QUc]=dJ_4$$
$$=K_4\;dx\;dy\;dz\;dl$$
$$=i\lambda\;dx\;dy\;dz\;dl$$
$$=id\xi/c$$

and simultaneously we have above mentioned that

$$id\xi/c=dJ_4$$
$$=K_4\;dx\;dy\;dz\;dl$$
$$=i\lambda\;dx\;dy\;dz\;dl$$

Now that is explained also in the following.

We have by differentiating with respect to the light time of any considered particle that,

$$J_\alpha=\int K_\alpha dx\;dy\;dz\;dl$$

It follows immediately in imaginary differential forms and by the general theory of relativity and tensor rules from above, the energy of any particle may be expressed in terms of charge Q, as with accordance to the velocity U, which that particle has moved with, and being under the effects of the that charge Q, as follows such that

$$dJ_4=K_\alpha dx\;dy\;dz\;dl$$
$$=i\lambda\;dx\;dy\;dz\;dl$$
$$=id[Mc]$$
$$=[id[QUc]$$
$$=id\xi/c$$

Hence, we must have upon integration of the above form, the kinetic energy of any particle is such that

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$$\xi = Mc^2$$
$$= QUc^2$$ or we may write from the above equation that,

$$\xi = Mc^2$$
$$= QUc^2$$
$$= \gamma QVc^2$$

Further, where above, in magnitudes, the relativistic velocity U of any particle is related to the ordinary velocity v, of the same particle, such that $U = \gamma V$

Considering the rest energy of any particle in a rest frame we must obtain from above that

$$\xi_{\circ} = \gamma m_{\circ} c^2$$
$$= QU_{\circ} C^2$$
$$= \gamma QV_{\circ} c^2$$

That is where the relativistic rest velocity $U_{\circ}$ of any particle is related to the ordinary rest velocity $v_{\circ}$ of the same particle such that [7],

$$U_{\circ} = \gamma V_{\circ}$$

That is explained in a more explicit way in this analysis. Later, we will consider the transformation of the Albert Einstein energy tensor $T_{\alpha\beta}$ which is any reference inertial frame S into the transformed energy tensor $T_{\alpha\beta}$.That is, after many transformations of fields. Now we stress in Cartesian frames, that the energy tensor $T_{\alpha\beta}$ is expressed in terms of the electromagnetic field tensor $\phi_{\alpha\beta}$ such that

$$T_{\alpha\beta} = -(1/4)\phi_{\alpha\beta^2}\delta_{\alpha\beta} + \phi_{\alpha\theta}\phi_{\beta\theta}$$

α,β,θ=1,2,3,4

Again, from above we present the physical meaning of the energy tensor $T_{\alpha\beta}$, when it becomes evident, when we obtain the 4-vector volume force K_{α}, where, the covariant 1st rank components S_{α} of the 4-pointing vector are obtained by the cross product of the 4 electric field vector E, and the 4 magnetic field vector H, as follows such that,

$$S_{\alpha} = {}_{\varepsilon\alpha\beta\lambda} E_{\beta} H_{\lambda}$$
$$= (S_i, S_4)$$
$$= (S_i, iS/c)$$

Where, in 0 rank 4 vector we have by using the cross product of the electric and magnetic field 4 vectors, in space-time that the 0 rank 4 pointing vector S such that,

$$S = E \times H$$
$$= (S, {}_i e \times h/c)$$
$$= (S_i e^i, {}_i e \times h/c) = S_{\alpha} e^{\alpha}$$

Relative to the 4-unit vector basis e^{α} [5,14]. Further, the 3 pointing vector is obtained by the cross product of the 3 vectors of the fields, as follows,

$$S = e \times h$$

Then it is important to mention that, we have resolved the 4-Poynting vector S_{α} into 4 components (which are the 3 pointing vector S_i and the imaginary 4th component S_4), where above,

$$S_i = {}_{\varepsilon ijk} e_j h_k$$
$$i, j, k = 1, 2, 3$$

ENERGY AND CHARGE BY INVARIANCE

By invariance of the charge Q, upon rotation from any coordinate system S to any other coordinate system or a frame S, we can

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express the energy ξ of any particle in any system S, as being the charge Q in any other system S such that,

$$\xi = Q = \text{invariant 0-rank scalar}$$

Using 1st rank components, we can express the 4 energy ξ of any particle with respect to any basis e^α, in any system S as being the 4 charge Q, relative to the basis e^λ, in any other system S as follows such that,

$$\xi = \xi_\alpha e^\alpha = Q_\lambda e^\lambda = Q = \text{invariant 0-rank}$$

α=1,2,3,4

By covariance of mechanics, electrodynamics, the components of the 4-flux vector D_α, in Cartesian orthonormal coordinates may be obtained and we can express the energy of any particle, under the effects of a field as being the invariant charge in any other system. So, to explain the preceding mathematically in a clear way we go as follows: Let us select in the continuum, space-time, any arbitrary volume (i*d*V), in any arbitrary reference frame S of the coordinates x, y, z, i c t. The real light time $l = c\ t$ of any particle in space-time, while the imaginary light time $T = i\ l$ is such that, $T = i\ l = i\ c\ t$ that is, the 3 coordinates *x, y, z* expresses the position of any particle while the 4th coordinate *i c t* is to denote the proper imaginary light time. Further, t denotes the proper real time of the same particle in frame S.

$$dV = dx\ dy\ dz\ dt$$

$$idV = dx\ dy\ dz\ d(i\ t)$$

$$= idx\ dy\ dz\ dt$$

Then, if we select in the same volume and the same region, any arbitrary area element da, having the 4-area element vector da^α and having the normal 4-unit vector n^α, where, $da^\alpha = n^\alpha da$

It is then seen by Stoke's theorem, the divergence theorem and invariance of the charge, in any Cartesian orthonormal frame in space-time, that the form, [1,12,19] where we have the 4-area element da is expressed in 0–rank bold vector notation, such that $da = nda$ n, being the 4 unit vector normal to the area element da. Further, once again by using the 1st rank contra variant components of any area element da^α, we have $dQ = divDdV$, having the 1st rank contra variant unit vector n^α, as being the 4-unit vector normal to the area element da. So that from the preceding, we must get by employing the divergence theorem, in Cartesian orthonormal coordinate systems that

$$dQ = divDdV$$

$$= \nabla.DdV = D_\alpha n^\alpha da = \text{invariant}$$

So that by using integration, we must obtain,

$$\int dQ = \int \nabla.DdV = \int divDdV$$

$$= \iint D_\alpha n^\alpha da$$

$$= \iint D_\alpha da^\alpha$$

$$= \text{invariant}$$

Hence, using the 4-divergence theorem upon rotation of the energy of any particle to the charge we get space-time from above in invariant 4-forms and 1-forms in the continuum space-time, the invariant 0- rank scalar quantity [1,3,14,19]

$$\int d\xi = \iint D_\alpha n^\alpha da$$

$$= -i\int i\ divDdV = \int divDdV = \int dQ$$

$$= \text{invariant}$$

Using pure imaginary functions, we arrive at obtaining, by invariance 4 forms and 1 forms, that

$$\int d[i\xi] = \int d[iQ]$$
$$= \iint iD_\alpha n^\alpha da$$
$$= \int div[iD]dV \quad \text{where, } i = \sqrt{-1}$$

ABOUT 2nd RANK TENSORS IN MINKOWSKI'S TENSOR SPACE $M^{\alpha\beta}$

It is worthy to mention about the 4×4 Makowski's tensor space $M^{\alpha\beta}$. If A is any arbitrary 0-rank invariant tensor then we can express A as the same invariant in the Makowski's tensor space $M^{\alpha\beta}$, which is a 4×4 tensor, and has 16 entries, by using its 2nd rank covariant components $A^{\alpha\beta}$, relative to the 2nd rank contra variant basis $M^{\alpha\beta}$. The 2nd rank components $A^{\alpha\beta}$ of any invariant A may be described as follows such that,

$A_{ij} = a_{ij}$ (Where the 9 entries a_{ij} are all real i,j=1,2,3)

$A_{i4} = -iA_i / c$ (Where the A_i are the 1st rank real components of A, in 3-space; i,j=1,2,3)

$$A_{4i} = iA_i / c$$
$$A_{44} = 0$$

It may be seen that any 2nd rank tensor components may look in the Minkowski's tensor space $M^{\alpha\beta}$ as follows,

$-iA_1 / c, -iA_2 / c, -iA_3 / c = 0$

$$-iA_1 / c, -iA_2 / c, -iA_3 / c = 0$$

The components $A_{\alpha\beta}$ of any arbitrary invariant 0-rank tensor are not ski-symmetric. However, again we have the entries $A_{i4} = -A_{4i}$ [3].

THE FLUX OF THE CHARGE

Let us pause here momentarily to introduce the notion of the flux of the charge Q. It is important to recall the concept of the flux in the theory of relativity. As an example we present the idea of the flux of charge. Under the effects of an electric field E, then, that is presented and perceived when the existence of electricity generate electric field lines crossing any associated area 'a', the cross sectional of any wire carrying electricity or the electric field lines existing in any region under the effects of such electric field. In such cases we treat electric field lines in a quantitative way. In considering an electric field E that is uniform in both magnitude and direction [6].

As shown in **Figure 2**, the field lines penetrate a rectangular surface of area 'a', whose plane is oriented perpendicular to the field. In general the product of the magnitude of the electric field E and surface area 'a' perpendicular to the field is viewed by some physicists as the electric flux Φ_e. By using the 4-flux density D_α, in space-time we obtain in differential forms upon invariance of the charge by Gauss' divergence theorem,

$$d\Phi = D_\alpha N^\alpha dS$$
$$= E_\alpha n^\alpha da$$

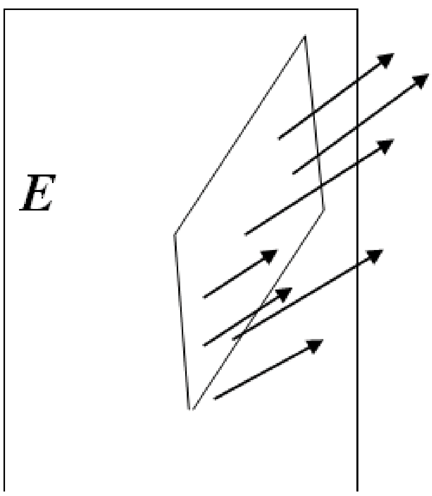

Figure 2. Electric field lines crossing an associated area 'a'.

It must be mentioned that, by Maxwell's equations we have in space-time, by using the 4 electric field vector that, $divE = \nabla_{\alpha} E^{\alpha}$ [1,7,19].

α=1,2,3,4. In Cartesian orthonormal coordinate systems (frames), the above divergence will be evaluated, only by using imaginary functions, while employing partial derivatives, as follows,

$idivE = i\partial_{\alpha} E^{\alpha}$ [1,7,9,12].

