

# The New Calculus

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## Short Communication

**Received:** 22-Mar-2021 Manuscript No. JSMS-23-94907; **Editor assigned:** 25-Mar-2021, Pre QC No. jsms-23-94907 (PQ); **Reviewed:** 08-Apr-2021, QC No. JSMS-23-94907; **Revised:** 31-Mar-2023, Manuscript No. JSMS-23-94907 (R) **Published:** 28-Apr-2023, DOI: 10.4172/J Stats Math Sci.9.1.001.

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**Citation:** Mora C. The New Calculus. J Stats Math Sci. 2023;9:001.

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## DESCRIPTION

The basic problem of differential calculus is the problem of tangent lines and calculating the slope of the tangent line to the graph at a given point P and the less seemingly important problem of defining the vertical asymptote line and its derivative. L'hopitals indeterminate forms and extracting real value of asymptotic equations are easily solved by applying "the logical derivative". These are new derivatives developed using a method of direct proportions. By reversing the decrement, all derivatives derived are of the same dimension as their functions.

**Keywords:** Tangent lines; Derivatives; Asymptotic equations; Logical derivative

Derivatives of increasing functions are similar in nature to increasing functions; and decreasing functions generate derivatives of decreasing nature-logical, but most of all sensible; It is now also possible to compute tangent line equations and also of those with vertical asymptotes <sup>[1]</sup>.

Newton's power rule based on lesser reasoning than required is now extended; summative expressed as a sum of linear terms; and the difference quotient is logically analyzed linear and directly proportional; flat tangent line equations of all functions are computed with the "logical derivative".

Tangent lines for all points on a circle, ellipse, hyperbolas are derived viz. the method presented in my work. A proof for the interdependency between minimum and maximum points for all curves is employed and readily available should you ask for it and require publishing. Newton's hypothesis of "instantaneous rate of change"

remains the solid scaffold on which all calculus is based. The motivation and obsession for seeking further precision in Newton’s voluminous and monumental work [2-6].

Newton’s calculus revised

Logical derivatives

1.  $\frac{d}{dx}(\sin x) = \cos x - \Delta x * \sin x$
2.  $\frac{d}{dx}(\cos x) = \Delta x * \cos x - \sin x$
3.  $\frac{d}{dx}(\tan x) = \frac{(\cos 2x - \sin 2x)}{\cos^2 x}$
4.  $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2} - \Delta x \cdot x}$
5.  $\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2} + \Delta x \cdot x}$

Power rule

$$\frac{d}{dx}(x^n) = \frac{n! \cdot x^{n-k} \cdot \Delta x^{k-1}}{k! \cdot (n-k)!}$$

Irrational functions

$$\frac{d}{dx}(x^{\frac{n}{m}}) = \sum_{k=0}^{k=n} \frac{n! \cdot x^{n-k} \cdot \Delta x^{k-1}}{k! \cdot (n-k)!} / \sum_{K=0}^{K=m} \frac{K! \cdot (m-K)!}{m! \cdot x^{m-K} \cdot \Delta y^{K-1}}$$

The natural logarithm function

$$\frac{d}{dx}(\ln x) = \ln\left(\frac{x+1}{x}\right)$$

Convergence of Newton’s decrement

Let.  $f(x) = \sin x$ .

And its derivative of limits:

$$\frac{d}{dx}(\sin(x)) = \lim_{\Delta x \rightarrow 0} \left[ \frac{\sin(x+\Delta x)}{\Delta x} - \frac{\sin(x)}{\Delta x} \right] = \lim_{\Delta x \rightarrow 0} \left[ \frac{\sin x \cdot \cos \Delta x}{\Delta x} + \frac{\cos x \cdot \sin \Delta x}{\Delta x} - \frac{\sin x}{\Delta x} \right]$$

Rule no. 1

$$\lim_{\Delta x \rightarrow 0} \frac{\cos \Delta x}{\Delta x} - 1 = 0$$

thus, because the

$$\lim_{\Delta x \rightarrow 0} \frac{\cos \Delta x}{\Delta x} = 1$$

then,

$$\cos \Delta x = \Delta x \text{ and } \Delta x^{-1} = 1$$

Rule no. 2

$$\lim_{\Delta x \rightarrow 0} \frac{\sin \Delta x}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{\frac{d}{dx}(\cos \Delta x)}{1} = 1. \text{ ,because of L'hopita's Rule of Indeterminate forms.}$$

Substitution of these values into the difference quotient gives me:

$$\frac{D}{dx}(\sin(x)) = \lim_{\Delta x \rightarrow 0} \left[ \frac{\sin(x+\Delta x)}{\Delta x} - \frac{\sin(x)}{\Delta x} \right] = \cos x + \Delta x * \sin x$$

Derivative of cos x

$$\text{Let } f(x) = \cos x.$$

And its derivative of limits-

$$\frac{d}{dx}(\cos x) = \lim_{\Delta x \rightarrow 0} \left[ \frac{\cos(x+\Delta x)}{\Delta x} - \frac{\cos(x)}{\Delta x} \right] = \lim_{\Delta x \rightarrow 0} \left[ \frac{\cos x * \cos \Delta x}{\Delta x} - \frac{\sin x * \sin \Delta x}{\Delta x} - \frac{\cos x}{\Delta x} \right]$$

Substitution of these values into the difference quotient gives me-

$$\frac{d}{dx}(\cos x) = \lim_{\Delta x \rightarrow 0} \left[ \frac{\cos(x+\Delta x)}{\Delta x} - \frac{\cos(x)}{\Delta x} \right] = \Delta x * \cos x - \sin x$$

Derivative of tan x

$$\begin{aligned} \frac{d}{dx}(\tan x) &= \lim_{\Delta x \rightarrow 0} \left[ \frac{\sin(x+\Delta x)}{\cos(x+\Delta x)} - \frac{\sin x}{\cos x} \right] = \lim_{\Delta x \rightarrow 0} \left[ \frac{\frac{\sin x * \cos \Delta x + \cos x * \sin \Delta x}{\Delta x}}{\frac{\cos x * \cos \Delta x - \sin x * \sin \Delta x}{\Delta x}} - \frac{\frac{\sin x}{\cos x}}{\Delta x} \right] = \\ \frac{d}{dx}(\tan x) &= \lim_{\Delta x \rightarrow 0} \left[ \frac{\frac{\cos x * \sin x * \cos \Delta x}{\Delta x} + \frac{\cos x^2 * \sin \Delta x}{\Delta x}}{\frac{\cos x^2}