Then by using purely imaginary function for the above divergence we have in general that, $divE = \nabla_{\alpha} E^{\alpha}$

So, upon rotation in space-time of the area differential element da, to the area element dS, which has the 4-unit normal N^{α}, then it is obvious by invariance of the charge Q, in terms of differential forms in the above that the flux is characterized by the equation above. So that upon integration we must get,

$$\iint D.NdS = \iint D_{\alpha} N^{\alpha} dS = \int divEdV$$
$$= \int \nabla_{\alpha} E^{\alpha} dV = \int dQ = \text{invariant}$$

It follows by using pure imaginary functions, in space-time, we must obtain from the flux density of the imaginary charge iQ in the above by integration, that,

$$\iint iD.NdS = \int i\ divE\ dV = \iint iD_{\alpha} N^{\alpha} dS = \int i\nabla_{\alpha} E^{\alpha} dV$$

$$\int d[iQ] = \text{invariant}$$

If considering any frame S of the coordinates *x, y, z,* and *il*, one can express the electric, magnetic, electrodynamics properties of any particle in space [1,10]; where, by covariance, we have in any frame S in the 4- dimensional continuum space -time, that the coordinates

$$x_a = (x_1, x_2, x_3, x_4)$$
$$= (x, y, z, il)$$
$$= (x, y, z, T)$$

Where, in any frame S we have, $x_1 = x, x_2 = y, x_3 = z, x_4 = T = i\ l = i\ c\ t$, *l* denotes the real light of any particle in space-time, T denotes the imaginary light time of any particle in space-time, and $T = il = i\ c\ t$. Further, t denotes the proper time of any particle in frame S in space-time. That is, while T characterizes the 4th dimension of the positional vector x_{α} any particle specified by the imaginary light time of the same particle in space-time. In the following we use 0- rank four vectors, being relative to the 4-basis

$$e^{\alpha}, \alpha = 1,2,3,4$$

Thus, in any system or frame S, the 3 coordinates x, y, z will express the position of any particle, in any electric, electromagnetic, process in space-time, while the 4th coordinate *i c t* is to denote the proper light time of the same particle. So, by expressing the four electric field intensity E, (also called the electric field), the magnetic field intensity H, (also called the magnetic field), we have, by covariance that,

$E = E_{\alpha} e^{\alpha}$, relative to the basis e^{α}, while we have for the 4-magnetic field, that: α=1,2,3,4

$H = H_{\alpha} e^{\alpha}$, relative to the basis e^n as well. That is, by expanding the electric and magnetic field into the 3 electric and magnetic fields e_i and h_i, that

$$E_{\alpha} = (e_i, iH / c)$$
$$= (E_i, E_4)$$

The 4 vector field E_{α}, in the above, in any Cartesian orthonormal inertial frame S, of the coordinates *x, y, z, i l*, are considered as continuously differentiable functions (at least differentiable twice), with respect to the said coordinates such that,

$$E_{\alpha} = E_{\alpha}(x, y, z, i\ l)$$

For the magnetic field 4 vector we have,

$$H_\alpha = (h_i, iE/c)$$
$$= (H_i, H_4)$$
$$i = 1,2,3$$
$$\alpha = 1,2,3,4$$
$$n = 1,2,3,4$$

That is again relative to the contra variant 1st rank basis e^α. By the Biot-Savart law and upon transformations, it may be seen that the 4 magnetic flux density vector B is generated experimentally at any associated distance 's', from any measured 4 electric current I, in any continuous charge electric circuit or charged path, where in 0 rank 4 vectors in space-time $B = \mu I / 2\pi s$

where, μ=permeability of free space,

ε=permittivity of free space, relative to the basis e^α.

Similar to the above, the 4-vector field B^α, in any Cartesian orthonormal inertial frame S, of the coordinates *x, y, z, i l*, are considered as continuously differentiable functions (at least differentiable twice), with respect to the said coordinates, such that $E_\alpha = E_\alpha(x, y, z, i\ l)$

By covariance, the 1st rank components B^α of the 4 magnetic flux density vector B in any circuit are measured and formed in space-time. That is, by resolving into 3 vectors, we obtain the 3 magnetic field density b_i. So, from the measured 4-electric current I_α, we obtain $B_\alpha = \mu I_\alpha / 2\pi s$ [5,13].

It is then stated again, by measuring the electric current I in any electric circuit, it follows, the covariant 1st rank components I_α of the 4 electric current vector I in any circuit are formed in space-time. So, by using the same method of resolving scalars into 3 vectors, we obtain the 3 electric current I_i, for the 4-electric current vector I_α where, $I_\alpha = (I_i, I/c)$ [5].

Similarly, by measuring the electric potential difference ϕ, in any electric circuit then the covariant 1st rank components ϕ_α of the 4 electric potential difference vector ϕ in any circuit are formed in space-time. So, by using the same method of resolving scalars into 3 vectors, we obtain the 3 electric potential current ϕ_i, for the measured 4-electric potential difference vector ϕ_α, where, $\phi_\alpha = (\phi_i, \phi/\mathrm{c})$ where we have, in any frame S, of the coordinates *x, y, z, i l*, that $\phi_\alpha = \phi_\alpha(x, y, z, i\ l)$ [5].

By covariance, employing the 1st rank components E_α of the 4 electric field vector, we obtain the 1st rank covariant components ϕ_α of the invariant 4 transformed relative electric potential difference vector ϕ are obtained, along any electrically charged path element dL, as in the following. The electric potential difference ϕ of the electric field E. The 1st rank covariant components ϕ_α, which are obtained and evaluated at any time and any positional point *x, y, z, i l*, from the origin O of any coordinate system S, in space-time, are seen to be generated from the rotation of the electric potential difference, where, $\phi_\alpha = \phi.\hat{E}_\alpha$ [3,18].

So, from above it is seen that we get the 0-rank electric field ϕ is such that, $\phi = \phi_\alpha e^\alpha$, relative to the 4-unit vectors basis e^α. By invariance of the DOT product we have, $\phi = \phi_\alpha e^\alpha = \phi^\lambda e_\lambda$. By using the space metric $\eta^{\alpha\lambda}$, in space-time we get, $e^\alpha = \eta^{\alpha\lambda} e_\lambda$ [3,12,18]. Further, 1 if α=β

$$\eta^{\alpha\lambda}\eta_{\lambda\beta} = \delta_{\alpha\beta} = 0 \text{ if } \alpha \neq \beta$$ [9].

It must be mentioned at any positional point *x, y, z, i l*, from the origin O of any coordinate system S in space-time, it is seen by the naked eye that the cross product of the 4-vectors H and ϕ, generate the components of a 0- rank 4 vector potential A=p in space-time, where

$$H \times \phi = A$$ [1,3,5].

Further, the components of the 4 potential A are obtained and evaluated at any time and any positional point *x, y, z, i l* from the origin O of any coordinate system S, in space-time. Then, by the 1st rank covariant components of the potential are such that

$A_\alpha = (A_i, iA/c)$, relative to the basis e^α the 3 components Ai as purely real, while, the 4th component A_4 is purely imaginary. That is, the 1st rank components A_α are the covariant components of the 0 rank vector potential A, in space-time. Moreover, by invariance of the vector A, then we can express the same 0 rank vector potential A as being invariant, in space-time, in the following manner, $A = A_\alpha e^\alpha$, relative to the basis e^α. By employing the ordinary volume (i.e. the density per unit volume) density ρ of the charge Q, of any particle under consideration and being under the effects of a field, the 4-electric current density J^α, where, $J_\alpha = \rho U_\alpha$, [5,13].

U is the relativistic velocity of the particle under consideration and U_α is the relativistic 4 velocity of the same particle, relative to frame S. Further, by using the ordinary velocity v of the same considered particle, we have $U = \gamma v$ and in 1st rank in Cartesian orthonormal systems, we have $U_\alpha = \gamma v_\alpha$. Now, using 3rd rank components of the measured electric potential ϕ, we have, $\phi^\lambda{}_{\alpha\beta} = \eta_{\alpha\theta}\phi^{\lambda\theta}{}_\beta$ [18].

Where, in any frame S, that $\phi^{\lambda}{}_{\alpha\beta} = \phi^{\lambda\theta}{}_{\beta}(x, y, z, i\ l)$

It is noted that the 3rd rank tensor components $J^{\alpha\beta}{}_{\theta}$, of the above-mentioned current density vector $J_{\alpha} = \rho U_{\alpha}$, generate the current tensor components Jαβμ, where $J^{\alpha}{}_{\beta\gamma} = \eta_{\beta\mu} J^{\alpha\mu}\gamma$ [18]

Further, in any frame S, $J^{\alpha\mu}\gamma = J^{\alpha\mu}\gamma(x, y, z, i\ l)$

Now, by the equation of continuity, we must have in any Cartesian orthonormal inertial frame S of the coordinates *x, y, z, i c t,* in space-time that, $i\partial\rho = i(-\nabla.J)\partial(i\ t)$ [13].

So, by continuity of the charge Q, of any considered particle, it is deduced from above, by the equation of continuity in Cartesian orthonormal coordinate systems in space-time that, $\frac{\partial\rho}{\partial t} = -\nabla.J$ [13].

Considering any arbitrary volume in space-time, it is then found by invariance of the charge Q, of any considered particle under effects of a field in any volume element dv, that the scalar quantity characterizing the charge Q, in a differential form is invariant, where $dQ = \rho dV = \text{invariant}$ [3,13]. Consequently, by considering the rest charge $Q_{\circ}$, and the rest charge density $\rho_{\circ}$ of any particle in the rest volume element $dv_{\circ}$, then the scalar quantity characterizing the charge Q in differential form is invariant such that,

$$dQ = \rho dV = \rho_{\circ} dV_{\circ} = \text{invariant}$$

In relation to dielectrics, we take into consideration the electric current displacement 4-vector D and the 4- magnetic field density B are expressed in the same way [3,13],

$$D = D_{\alpha} e^{\alpha}$$

$B = B_{\alpha} e^{\alpha}$, relative to the basis e^{α}. For the electric current displacement 4-vector D_{α} (which will be called later the flux density of the charge Q) and magnetic field density 4 vector D_{α} we have,

$$D_{\alpha} = (D_i, iB/c)$$
$$= (D_i, D_4)$$

Now, in any frame S, of the coordinates *x, y, z, i l,* $D_{\alpha} = D_{\alpha}(x, y, z, i\ l)$ and the 3 components D_i are purely real while the 4th component D_4 is purely imaginary. That is, while we have also, for the magnetic flux density,

$$B_{\alpha} = (b_i, iD/c)$$
$$= (B_i, B_4)$$

Further, in any frame S, $D_{\alpha} = D_{\alpha}(x, y, z, i\ l)$ relative to the basis e^{n} where, the 3 components b_i are purely real while the 4th component B_4 is purely imaginary, i=1,2,3 [3,13,18]. In a similar fashion, in any frame S, the magnetic flux density $B_{\alpha} = B_{\alpha}(x, y, z, i\ l)$. Other 4 vectors of electrodynamics, magnetism, etc. are expressed in similar manners in space-time. When speaking of dielectrics, one has to make sure that there are sufficient methods to express free charges, if they exist in space in any way [13,14].

By using the 3rd rank components of the electric displacement vector D (which will be called later as the flux density of the charge Q) and also using the current density tensor $J^{\alpha\beta}{}_{\gamma}$, it is found that the following tensor product generate the components of a tensor. That is the tensor product $\varsigma^{\alpha\beta\theta\mu}{}_{\gamma\lambda} = J^{\alpha\beta}{}_{\gamma} D^{\theta\mu}{}_{\lambda}$ [18].

α,β,θ,μ,γ,λ= 1,2,3,4

In S frame we have, $\varsigma^{\alpha\beta\theta\mu}{}_{\gamma\lambda} = \varsigma^{\alpha\beta\theta\mu}{}_{\gamma\lambda}(x, y, z, i\ l)$

It follows that if we introduce 4 potential Ω_{α}, in any system or frame S to incur its electric effects on any particle being in any electric, electromagnetic and so on process, in space-time, we have by expressing the four potential E, (also called the electric field), the magnetic field intensity H, (also called the magnetic field) we have by covariance that, $\Omega = \Omega_{\alpha} e^{\alpha}$ relative to the basis e^{α}. The 1st rank components Ω_{α} are obtained by covariance where,

$$\Omega_{\alpha} = (A_i, i\phi)$$
$$= (\Omega_i, \Omega_4)$$

It is important to mention, by using the D'alembertian of the 4-potential Ω_{α}, we have that the 4-electric current density vector [5,12], such that,

$$^{2}\Omega_{\alpha} = J_{\alpha} = (J_i, iJ/c)$$
$$= (J_i, J_4)$$

By using 3rd rank components for the 4-electric current density vector J_α that, $J_{\mu\alpha\beta} = \eta^{\mu\theta} J_{\theta}^{\ \alpha\beta}$

where, $\eta^{\mu\theta}$ is the space metric in space-time. That is, while by using partial derivatives for the curl $_{\alpha\lambda}D$ in any Cartesian orthonormal inertial frame S of the coordinates *x, y, z, i l,* for the curl of the electric displacement vector D (which may be considered as the flux density D of the charge Q) that,

$curl_{\alpha\beta} D = \partial_\alpha D - \partial_\beta D_\alpha$ [1,3].

By employing the 2nd rank curl $_{\alpha\beta}D$ of the 0 rank electric flux density tensor D, of the charge then the electric current density field tensor $j_{\theta\mu\varphi}$ is generated, where, $j_{\alpha\lambda\beta} = \Omega_\alpha curl_{\beta\lambda} D$ (3), p.25-50, 60-72, (1), p.311-325.

In frame S we have, $j_{\alpha\lambda\beta} = j_{\alpha\lambda\beta}(x, y, z, i\ l)$

By covariance, the electric displacement vector in direction of the 4-potential Ω, generates the following magnetic flux density vector components B_α, which are obtained and evaluated at any time and at any positional point *x, y, z, i l*, from the origin O of any coordinate system S, in space-time, where $B_\alpha = D.\hat{\Omega}_\alpha$ and $B_\alpha = B_\alpha(x, y, z, i\ l)$

Making use of the 3rd rank components of the current density tensor $j_{\alpha\lambda\beta}$, it is found that the following tensor product generate the components of a tensor. That is, the 2nd rank flux density $D^{\lambda}\alpha$ mixed tensor is generated such that,

$D_\alpha^{\ \varphi} = j^{\alpha\lambda\beta}\phi^{\lambda\beta\varphi}$ (18), p.73-77, $\alpha,\beta,\theta,\mu,\gamma,\lambda=1,2,3,4$

We have the magnetic field vector H in direction of the 4-potential E generates the 4-vector potential $\wedge_\alpha$ where, $\wedge_\alpha = H.\hat{E}_\alpha$ [3,5,18].

It is important to state in the above that, the components of the 4 potential are arbitrary, such that the 3 potential components A_i are real while the 4th component Ω_4 is purely imaginary. Further, the 1st rank covariant component Ω_α are obtained and evaluated at any time and at any positional point, *x, y, z, i l,* from the origin O of any coordinate system S, in space-time. We have the magnetic field vector H in direction of the 4-potential E generates the 4-vector potential $\wedge_\alpha$ where, $\wedge_\alpha = H.\hat{E}_\alpha$ [3,5,18].

The 2nd rank components $_{\alpha\beta}$ of the tensor $\wedge$ in the Minkowski's Tensor Space $M^{(\alpha\beta)}$ generate the charge field tensor $Q^{\beta\alpha}$, where $Q^{\lambda\beta}{}_{\alpha} = \wedge_{\alpha\theta}\Omega^{\lambda\beta}{}_{\theta}$

The 4 magnetic field vector H in direction of the 4 electric current vector I generates the 4-vector magnetic field H_α, where, $H_\alpha = H.\hat{\Omega}_\alpha$ [3,18].

Further, by expressing the 4-magnetic vector field H, as invariant in space by using 1st rank components H_α, we have from covariance, that $H = H_\alpha e^\alpha = \text{invariant}$, relative to the basis e^α. Further, by invariance of the magnetic field, we must have,

$H = H_\alpha e^\alpha = H^\alpha e_\alpha \text{invariant}$ [3,18]

By using the space metric $\eta_{\alpha\beta}$ of space-time, the 3rd rank components of the invariant magnetic field tensor H are such that, $H^{\alpha}{}_{\lambda\beta} = \eta_{\lambda\mu} H^{\mu\alpha}$ [18],

Now the 4 electric field vector E in direction of the 4-potential Ω generates the 4- vector electric field Θ_α where, $\Theta_\alpha = E.\Omega_\alpha$ [3].

By expressing the 4-potential vector field$\wedge$, as invariant in space while using 1st rank components $\wedge_\alpha$ of the 4- vector potential $\wedge$ and by covariance,

$\wedge = \wedge_\alpha e^\alpha = \wedge^\alpha e_\alpha$ [1]. Then by expressing the field vector Θ and by using 1st rank components and covariance, $\Theta = \Theta_\alpha e^\alpha$, relative to the basis e^α. By considering the cross product of the 4-vectors $\wedge$ and Θ, generate the components of a 0- rank magnetic flux density vector b in space-time,

$$b = \wedge \times \Theta$$
$$= b_\alpha e^\alpha$$

The 1st rank components b_α is obtained as follows, by using indices $b_\alpha = {}_{\varepsilon\alpha\beta\gamma}\wedge_\beta\Theta_\gamma$ where, εαβγ are the 3-permutation tensor in space-time [1,3,5]. The electric field vector C is generated as seen below, from the inner product of 4 vectors in space-time. That is, the 4 potential Ω in direction of the electric potential ϕ generates the electric field vector C_α. So that, the 1st rank covariant components C_α which are obtained and evaluated at any time and at any positional point, *x, y, z, i l*, from the origin O of any coordinate system S, in space-time are seen to be generated from the rotation of the electric potential difference, $C_\alpha = \Omega.\hat{\phi}_\alpha$ [3,18], relative to the basis e^α. Consequently, it may be seen, that the following 4 unit vector is generated, thus, forming a direction in space-time where, the unit vector

$$\hat{C}\alpha = \frac{C.e_\alpha}{C}$$

Where, C is the magnitude of the invariant 0 rank vector C. The 3rd rank components $C^{\alpha}_{\lambda\beta}$ of the invariant tensor C are found to be such that, $C^{\mu}{}_{\gamma\alpha} = \eta_{\gamma\lambda} C\lambda\mu_\alpha$ where, the components $C\lambda\mu_\alpha$ are not skew-symmetric. α,β,γ,λ,μ=1,2,3,4. The cross product of the 4-vectors E and Ω, generates the components of a 0 rank 4 vector, in space-time, where, $E \times \Omega = P$ [1,3,5], relative to the basis e^α. Consequently, it may be seen that, the following 4 unit vector is generated, thus forming a direction in space-time, such that, the unit vector

$\hat{P}\alpha = \frac{P.e_\alpha}{P}$, where, P is the magnitude of the invariant 0 rank vector P. The 3rd rank components $P_{\alpha\lambda\beta}$ of the 0 rank tensor P, which is invariant, are found to be such that,

$P^{\mu}{}_{\gamma\alpha} = \eta_{\gamma\lambda} P\lambda\mu_\alpha$, where, the components $P\lambda\mu_\alpha$ are not skew-symmetric. *α,β,γ,λ,μ*= 1,2,3,4. It follows by multiplying on both sides by the space metric $\eta^{\alpha\gamma}$ which is the space metric in space-time, it may be seen from above that, the following 3rd rank components are the components of tensor, where,

$\eta^{\alpha\gamma} P\mu_{\gamma\alpha} = P\alpha\mu_\gamma$ [18]. The above is explained further. Next, it is found that the following tensor product generate the components of the 4th rank electric field tensor $E_{\beta\lambda\alpha\mu}$ where, $E_{\beta\lambda\alpha\mu} = H^{\beta\lambda}\gamma C^{\alpha\mu}\gamma$

The covariant components $E_{\theta\phi\psi\varphi}$ of the electric field tensor are obtained by using the space metric $\eta_{\theta\phi}$ of the 4-Dimensional continuum space-time such that, $E_{\theta\phi\psi\varphi} = \eta_{\theta\beta}\eta_{\phi\lambda}\eta_{\psi\alpha}\eta_{\varphi\mu}E^{\beta\lambda\alpha\mu}$ [3,18].

Thus, we arrive by covariance, at obtaining the 4th rank flux density tensor $D_{\theta\phi\psi\varphi}$ of the charge Q where, $D_{\theta\phi\psi\varphi} = E_{\theta\phi\psi\varphi}$ that is stated once again that the 4th rank flux density tensor $D_{\theta\phi\psi\varphi}$ of the tensor D, which is invariant in space. Further, the 4th rank flux density tensor characterizes the flux density of the charge Q is obtained as follows,

$$D_{\theta\phi\psi\varphi} = \eta_{\theta\beta}\eta_{\phi\lambda}\eta_{\psi\alpha}\eta_{\varphi\mu}E^{\beta\lambda\alpha\mu} = E_{\theta\phi\psi\varphi}$$

It is important to mention that the 1st rank components of the invariant flux density in space-time are the components D_α, where by using 1st rank components we must have the 0-rank tensor may be expressed as an invariant, in the 4-D space-time. That is, the 0-rank tensor D, which is invariant upon any transformation and change of basis is expressed as follows such that, $D = D_\alpha e^\alpha = \text{invariant}$, relative to the basis e^α. Further, by invariance we have, $D = D^\alpha e_\alpha = \text{invariant}$. So, if we select any 4-area element vector by the divergence theorem we must have therefore, by invariance of the charge Q and using invariant forms, that $dQ = D.dA = D.NdA = \text{invariant}$ where, N is the 4-unit vector which is normal to the area element dA and in 4 vector bold notation we have, $dA = NdA$. It may be seen by invariance of the divergence theorem, by invariant forms in any volume dV that, $dQ = \nabla.DdV = D.NdA = \text{invariant}$ [1,19].

Proceeding further, to account for the 2nd rank skew-symmetric components curl $_{\alpha\beta}$A of curl of the field C, *T=i l*, it must be noted in any inertial frame S of the coordinates *x, y, z,* we have, $curl_{\alpha\beta} C = \partial_\alpha C_\beta - \partial_\beta C_\alpha$ [1,3].

In a similar fashion we can obtain the 4-unit vectors of any other corresponding 4 vectors, in space-time, which form directions there. It is seen then in a similar way, to that presented, by using the cross product of 4 vectors, in space-time, we have the cross product of the 4- vectors Ω and H, as generating 0- rank 4 vector in space-time where,

$\Omega \times H = A$, relative to the basis e^α. Accordingly, we obtain the following direction 4-unit vector which is generated in space-time, where the unit vector

$$\hat{A}_\alpha = \frac{A.e_\alpha}{A}$$

Where, A is the magnitude of the invariant 0 rank vector A in the 4- D space-time. Recalling the curl vector function and by using 1st rank components of the curl, by covariance the components curl $_\alpha$A of curl of the vector field A in any inertial frame S of the coordinates x, y, z, T=i l, that, $curl_{\alpha\beta} E = \partial_\alpha E_\beta - \partial_\beta E_\alpha$ [5], where εαβγ is the permutation tensor in space-time. The 2nd rank skew-symmetric components curl $_{\alpha\beta}$E of curl of the electric field E, in any inertial frame S, of the coordinates x, y, z, T=i l, that $curl_{\alpha\beta} E = \partial_\alpha E_\beta - \partial_\beta E_\alpha$ [3].

Where, l=c t is the real light time and *T=i l* is the imaginary time of any particle in space-time. Using the 1st rank components Ω_α are obtained by covariance. By using the 2nd rank skew-symmetric components curl $_{\alpha\beta}$E of curl of the electric field E in any inertial frame S of the coordinates *x, y, z, T= i l*, that

$curl_{\alpha\beta} E = \partial_\alpha E_\beta - \partial_\beta E_\alpha$ [3], where, *l=c t* is the real light time and *T=i l* is the imaginary time of any particle in space-time. That is, for the magnetic field H, the 2nd skew-symmetric components curl $_{\alpha\beta}$H, in any inertial frame S, with respect to the coordinates *x, y, z, T= i l* of S. So that is,

$$curl_{\alpha\beta} H = \partial_\alpha H_\beta - \partial_\beta H_\alpha$$ [1,3].

Now by using the inner product of 4 vectors in space-time and considering the electric field in direction of the magnetic field H, we have the 4-electric field θ_α such that, $\theta_\alpha = E.H_\alpha$ that is, the 0-rank electric field θ, such that $\theta = \theta_\alpha e^\alpha$, relative to the 4-unit vectors basis e^α. It must be mentioned, by invariance of the DOT product we have,

$$\theta = \theta_\alpha e^\alpha$$
$$= \theta^\alpha e_\alpha$$

We have the 2nd rank skew symmetric components $curl_{\alpha\beta}\theta$ that, $curl_{\alpha\beta}\theta = \partial_\alpha \theta_\beta - \partial_\beta \theta_\alpha$ [3,12,18].

Now, by employing the 3rd rank components $\theta^\beta_{\mu\alpha}$ of the invariant tensor field θ, we have $\beta_{\mu\alpha} = \eta_{\mu\lambda}\theta^{\lambda\beta}{}_\alpha$ [1,3,18], where, the components $\theta^{\lambda\beta}{}_\alpha$ are not skew–symmetric. Using the cross product of 4 vectors in space-time, we obtain the flux density D of the charge Q, where,

$$D = A \times curl\theta$$
$$= e^\alpha{}_{\varepsilon\alpha\beta\gamma} A_\beta curl_\gamma \theta$$
$$= D_\alpha e^\alpha$$

Recalling again, the 4 electric current density J_α, it is noted that $J_\alpha = \rho U_\alpha$ By employing the 4 cross product of vectors in space-time, then it is seen that the 4 current density j is such that,

$$= \iint iD.nda$$
$$= \iint iD_\alpha.n^\alpha da$$
$$= \iint iD.da$$

By means of what has been explained above we can now speak in a similar manner about the rotation of the energy ξ of any particle to the charge Q [3,5]. We have upon rotation in space-time of the area differential element *d*a, which has the 4-normal unit vector n^α, in any frame (coordinate system) S of the coordinates x, y, z, t to the area element, dS, which has the 4-unit normal vector N^α, existing in the frame (coordinate system) S of the coordinates, x, y, z, t, then it becomes clear and obvious by invariance of the charge Q, in terms of differential forms as noted earlier in the above that, the flux of the charge Q is characterized by the equation above. So that upon integration we must get,

$$\int d[i\xi] = \int d[iQ]$$
$$= \int i\ divDdV$$
$$= \iint iD.nda$$
$$= \iint iD_\alpha.n^\alpha da$$
$$= \iint iD.da$$

By means of what has been stated earlier it is then seen in space-time above,

$$\int d[i\xi / c] = \iint iD.nda / c = \iint iD_\alpha.n^\alpha da / c$$

And simultaneously, we must have from the invariance of the charge Q, that

$$\int d[iQUc] = \iint iD.nda / c = \iint iD_\alpha.n^\alpha da / c$$

Hence, by using the charge Q and the relativistic velocity U of any particle, we must obtain in the preceding 2 equations, by the divergence theorem, that the total energy ξ of the same particle being under the effects of one or more fields is such that, $\xi = QUc^2$

We have from invariance of the divergence theorem, in 4-forms that,

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$$\int d[i\xi] = \int d[iQ]$$
$$= \int i\ divDdV$$
$$= \iint iD.nda$$
$$= \iint iD_{\alpha}.n^{\alpha} da$$
$$= \iint iD.da$$

From the above we recall once again, by invariance of the charge Q and also by using the divergence theorem in any volume dV that,

$$dQ = \nabla.DdV = D.NdA = \text{invariant}$$

[1,19]. It follows by using pure imaginary forms that, we must obtain by the divergence theorem that,

$$d[iQ] = i\nabla.DdV = iD.NdA$$

By invariance of the charge Q as well we have in differential forms that the 0-rank scalar quantity

$$d(QUc) = dQ / c$$
$$= \nabla.DdV / c$$
$$= D.NdA / c$$
$$= \text{invariant}$$

Consequently, in imaginary forms that the following imaginary 1-form $w = d(QU)$ is invariant, such that

$$iw = d(iQU),$$
$$= d[iQ / c^2]$$
$$= i\nabla.DdV / c^2$$
$$= iD.NdA / c^2$$

Further, by invariance of the charge upon any rotation, $\int dQ = \text{invariant}$

Hence, by using the energy dξ of any particle, presented in a differential form and stated as in the above, we must get the scalar quantity [1,5,19].

$$\int d\xi = \int dQ = \text{invariant}$$

Then, in space-time, the 1- form scalar quantity, characterized by the form

$$Q.U = Q^{\alpha}U_{\alpha} = \text{invariant}$$ [5,19]

Next, we recall the relativistic 4-velocity U_{α} of any particle in any inertial frame S, of the coordinates x, y, z, t (where t denotes the proper time of the considered particle) and the 4- charge vector Q^{α}, in the same system S. It is clearly evident by invariance of the scalar product of 4 vectors in orthonormal cartesian frames in the continuum space-time that the scalar product of the 4-vector Q^{α} and the 4-vector quantity U_{α} of any particle under the effects of the charge Q, such that; $Q.U = Q^{\alpha}U_{\alpha} = \text{invariant}$

It follows immediately, by invariance of the energy ξ upon rotation and upon having a 2nd rotation from the energy to the charge, we must obtain, by invariance of differential forms, that the energy ξ, of any particle under the effects of the charge Q is

$$d(Q.U) = d(Q^{\alpha}U_{\alpha})$$
$$= (1/c^2)divDdV$$
$$= (1/c^2)\int \nabla.DdV$$
$$= (1/c^2)D.nda$$
$$= d\xi / c^2$$
$$= \text{invariant}$$

[1,9,12,19] α, λ=1,2,3,4. That is, further we have,

$$d(QU) = d(Q.U)$$
$$= d(Q^{\alpha}U_{\alpha})$$
$$= (1/c^2)\nabla.DdV$$
$$= (1/c^2)D_{\lambda}n^{\lambda}da$$
$$= \text{invariant}$$

Simultaneously, by invariance of differential forms using the scalar product which is invariant upon any rotation or transformation that the 1-form is invariant upon any rotation such that,

$$d(QU) = d'(Q'.U')$$
$$= d'(Q'^{\alpha}U'_{\alpha})$$
$$= d(Q.U)$$
$$= d(Q^{\alpha}U_{\alpha})$$
$$= \text{invariant upon any transformation}$$

It follows immediately, by invariance of the charge that we must get,

$$d(QU) = d(Q.U)$$
$$= d(Q^{\alpha}U_{\alpha})$$
$$= (1/c^2)\nabla.DdV$$
$$= (1/c^2)divDdV$$
$$= (1/c^2)D_{\alpha}n^{\alpha}da$$
$$= d\xi/c^2 = \text{invariant}$$

Using the 4th imaginary coordinate Q_4 of the 4-charge vector Q in the continuum 4-D space-time we must then obtain from above, by invariance of the charge Q that,

$$\zeta_4 = iQUc$$
$$= i\xi/c = iQ/c$$

CHANGE OF BASIS

In the following, the 4-energy vector in space-time that is to investigate the invariance of the charge Q, upon any change of basis. It is important to mention that the 4-energy vector ζ is invariant in space-time,

$$\zeta = \zeta^{\alpha}e_{\alpha}$$
$$= \zeta_{\alpha}e^{\alpha}$$
$$= Q_{\alpha}e^{\alpha} = Q$$
$$= \text{invariant upon any change of basis}$$

That is the 4-charge vector is expressed by covariance in space-time such that,

$$Q_{\alpha} = (Q_i, Q_4)$$
$$= (Q_i, iQ/c)$$

By covariance, we have the 1st rank vector energy vector is such that,

$$\zeta_4 = i\xi/c$$
$$= iQ/c$$

Further, we have resolved energy into 3 energy ζ_i and,

$$\zeta_4 = i\xi / c = iQ / c$$ and simultaneously we must have from invariance of the charge Q, that $$\zeta_4 = iQUc = iQ / c$$

Using the 4[th] imaginary coordinate Q_4 of the 4-charge vector Q in the continuum 4-D space-time, we must then obtain from above, by invariance of the charge Q,

$$\zeta_4 = iQUc$$
$$= i\xi / c$$
$$= iQ / c$$

Hence, we must arrive at obtaining the energy ξ of any particle in terms of the charge Q attained by the same particle and the relativistic velocity U of the same particle, where by tensor rules of transformations in the 4-D space-time we must have the energy of any particle $\xi = QUc^2, \text{ for} -\infty < Q < +\infty$

That is specifically for any positive or negative real values of the charge Q. To be more concise it must be stated that the charge Q, of any considered particle is permitted and defined as follows in the following interval $-\infty < Q < +\infty$. If the mass m of any considered material particle exists it may be seen immediately by invariance of the charge that we must get

$$\xi = QUc^2$$
$$= mc^2 = \gamma mc^2$$
$$= (1-\beta^2)^{-1/2} mc^2$$

where above is the relativistic mass of the same material particle under consideration, $m = \gamma m = (1-\beta^2)^{-1/2} m$

$$\gamma = (1-\beta^2)^{-1/2}$$
$$i = \sqrt{-1}$$
That is, $\beta = v / c \text{ and } i = \sqrt{-1}$

MOMENTUM OF A PARTICLE IN SPACE-TIME

By using 4 vectors in any Cartesian orthonormal coordinate system S in space-time, the 4-ordinary velocity v_α of any particle in motion in space-time is expressed as follows, $v_\alpha = (v_i, i_c)$

where above we have considered the 3 velocity v_i being presented as stated in the previous and c is the speed of light measured in a vacuum. It follows that the 4-relative velocity U_α of any particle, in motion in space-time is expressed as follows,

$$U_\alpha = \gamma v_\alpha$$
$$= (U_i, i\gamma c)$$

where we have resolved the velocity into the relativistic 3 components U_i. [5]. The 4- momentum P_α of any material particle having an ordinary mass m, in motion in space can be expressed in terms of its relativistic 4-velocity vector U_α such that, $P_\alpha = mU_\alpha$ by resolving the considered particle's momentum into the 3 components we get, the 4- momentum that,

$$P_\alpha = mU_\alpha$$
$$= (p_i, i\gamma mc),$$
$$\gamma = (1-\beta^2)^{-1/2}$$
$$i = \sqrt{-1}$$

Where, $\beta = v / c \text{ and } i = \sqrt{-1}$ We have v, the ordinary magnitude velocity of any particle under consideration while the relativistic velocity of the same particle is obtained as follows such that, $U_\alpha = \gamma v_\alpha$ .By using the 3[rd] rank components $P^\alpha{}_{\beta\gamma}$ for the momentum of any particle in motion we have, $P^\alpha{}_{\beta\gamma} = \eta_{\beta\lambda} P^{\alpha\lambda}{}_\gamma$ [9], α,β,λ,γ,θ=1,2,3,4, where, $\eta_{\alpha\theta}$ is the space-metric, in space-time. The relativistic 3[rd] rank components $E^\alpha{}_{\beta\lambda}$ for the electric field tensor affecting the dynamic or electrodynamic behaviors of any particle in motion or at rest under the electric current and electric potential along any circuit is expressed where, $E^\beta{}_{\lambda\alpha} = \eta_{\lambda\theta} E^{\beta\theta}{}_\alpha$ [9].

Multiplying both sides above by $\eta^{\theta\beta}$, we get,

$$\eta^{\theta\beta}P^{\alpha}{}_{\beta\gamma}=\eta^{\theta\beta}\eta_{\beta\lambda}P^{\alpha\lambda}{}_{\gamma}$$
$$=\delta^{\theta}{}_{\partial}P^{\alpha\lambda}{}_{\gamma}$$
$$=P^{\alpha\theta}{}_{\gamma}$$

It is seen from above that, $P^{\alpha\theta}{}_{\gamma}=P^{\alpha}{}_{\beta\gamma}\eta^{\theta\beta}$, summation is implied over indices θ, γ, [9]. It follows in a similar manner, by using 3rd rank components for the magnetic field we get,

$$H^{\alpha\theta}{}_{\gamma}=H^{\alpha}{}_{\beta\gamma}\eta^{\theta\beta}$$

MAXWELL'S EQUATIONS

After Maxwell first presented Maxwell's equations in 0-rank vector notation it is known that in addition to electric charge and current densities, ρ_e and J_e, there existed magnetic charge and current densities, ρ_m and J_m. Now, by expressing densities in terms of 0-rank vectors in the 4-D space-time we can speak of the 4-electric current density.

$J_m=J_{m\ \alpha}e^{\alpha}$ while we may refer to any 4-magnetic current density: $J_m=J_{m\ \alpha}e^{\alpha}$, α=1,2,3,4, relative to the basis e^n. That is where, by expanding the electric current density and using the 3-current density J_{ei}, we obtain:

$$J_{e\alpha}=(J_{ei},iJ_e/c)$$
$$=(J_{ei},J_{e4})$$

Meanwhile, when speaking of any existing magnetic current density by expanding the electric current density and using the scalar density J_m and the 3-current density J_{mi}, we obtain:

$$J_{m\alpha}=(J_{mi},iJ_m/c)$$
$$=(J_{mi},J_{m4})$$

α=1,2,3,4, i=1,2,3

By using the scalar DOT product and the gradient operator ∇ to express divergences in cartesian orthonormal frames (coordinate systems) in the 4-D space-time the Maxwell's equations for microscopic electrodynamics are then (in SI units) expressed in 0-rank 4 vectors as follows,

$$\nabla.E=\varepsilon\rho e,$$
$$\nabla.Bm=\mu\rho m,$$
$$-c_2\nabla\times\varepsilon E=\frac{\partial}{\partial t}\frac{B_m}{\mu}+J_m$$
$$-c_2\nabla\times\frac{B_m}{\mu}=\frac{\partial}{\partial t}(\varepsilon E)+J_e$$
$$\text{where, } c=1/\sqrt{(\varepsilon\mu)}$$

s, the speed of light in vacuum. It is noted in macroscopic electrodynamics we take into consideration media that contain volume densities of electric- and magnetic-dipole moments, P_m and M_e respectively (often P is called the densities of polarization and M is called the magnetization). Supposing that magnetic charges exist the media could also contain volume densities of (Gilbertian) electric- and magnetic-dipole moments M_m and M_e, respectively [14]. It follows upon transformation, by using relativistic values for electric ρ_e and magnetic ρ_m charge densities and further electric current densities then we can associate between free charges and bound charges in a manner similar to the following below. So that, if considering bound charges and current densities J'_e, which together can be associated with any existing free charge (if they exist in space) and current densities ρ'_e, J'_e, ρ'_e and J'_m comprise the total charge and current densities then these will take the form as follows,

$$\rho_e = \rho'_e - \nabla.P_e,$$
$$\rho_m = \rho'_m - \nabla \times M_m,$$
$$J_e = J'_e + \frac{\partial P_e}{\partial t} + \nabla \times M_e$$
$$J_m = J'_m + \frac{\partial M_m}{\partial t} - c^2 \times P_m$$

By means of the 4-vectors stated above, one can recognize these equations as the same ones developed for plane waves [13]. The permissible wave solutions found for transmission lines, or transmission of a signal can therefore be permissible by using 4-vectors. However, when talking about the transmission of a signal, it becomes more convenient to introduce circuit variables of voltage and current along the transmission line or transmission media, which will depend on the variables x, y, z and t. Kirchhoff's voltage and current laws will not hold along the transmission line as the electric field has nonzero curl and the current along the electrodes will have a divergence due to the time varying surface charge distribution. Because E has a curl, the voltage difference measured between any two points can be measured and calculated by the use of 4-vectors in space-time. This can be illustrated in elementary circuit experiment. So, when we have varying magnetic flux passing through any contour of lines (or charged paths). In general, the potential difference may be measured between the any two points A and B, along any path dL^{α} of any charged electric circuit wire. That is, in terms of the same coordinates x, y, z and t, in any frame S (x, y, z, t). Thus, the electric potential ϕ can be uniquely defined between the points A and B in terms of the coordinates x, y, z and t, where,

$$\phi(x,y,z,t) = -\int E.dL$$
$$= -\int E_{\alpha}(x,y,z,t)dL^{\alpha}$$ [1,9,10]

where, the 4-electric field vector E_{α} is denoted upon rotation, by E_{α}. There are 3 points we wish to draw the attention of readers to:

1. It must be mentioned that, in uniformed media of three spatial dimensions the electric and magnetic fields of electric and magnetic charges have the same character such that a single electric and a single magnetic field could describe the effects of both types of charges. So, the 4th dimension of the above vectors, in space-time is obtained in the same conventional manner used in relativity textbooks.

2. Magnetic charges seemed, earlier to be mysterious and difficult to be traced. However, at latter times, that has become to be an important issue in electric, electrodynamics and electromagnetism.

3. If the interaction of magnetic charges with magnetic moments which is due to electrical currents, is to conserve energy, the magnetic charges may be accounted for in the continuum 4-D space-time.

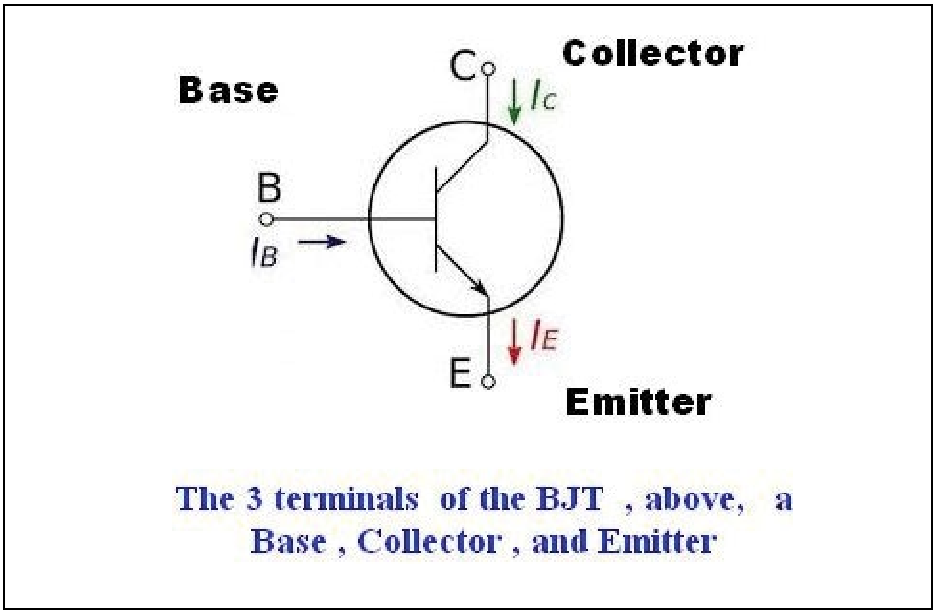

Figure 3. The 3 terminals of BJT: base, collector and emitter.

Going back to continue our discussion in cartesian orthonormal frames, we obtain the potential difference V, in similar manners 3 vectors. That is, by using the gradient of a scalar that we get

$$E+\frac{\partial A}{\partial t}=-v$$

where, the 4-potential A may be introduced in space to get the 4-potential difference V, as mentioned in the above. Then, by covariance we have, iu 4-vectors that,

$$E_\alpha+\frac{\partial A_\alpha}{\partial t}=-v_\alpha$$

where, the 4 potential A_α is expressed as follows, $A_\alpha=(A_i,i\phi)$

α=1,2,3,4, i=1,2,3

It is noted above that, we have introduced the negative sign on the right-hand side so that V becomes the electric potential in any static event when A is independent of time; for any substantiate situation.

PRINCIPLES OF RELATIVITY IN BIPOLAR JUNCTION TRANSISTORS USING ENERGY AND CHARGE

In this section, we use the principles of relativity and concepts in Bipolar Junction Transistors (BJT). That is in general, by employing energy and charge of particles in electric systems, electronics and microelectronic systems. So, as we know that we have the Bipolar Junction Transistors (BJT). I find it important to give a quick idea about some specific transistor such as the BJT. That is done before we get into the calculations of the charge Q or the relativistic velocity of any considered signal by using energy and charge in electronic or microelectronic component.

It must be mentioned that BJT or FET both share mutually common similarities. Any of them shares the same category of transistors. These transistors are known to have the property of both conduction as well as insulation. BJT as well as FET consists of three basic terminals in it. Moreover, these transistors are found everywhere as the basic components of electronic systems. In the following we discuss in brief, the basics and comparison between BJT and FET. We shall also show how to pick up any output relativistic velocity U of any signal under consideration in any electronic and microelectronic circuit. That is shown in the following examples and illustrations by using the output voltage V output where, V output may be measured from any output terminal of the BJT, JFET or any electronic, microelectronic, VLSI (Very Large Signals), circuits. The same method is used in various resonance, sensors, circuits. The BJT or the JFET, comes under various sizes and with multiple shapes. The main criteria of this transistors are to control the flow of current that passes through one channel by making the variations in the intensity of the currents that are very smaller which flows through the second channel. BJT or FET both have the features resembles to switch as well as amplifiers.

BASICS OF BJT

BJT stands as an abbreviation, for Bipolar Junction Transistor. It consists of one p-type and both n-types referred to as n-p-n or one n-type and both p- types referred to as p-n-p. In general, it is known that the ideology behind this is two diodes with the p-n junction can be connected in such a way that a bipolar junction has been formed. The three terminals in BJT can be classified as the three terminals of a transistor are labeled the base (b), collector (c), and emitter (e) as shown in **Figure 3**. They are named after their roles in the transport of charge Q within the device.

The h parameters of a bipolar junction transistor are typically measured with the emitter terminal grounded also known as the common-emitter configuration, the base is then designated as the input and the collector as the output. In the following, we present the symbolic representations of both the n-p-n and p-n-p transistors are shown here respectively in **Figure 4**.

It follows the non-linearity of the device that the definition of h parameters which are valid for all voltages and currents is not possible. It is therefore common practice to quote h parameters at a specific value of collector current IC and collector-emitter voltage VCE. Another consequence noted about the nonlinearity of the device is that AC h parameters and DC h parameters are often quite different in value. There are many types of instruments which may be employed to obtain the h parameters for a particular transistor.

As an example, one may find an electronic analyzer, a semiconductor parameter analyzer. This instrument sweeps the desired current which may be (plotted on the vertical axis) against a specified voltage (plotted on the horizontal axis). A family of curves is produced by varying a third parameter, often the base current, in discrete steps. As an example, the manufacturer of the 2N3904 n-p-n silicon transistor quotes h parameters. It must be noted that the specific parameters are given alternative designations by transistor engineers. The measurements were made at IC=1.0 mA, VCE=10 Vdc, and f=1.0 kHz.

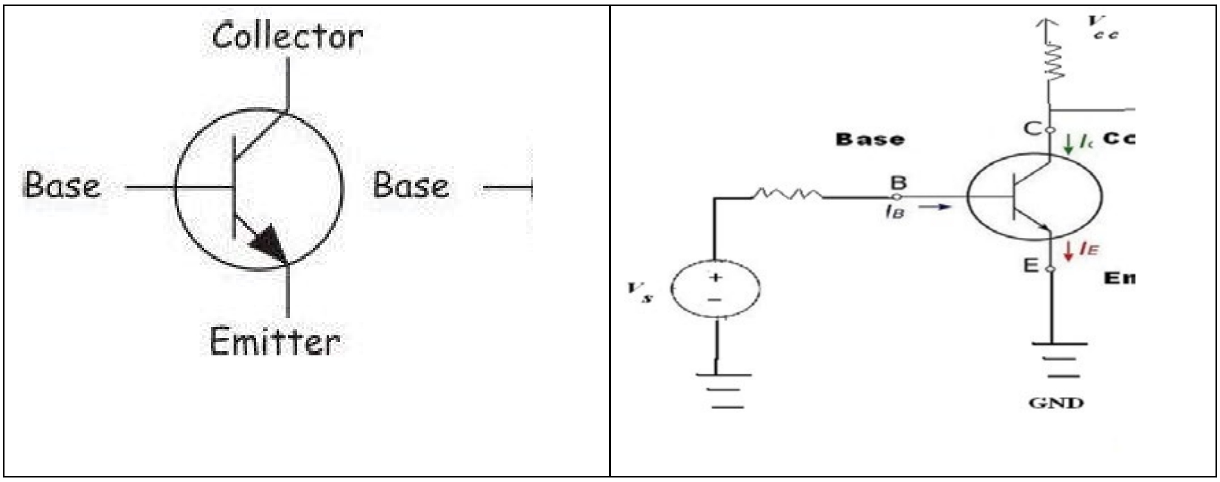

Figure 4. Symbolic representations of both the n-p-n and p-n-p transistors.

THE WORKING PRINCIPLE OF THE BJT

The three terminals present in the BJT are responsible for the formation of the junctions of the emitter and the base as well as a collector and the base. As considered the junction of base and the emitter is in the forward bias and the collector-base junction is in revere bias. Because of the forward biasing at the base and the emitter the flow of the majority carriers takes place from the emitter to the base.

It is necessary to mention that, as the region at the base is of light doping concentration, not all the majority carriers combine some of them tends to flow towards the collector. So, in this manner, the currents at the emitter, base and the collector are generated.

We note that the emitter current I_E generated is the sum of the base I_b current (at the base of the BJT) and the collector I_c current (at the collector of the BJT). The amount of generated base current is less compared to that of the currents generated at the emitter and the collector. The working principle is seen to remain the same for both P-N-P and the N-P-N transistor. However, the only difference between them is there majority charge carriers. Further, in P-N-P the majority of the carriers are holes while in N-P-N the majority of the carriers are the electrons.

THE EQUIVALENT CIRCUIT OF BJT

In discussing transistors, it is noted in a clear way that the formation of the transistor is due to the involvement of the two diodes being connected back to the back of it. So, these diodes are seen to lead to the formation of the two respective junctions of the BJT which in turn, is found to relate to the presence of terminals in the BJT.

The circuit of BJT can be represented by the two diodes with the junction P-N. This represents the equivalent circuit of BJT in **Figure 5**.

BIPOLAR JUNCTION TRANSISTOR BIASING

The process of biasing the bipolar junction transistor is nothing but just the application of the external supply of the voltages to the respective junctions involved in it. This biasing leads further to the major process of the transistor based on which the regions are classified.

Cut-off region

If both the junctions of the transistors are not supplied with any external voltage supply, then there is no evident supply of the voltages seen. The region formed is defined as the cut-off region.

Active region

In the active region, the one junction must be kept in the mode of forwarding bias. That is, the other will be at the reverse bias. So, once again this type of region is referred to as the active region. In this, the q-point will be at the center of the characteristics curve so that it is most frequently used during the operations.

Saturation region

In the saturation region, both the junctions must be at the forward mode that is on highly conducting mode. The transistor can be used as a switch in electronic circuits. During the application of the transistor as a switch, it is noted that the cut- off mode and the saturation modes are preferred. In other words, it is either should function in completely ON mode or the OFF mode.

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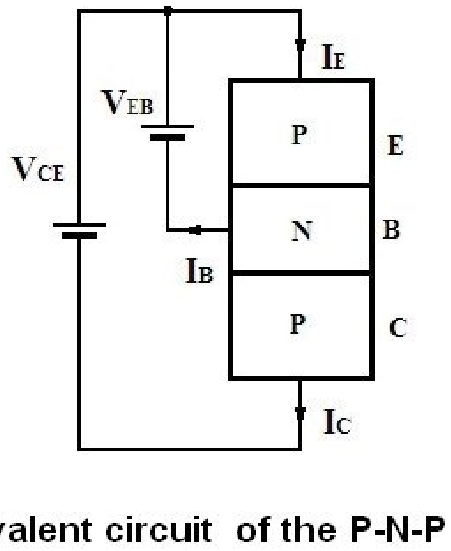

Figure 5. Equivalent circuit of the P-N-P.

Fixed bias

This bias is also referred to as the base bias. In this type of bias, the connection of the single power supply will be maintained between the base and the collector with the help of the two resistors. If the values of the resistance are varied based on it the current at the terminal base can be adjusted. In this way, the Q-point can also be monitored.

Collector-Base bias

In the Collector-Base bias region, the resistor of the base is collected across the collector rather than connecting it to the supply. It is preferred to use this type of bias in the stabilization of the Q-point against the temperature changes. It is further noted that if the collector current I_c tends to increase then there can be the voltage drop at the resistor resulting in the reduction at the base of the resistor. So, which means that incurs a reduction in the value of the voltage across the resistor of the base. That further means that, the current at the base gets reduced simultaneously, the current I_c value at the collector is reduced. Thus, this reduces the effect of the temperature on the Q-point by making it stable.

Self-bias

This self-bias is also referred to as the voltage divider bias. This type of bias is the type most frequently used. Further, in this bias the resistors arranged in the form of potential divider circuitry. Hence, that in turns leads to the fact that, the equal or the fixed amounts of the voltages are supplied to the base terminal. In this way, the biasing techniques for the transistors are classified.

BIPOLAR JUNCTION TRANSISTOR CHARACTERISTICS

The characteristics of BJT depend on their configurations. So, in this way they are classified whether it is of the common emitter, common base, and the common collector. It follows in this manner that one can compare the characteristics for the various configurations of the bipolar junction transistor.

Voltage gain

The voltage gain is found to be as the ratio between the output voltages to the applied input voltage. This voltage gain depends, to a great extent, on the currents generated which are based on its configurations and also the resistors connected across it.

Current gain

The Current gain is the ratio of the currents generated at the output to the input value of any particular transistor being used. One can possibly obtain the highest current gain at the common collector. Further to mention that, the highest current gains are obtained in the common collector configuration with very lesser voltage gain value.

APPLICATIONS OF BJT

BJT may be used in many different electronic purposes. The applications of the bipolar junction transistor are such as being used in:

1. The circuits of amplification.
2. The logic circuits.
3. Multi-vibrator circuits.

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4. The clipping circuits, these are preferred for wave shaping circuits.
5. The oscillation circuits.
6. Switching circuits.

These bipolar junction transistors are constructed in a simpler manner. The above are considered as being the basic classification of the transistors. Basic application of these transistor is frequently in circuit switching.

THE CHARGE AND RELATIVISTIC COMPLEX POWER IN RELATIVISTIC TRANSIENT CIRCUITS

In the following, we investigate how to employ the charge Q when using complex power in transient steady-state and in general, in sinusoidal circuits. That is done by using SI units or alternatively, we can use British system units for conversion of power and energy from one system to another. The same principle is applied to resonant circuits. It must be recalled that, by depending on the charge Q in relativistic circuits, we have by the theory of relativity that the Moaqat Energy-charge equation for any electric circuit, electro-dynamic energy transfer circuits, heat transfer circuits, etc, is:

$\xi = QUc^2$ That is, ξ (in joules) is the energy of any considered particle (s), Q is the charge, U is the relativistic velocity of the same particle (s), under consideration. Further, c is the speed of light, as measured in a vacuum. It must be mentioned, in taking units of the SI system we can express the energy of any particle in terms of the charge Q such that by showing units we have:

$$\xi(\text{in joules}) = Q.U.c^2(\text{m}^3/\text{sec}^3)$$
$$=Q.v.c^2(\text{in gamma. m}^3/\text{sec}^3)$$

It is noted above that the ordinary velocity of any considered particle is associated with the relativistic velocity of the same particle as follows,

$$U = v\gamma = v \text{ gamma meter/second,}$$

Where, we have denoted the coefficient γ of relativity as follows such that,

$$\gamma\text{=gamma}$$
$$\gamma = (1-\beta^2)^{-1/2}$$
$$= [1-(v^2/c^2)]^{-1/2}$$

Further, $\beta = v/c$ [3,5,16]

If the mass m of any particle exists, one can use the Albert Einstein energy-mass equation, which is:

$$\xi = m\gamma c^2$$
$$= Mc^2$$

Where, M (in Einstein Kg) is the relativistic mass of any considered particle [3]. That is the relativistic mass M of any particle is measured in Einstein, kilogram units.

Thus, it is obvious that the charge Q (in Tarek), which has the unit of Tarek, is associated with the energy ξ (in joules) of any particle in space such that,

1 joule=1 Tarek. meter^3.gamma/sec^3

=1 Einstein Kg. meter^2/sec^2

THE CHARGE Q AND COMPLEX POWER S

It is important to recall in any sinusoidal circuit that the electric power S may be expressed in a complex form vector, such that $S = P + jQ$ [25]

The complex power vector S, stated above consists of the real part P and the imaginary part jQ. Now, the real part P expresses the real power part of the complex power vector S. However, the imaginary part jQ, above expressed the real power part of the complex power vector S. That is, from trigonometric functions and 2 dimension-vectors [1,2], we have,

$$\tan\lambda = Q/P$$

It is important to state that the complex power vector can be represented as follows by using the following notation:

$$S = P + jQ$$
$$= P\lambda$$

where, P is the maximum power that can be obtained at an angle λ, which may be referred to as the phase angle [25]. The ordinary proper time t, of any particle is related to the relativistic time of t, in the following manner, such that

$$dt = \gamma dt$$
$$= [1-\beta^2]^{-1/2} dt$$
$$= [1-(v^2/c^2)]^{-1/2} dt$$

In the following, we use 0-rank vectors to express the complex power in any RLC transient, resonant and steady state circuits [5,16]. It becomes evidently apparent by using the charge Q, of the considered particle (s), in any transient or resonant circuit, the complex power vector S is expressed as invariant under any change of basis in space. Thus, the complex power vector is found to be associated with the charge Q as follows such that in invariant 0 rank vectors we have,

$$\frac{S}{c} = \frac{P}{c} + j\frac{Q}{c}$$
$$= \gamma \frac{d}{cdt}[QU\widehat{U}c]$$
$$= \frac{d\xi}{cd\tau} = \gamma \frac{d\xi}{cdt}$$
$$= \text{invariant}$$

We have, $\text{U}=\text{U}\widehat{\text{U}}$

By covariance the previous complex power vector can be written as follows by using 1st rank covariant components, it is found in my research mentioned above that

$$\frac{S}{c} = \gamma \frac{d[QUc]}{dt}$$
$$= \gamma \frac{d\xi}{cdt}$$

Thus, it may be seen in the above that, we can operate with the energy-charge equation listed above by using the charge Q, when dealing with material particles (i.e., particles that have a mass m, such as electrons, etc), or non-material particles (i.e., particles that have no-mass m such as photons, etc.). That is, whether, the mass of any considered particle exists or not. However, if the mass m of any considered particle exists, then we can also use the Albert Einstein energy-mass equation,

$$\xi = m\gamma c^2$$
$$= Mc^2$$

It may be deduced then, by using the above complex power vector that,

$$\frac{S}{c} = \gamma \frac{d[Mc]}{dt}$$
$$= \gamma \frac{d[QUc]}{dt}$$
$$= \gamma \frac{d\xi}{cdt}$$

Regarding the angle λ in the above, to be the phase angle then the frequency is denoted ω, unless otherwise specified [3,18]. That is, for an infinite phase angle λ, ending or non-ending processes, which end at a certain point of time we have,

$$\lambda = \omega / T = 2\pi f$$

having the corresponding radian frequency f. It must be mentioned above that the ordinary time period is related to the relativistic time period T, as follows

$T = \gamma T = [1-\beta^2]^{-1/2} T$ where as usual we have,

$$\gamma = gamma = [1-\beta^2]^{-1/2}$$
$$= [1-(v^2/c^2]^{-1/2}$$

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It is necessary to remember that, the function $P(t) = P_m \sin \omega t$, which occurs every period T of time repeats itself every 2 radians. That is where its period is evidently seen to be as 2π radians. Further, the recurring function $P(t) = P_m \sin \omega t$ may be represented graphically by being plotted as a function of the proper ordinary time t and the period is now T. It follows by recalling that sine wave functions, as having a period T must execute 1/T periods each second; it is then found that its frequency f is 1/T hertz, which may be abbreviated by the notation Hz [3,5,16]. Thus, the frequency

$$f = 1/T = 1/[1-\beta^2]^{-1/2} T$$

Using the ordinary proper time t of the transmission of a signal from any initial position to another and making use of the Albert Einstein's relativistic time t of the same signal from the same initial position to the same other we must recall that:

$$t = 1/[1-\beta^2]^{-1/2} t$$
$$= \gamma t$$

Now for a lengthy infinite parameter of λ, $-\infty < \lambda < +\infty$ [5,16]

(whether, ending at a certain point of time or not) we have,

$$\lambda = \omega t = \omega \gamma t$$
$$= \omega[1-\beta^2]^{-1/2} t$$

Thus, if we arrive to obtain the common relationship between frequency f and the frequency ω.

$$\omega = 2\pi f$$

RELATIVISTIC TRANSIENT CIRCUITS

In the following we recall the general idea of using the theory of relativity and the Moaqat energy-charge equation $\xi = QUc^2$

Once again, we have above, ξ being as the stored energy or the energy sought to be stored, consumed, etc. Further, Q is the charge U is the relativistic velocity of any considered particles and c is the speed of light as measured in a vacuum that is, by using the charge Q, the relativistic velocity U of the particles under consideration in capacitors, inductors, transient charged circuits, etc. In general, the ordinary current I is related to the ordinary voltage V across any capacitor by the following equation: $dV = (1/C)I(t)dt$

where, C is called the capacitance of any corresponding under consideration [25].

INTEGRAL VOLTAGE-CURRENT RELATIONSHIPS

In general, the capacitor voltage V may be expressed in terms of the current by integrating in the current equation stated earlier. We obtain in relativistic form, that the differential element of relativistic voltage dV, across any capacitor is such that, $dV = (1/C)I(t)dt$

[25], C is the relativistic capacitance of the capacitor under consideration and I is the relativistic current crossing along the same capacitor. We note that t is the relativistic time of the transmission of the relativistic signal (current), inside, along the same capacitor. That is, further, the real relativistic time τ, also denoted by t of the transmission of the signal from one position to another, inside the considered capacitor. That is viewed in a clear way from the theory of relativity, where again we note in the previous equation the ordinary time t of signal transmission is related to the real relativistic time t of signal transmission, as follows $dt = [1-\beta^2]^{-1/2} d\tau$

[5] or, we may write the above equation as follows,

$$dt = [1-\beta^2]^{-1/2} d\tau$$
$$= \gamma d\tau$$

[5], where, τ=t-unless otherwise specified.

Now, we can arrive at getting the relativistic power or energy across capacitors, inductors or any circuit element. That may be done by fully replacing ordinary values by relativistic values in any prospective equations, formulas, identities, etc. It follows by relativistic values for voltage and current across the same considered capacitor, we have from above the relativistic voltage equation for the relativistic voltage dV across the same capacitor, the above voltage equation which takes the form, as follows such that,

$$dV = (1/C)I(\tau)d\tau$$ [25]

C is the capacitance of the same capacitor under consideration and the relativistic current I which is crossing along the same capacitor. Further, we have used the Albert Einstein's relativistic time τ for the transmission of the relativistic signal (current) inside along the same capacitor; while t is the ordinary time, across any capacitor, as stated earlier, according to Albert Einstein and the theory of relativity. Then, integrating between the real relativistic time interval of τ_0 and τ_1 and between the corresponding voltages $V(\tau_0)$ and $V(\tau_1)$,

$V = (1/C)\int_{\tau_\circ}^{\tau_1} I(x)dx + v(\tau_\circ)$

where, we have used relativistic values, in the above equation of the relativistic voltage V. The voltage- current equation state in the previous may also be written in the following form as an indefinite integral plus a relativistic constant k of integration such that,

$V(\tau) = (1/C)\int I(\tau)d\tau + k$ [25]

Thus, it is obvious that we have completely expressed the relativistic voltage equation above with relativistic values. In one way or another, the charge Q stored is then calculated in relation to the relativistic voltage V, as follows such that,

$Q = cV$ [25].

Now, by using the theory of relativity and the Moaqat energy-charge equation, we have from above, that the total energy of any particle, $\xi = QUc^2$

Hence, by employing the relativistic velocity of any particle, in any RLC circuit, transient or resonant circuit, we then have, inside in capacitors, that the charge Q is such that, $Q = \xi / Uc^2$

It follows immediately from above that, we must obtain, in terms of voltage and energy stored inside capacitors by using the charge Q stored in capacitors that,

$Q = CV = \xi / Uc^2$

By using the relativistic voltage V, across any capacitor it is seen from above, for any desired energy ξ, sought to be stored in a capacitor we must have the required voltage V, which is necessary to store the amount of energy ξ, in such a capacitor such that:

$V = \xi / CUc^2$

where V is the relativistic voltage across the capacitor and C is the capacitance across the same capacitor. Moreover, we have in terms of the relativistic proper real time τ (unless otherwise specified), that $=\gamma V$ [25].

CHARACTERISTICS OF RELATIVISTIC SINUSOIDS

It must be mentioned, by measures of the ordinary velocity v and the relativistic velocity U of any particle, we have,

$U = V$ gamma

$=\gamma V$

[5], where, γ=gamma. In the same manner for the ordinary frequency ω and the relativistic frequency ω of any signal in space, if we introduce curvilinear coordinates, we arrive at obtaining that,

$\omega = \gamma\omega = \text{gamma } \omega$

Let us consider a sinusoidal varying voltage by using relativistic the frequency ω and the relativistic real time τ, as in the following: $V_{(\tau)} = V_m \sin \omega\tau$

In the above, we show the sinusoidal function $V_{(\tau)} = V_m \sin \omega\tau$ is plotted (a) versus ωτ [25].

Here, we present the sinusoidal function $V_{(\tau)} = V_m \sin \omega\tau$ is plotted versus τ in **Figure 6**.

The sinusoidal relativistic voltage V (τ) is shown graphically above. It must be noted that the amplitude of the relativistic sine wave is V_m, and the argument is ωτ. We refer to the radian frequency or angular frequency by the notation greek letter ω. In the sinusoidal figure shown above, the sinusoidal voltage V_m sin ω is plotted as a function of the argument ωτ and the periodic nature of the sine wave is seen as evident. That is, V_m denotes the maximum voltage. It is important to mention that the function $V(\tau) = V_m \sin \omega\tau$ repeats itself every 2π gamma (in radians) and its period is therefore 2π gamma radians. That is, $V(\tau) = V_m \sin \omega\tau$ is plotted as a function of the proper ordinary time t and the period is now T. So, we recall that, a sine wave having a relativistic period T must execute 1/T periods each second; its frequency f is 1/T hertz, which may be abbreviated by the notation Hz. Thus, the relativistic frequency f=1/T [25] and since, $\omega T = 2\pi f$. we arrive at obtaining the common relationship between frequency f and the radian relativistic frequency ω,

$\omega T = 2\pi f$ gamma

LAGGING AND LEADING IN RELATIVISTIC VOLTAGE

In general, a more used form of the relativistic sinusoid, $V(\tau) = V_m(\sin \omega\tau + \theta)$ where, we have V_m as denoting the maximum voltage. That is, in terms of the time τ, which is relativistic. The equation above includes a phase angle θ in its argument. So, by using graphical representation of sinusoidal lagging or leading voltages in electric circuits, the previous equation can be plotted

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as shown in the **Figure 7**. Hence, it is correct to describe the function curve on left is $V_m \sin\omega\tau$ and the function curve on the right is $V_m \sin\omega\tau$

Here, we present the sinusoidal function $V(\tau) = V_m \sin\omega\tau$ is plotted versus τ as a function of ωτ, and the phase angle appears as the number of radians by which the original sine wave is shifted to the left, or earlier in time. Since corresponding points on the sinusoid function $V(\tau) = V_m \sin\omega\tau$ occur θ rad, or θ/ω seconds earlier, it is the habit to say that $V_m(\sin\omega\tau + \theta)$ leads $V_m \sin\omega\tau$ by the angle θ rad

FORCED RELATIVISTIC SINUSOSIDAL FUNCTIONS AND THE STEADY STATE RESPONSE

It is necessary to recall the notion of forced relativistic sinusoidal functions and the steady-state response. The term steady-state response is usually used synonymously with the term or notion of forced response. That is, circuits of this form that are usually analyzed are commonly said to be in the sinusoidal steady state.

Many students carry in minds, the connotation of not changing with time, for the term steady state. This is true for dc forcing functions. However, the relativistic sinusoidal steady-state response is definitely changing with time τ. So, the steady state as

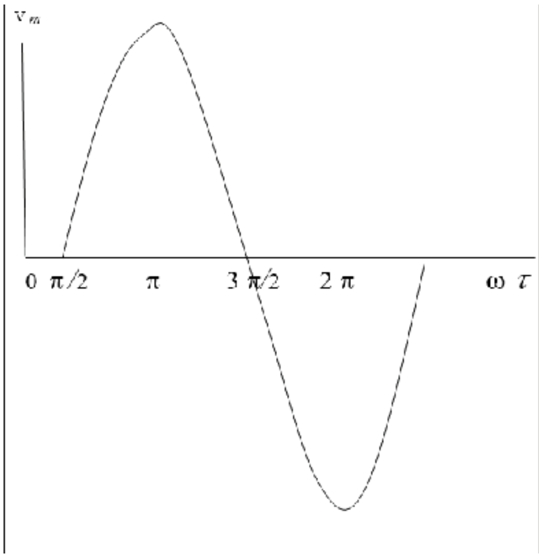

Figure 6. Representation of sinusoidal function.

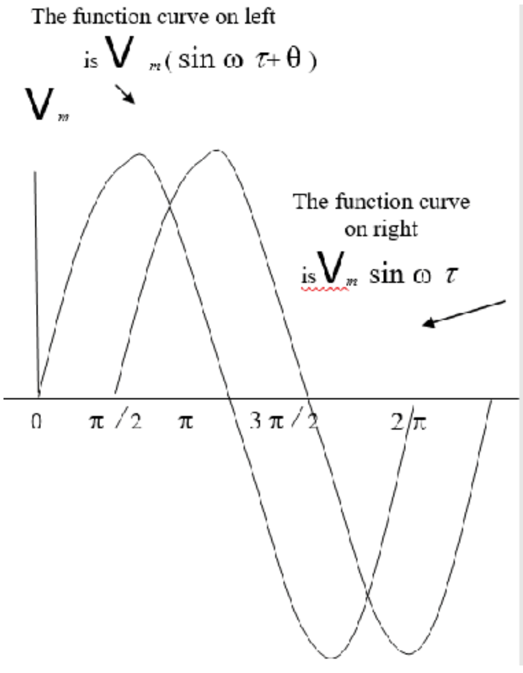

Figure 7. Presentation of sinusoidal function $V(\tau) = V_m \sin\omega\tau$ is plotted versus τ.

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simply referring to the condition which is reached after the transient or natural response has died out [25].

It must be noted that, the forced response has the mathematical form of the forcing function, plus all its derivatives and its first integral. By this knowledge, it is obvious that one of the methods by which the forced response may be found is to assume a solution composed of a sum of such functions, where each function has an unknown amplitude to be determined by direct substitution in the differential equation.

This can be a lengthy process, so we will be sufficiently motivated to seek out a simpler alternative. For example, let us consider the series RL circuit which can be represented in sinusoidal figure representation [25]. In the above figure, the sinusoidal source voltage Vs (τ), such that, $V_s(\tau) = V_m \cos\omega\tau$

Further, the sinusoidal relativistic voltage Vs(τ) has been switched into the circuit at some remote time in the past and the natural response has died out completely. Using the resistance, R of any electric wire, the relativistic inductance L, of inductors we can seek the forced (or "steady-state") response, which must satisfy the differential equation $Ldi/d\tau + Ri = V_m \cos\omega\tau$

where, i is the relativistic current in the loop. The equation stated above is now fully and completely in relativistic. Thus, such equation can be obtained in a relativistic differential equation as shown above by applying KVL (Kirchhoff's Voltage or current Law) around the simple loop [25]. These types of sinusoidal, transient, step-response, step functions circuit can be solved by using the principle of relativity. That is, by employing the energy-charge equation also called the Moaqat energy-charge equation, which is: $\xi = QUc^2$

That is obviously evident whether the mass m of electrons, protons, etc. exist or do not exist. That is further explained here in this analysis since we exclusively depend on the charge of particles, rather than depending on the mass m of any particle in space. For complete details on how to derive and obtain the Moaqat energy-charge equation $\xi = QUc^2$

Four-force and three-force

Let us consider any coordinate system S having the 4 coordinates x_α, in the 4-dimensional space-time, where we have explicitly that,

$$\begin{aligned} x_\alpha &= (x_1, x_2, x_3, x_4) \\ &= (x, y, z, i\ c\ t) \\ &= (x, y, z, il) \end{aligned}$$

where, the 4th component $x_4 = T = i\ c\ t = i\ l$ above expresses the ordinary imaginary light time T, of any particle c under consideration. That is while, l denotes the ordinary real light time $l = c\ t$, in frame S. Further, t is the ordinary proper real time of any particle under consideration in frame S and is the speed of light measured in a vaccum and as measured nowadays, $c = 3\times10^8$ (approx.)

The relativistic proper real time t is considered as being the relativistic time of the same particle under consideration where by using differential elements of time, we have, $dt = \gamma dt$.

Alternatively, we may write the above equation as follows, $dt = [1-\beta^2]^{-1/2} dt$

We shall use the notation τ to express the relativistic real time of any considered particle. So, the previous equation of the relativistic time of any particle under consideration, in space-time, may be express as follows,

$dt = [1-\beta^2]^{-1/2} d\tau$ or $dt = \gamma d\tau$

We note here that we shall use the notation $\begin{aligned} \tau &= t, \text{ and,} \\ d\tau &= dt \end{aligned}$

That is, by using the scalar velocity v, of any considered particle in the above, we have, $\beta = v/c$

Further, $\gamma = \text{gamma} = [1-\beta^2]^{-1/2}$ [5,16].

It must be noted that the only influence on the motion of particles that we have so far considered were collisions and we have come across without recourse to the concept of force. But it is seen that it plays an important role in relativistic mechanics as in the situation when fast charged particles move through an electromagnetic field. It is clear then that even without such practical need it would be desirable to have a relativistic version of the force concept, so that relativistic mechanics might contain all of relativistic Newtonian mechanics in a suitable limit. The ordinary velocity of any particle may be resolved into 4 components V_α. We note here that ordinary 4 velocity V_α, of any corresponding particle in motion in any cartesian orthonormal frame (coordinate system) S space, is obtained by differentiating the position coordinates x_α, of any considered particle with respect to the ordinary real time t, where

$$V_\alpha = dx_\alpha / dt$$
$$= (V_i, ic)$$
$$= (V_i, V_4)$$

α=1,2,3,4, and i=1,2,3

We note that the 4th component v_4 of the ordinary 4 velocity v_α of any particle in motion in space-time is such that $V_4 = dx_4 / dt = i\ c$

Further, we have the ordinary 4 velocity of any considered particle in motion expressed as follows,

$$V_\alpha = dx_\alpha / dt$$
$$= (dx_1 / dt, dx_2 / dt, dx_3 / dt, dx_4 / dt)$$
$$= (dx / dt, dy / dt, dz / dt, di\ c\ t / dt)$$
$$= [dx / dt, dy / dt, dz / dt, d(i\ l) / dt]$$

The 4th component v_4 of the ordinary velocity v_α, of any particle is $V_4 = dx_4 / dt = i\ c$

We note that the 4 relativistic velocity U_α of any considered particle in motion is related to the ordinary 4-velocity of the same particle such that, $U_\alpha = \gamma V_\alpha = dx_\alpha / d\tau$ [5,16]. Further, we can resolve the scalar relativistic velocity into 4 components U_α such that,

$$U_\alpha = (U_i, i\gamma c)$$
$$= (U_i, U_4)$$

where, the 4th component U_4 of the relativistic 4-velocity $U_\alpha = (U_i, i\gamma c)$ is such that $U_4 = i\gamma c$

α=1,2,3,4, and i=1,2,3, [5,16].

Using cartesian orthonormal coordinates, it is important to state by using bold notation for 0-rank vectors that the 4-momentum P of any material particle having a rest mass m, can be expressed by the 3-momentum p of the same particle as follows such that:

$$P = mU$$
$$= (mU, imc)$$
$$= (mU, i\gamma mc)$$

or we can use 1st rank covariant components P_α, in the 4-Dimensional space-time to express the momentum of any such material particle as follows:

$$P_\alpha = mU_\alpha$$
$$= (mU_i, imc)$$
$$= (mU_i, i\gamma mc)$$

By resolving the scalar momentum P of any considered particle into 4 components, we can write $P = (p, imc)$ [3,5,16]. However, by covariance we can write the 4-momentum in vector form as follows:

$$P_\alpha = (p_i, imc)$$
$$= (p_i, P_4)$$

α=1,2,3,4, and i=1,2,3

where, the 4th component of the 4-momentum of any considered particle is $P_4 = imc$ and the relativistic mass m of any particle under consideration is expressed as follows such that: $m = \gamma m = \gamma m_\circ$

When using a rest frame for any particle having the rest mass then we change the values of the above indices, as taking the following values such that, α=0,1,2,3 and i=1,2,3

For example, for the ordinary 4-velocity in a rest frame we will have

$$V_\alpha = dx_\alpha / dt$$
$$= (ic, v_i)$$
$$= (V_\circ, vi)$$
$$P_\alpha = (imc, p_i)$$
$$= (P_\circ, pi)$$

α=0,1,2,3 and i=1,2,3, etc. and so forth, for other 4-vectors when using the indices α and i in rest frame. That is, in rest frames indices, where, α and i will have the values, α=0,1,2,3 and i=1,2,3. In cartesian orthonormal coordinates, the relativistic 4-force resulting from the above 4-momentum vector, by using bold notation 0-rank vectors that,

$$F = m_\circ A$$

we may write the above equation as follows by using different notation, $F = mA$

It follows from above, by using 1st rank covariant components A_α of the 4 acceleration we can express the 4 force of any material particle, by using covariant 1st rank components F_α, where, $F_\alpha = mA_\alpha$ Now, Q is the ordinary charge of any particle in any chemical, electric, nuclear, quantum process. That is, m is the ordinary mass of the same particle in such a considered process. It follows by investigating the relativistic values, ξ for the relativistic energy, M for the relativistic mass and Q for the charge may write in cartesian orthonormal coordinates that the 4-force F of any considered particles is expressed as follows

$$F = (dP / d\tau)$$
$$= (f, if[Mc] / d\tau)$$

then we arrive at obtaining that,

$$F_\alpha = \{f_i, d[iMc] / d\tau\}$$
$$= \{f_i, d[i\gamma Mc] / d\tau\}$$
$$= \{f_i, d[iQUc] / d\tau\}$$
$$= \{f_i, d[i\gamma QUc] / d\tau\}$$
$$= (f_i, F_4)$$

Where, U is the relativistic scalar velocity of the same considered particle and, $U = \gamma V = \text{gamma } V$

where M is the relativistic mass of any considered particle such that, $M = \gamma M$

Further, Q is the relativistic charge of the same considered particle in any electric, chemical, quantum process, etc., such that $Q = \gamma Q$, α=1,2,3,4, and i=1,2,3, is the relativistic mass of any particle under consideration.

It is seen that there are at hand essentially only two reasonable definitions for the 4-force F on a particle of rest mass $m=m_o$, 4-acceleration A and 4-momentum P:

$$F = m_\circ A \text{ or}$$
$$F = dP / d\tau$$

But P is a more fundamental quantity than A, so we choose as the more promising equation of relativistic mechanics of motion for any particle [13,16] where,

$$F = \frac{dP}{d\tau} = \frac{d}{d\tau}(mU)$$
$$= mA + \frac{dm}{d\tau}U$$

So, by covariance we have from above in 1st rank vectors, in any cartesian frame S, that the above will take the form as follows,

$$F_1 + F_2 = \frac{d}{d\tau}(P_1 + P_2)$$

Then, as long as we have no knowledge of specific 4-forces in the previous equation must indeed be regarded as a mere definition. However, it certainly satisfies the desideratum that in the absence of a force, P remains constant. And when we later find that the Lorentz force of Maxwell's theory fits into this pattern, it will become a law. In Newton's theory, the conservation of momentum is a consequence of Newton's second and third laws. In our scheme, by contrast, a limited (though 4- dimensional) analog of Newton's third law is a consequence of momentum conservation and the definition as introduced above, that is, during the contact phase of a collision of two compound particles their proper times are the same, and of course, the sum of their 4-momenta P_1, P_2 remains constant. So, if F_1 and F_2 are the respective contact 4-forces on the two particles, we have

$$F_1 + F_2 = \frac{d}{d\tau}(P_1 + P_2)$$

From the above equations and by the principle of momentum energy we find that,

$$F = \frac{d}{d\tau}P$$
$$= \gamma \frac{dP}{dt}$$
$$= \gamma(u)\frac{d}{dt}(p, iMc)$$

In accounting for the relativistic power $id\xi / cd\tau$, the rate of change of the energy of any particle per unit time we must get by invariance of differential forms the following equation of the power, as stated below. That is, by using relativistic values of the energy ξ and the relativistic time τ, it is then found that we must arrive at obtaining [13]

$$F = (dP / d\tau)$$
$$= (f, id[i\xi] / cd\tau)$$
$$= (f, d[i\gamma\xi] / cd\tau)$$

Where, $\xi = \gamma\xi$, hence, we must obtain from above that

$$F_\alpha = \{f_i, d[iMc] / d\tau\}$$
$$= (f_i, d[i\gamma\xi] / cd\tau)$$
$$= \{f_i, d[i\gamma Mc] / d\tau\}$$
$$= \{f_i, d[iQUc] / d\tau\}$$
$$= \{f_i, \gamma d[i\gamma QUc] / dt\}$$
$$= (f_i, \gamma d[i\gamma\xi] / cdt)$$
$$= \{f_i, \gamma d[i\gamma Mc] / dt\}$$
$$= (f_i, F_4)$$

where, $Q = \gamma Q$

It is obvious that, we have introduced the relativistic 3-force f defined by

$$f = dP / d\tau$$
$$= d(mu) / d\tau$$

It is clearly apparent in the above equation that the power which is characterized by the rate of flow of the energy is obtained from the above equation as follows such that,

$$F_4 = \gamma d[i\gamma\xi] / cdt$$
$$= \gamma d[i\gamma QUc] / dt$$
$$= \gamma d[i\gamma Mc] / dt$$

where, at the existence of any material particle under consideration we have, M as denoting the relativistic mass of the same considered material particle. It may be seen upon integration and considering the limits of integration in the above equation we must then obtain for the energy ξ of any particle under consideration that, $\xi = QUc^2$ and also simultaneously, $\xi = Mc^2$ [3,5,16].

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ENERGY FLOW AND THE POYNTING VECTOR

In this section, we investigated the 4-poynting vector in the coordinate system S which was presented earlier in the analysis. It must be noted that energy flow of any system can be characterized by the existence of the Poynting vector. That may be done by the use of Gauss' theorem and the poynting theorem. The energy emitted as it crosses any cross-sectional area in a surface of any electrically charged wire or any electrically charged volume is measured by the use of the Poynting vector S stated below. In the 4-Dimension space-time, the Poynting vector S is the vector resulting from the cross product of the electric field 4-vector E and the magnetic field 4-vector B, taking the form as follows such that: $S = E \times B$

Again, by covariance we have in 0 rank vectors that the 0-rank vector S, resulting from the 4-cross product of the electric field E and the magnetic field B, taking the following form such that by using 1st rank covariant components S^α, we have the Poynting Vector $S = E \times B = S_\alpha e^\alpha$ relative to the 1st contravariant basis e^α or we may write the Poynting vector as follows, by using contravariant 1st rank components S^α as follows $S = E \times B = S^\alpha e_\alpha$ relative to the 1st rank covariant basis e_α [3,13].

That is, in 1st rank tensor form we can describe the Poynting vector as follows by using indices: $S_\alpha = {}_{\varepsilon\alpha\beta\gamma} E_\beta B_\gamma$, α,β,γ= 1,2,3,4, where εαβγ is the usual permutation tensor in the 4-Dimension space-time [1,12].

That is to emphasize that by using Gauss theorem and Poynting theorem, we obtain the energy per unit time crossing any vector surface da in any electric wire or any path in any volume v. So that, by the Poynting theorem, we have by integration that the power (i.e. the energy per unit time) generated from the Poynting vector is

$$\int S.da = \int S.nda,$$
$$da = nda$$

The above equation is written as follows, by using vector indices

$$\int S^\alpha da_\alpha = \int S^\alpha n_\alpha da,$$
$$da_\alpha = n_\alpha da$$

where, we have, n as being the 4-unit vector normal to any surface area element da, along the said path. So that, in 1st rank vector form, we can express the energy flowing per unit time through any surface da_α, any electrically charged wire or any volume as follows $\int S^\alpha da_\alpha = \text{invariant}$ [13].

Then the ordinary derivative of the Poynting vector, with respect to time τ, is noticed to be as follows:

$$\frac{dS}{dl} = \frac{d}{dl}(E \times B)$$

Considering the time variation of the Poynting in our 2nd coordinate system S, and differentiating covariant [3,4], we have by the intrinsic derivative that:

$$\frac{\partial S}{\partial l} = \frac{\partial}{\partial l}(E \times B)$$

Using an alternative notation for the above, we may write the previous equation in the following form, by using the gradient operator (the del operator):

$$\frac{\nabla S}{d\tau} = \frac{\nabla (E \times B)}{d\tau}$$

where B is the magnetic field density vector in 0-rank tensor form, and H denotes the magnetic field 0- rank vector. Using the permeability of free space constant μ, we have in isotropic media, $B = \mu H$ [13]. Now, in our 2nd coordinate system S in space-time M_4, the 4-vector momentum flux is associated with the Poynting vector which is the 4-vector S^α by the momentum density 4-vector g^α ith the 4-Poynting vector and is obtained as follows:

$$g^\alpha = \frac{S^\alpha}{c^2}$$ (14), p.232.

We have upon one transformation of the Poynting vector, in the direction of the field H, we have that: $S'_\alpha = S.\widehat{H}_\alpha$

It follows from above that the following integral quantity is invariant, such that $\int S^\alpha da_\alpha = \text{invariant}$

Then, by the Albert Einstein calculations, we must have upon using one transformation of the coordinates, or change of basis, that the energy per unit time crossing any surface of any electric wire, as invariant under any rotation such that: $\int S'^\alpha da'_\alpha = \int S^\alpha da_\alpha = \text{invariant}$

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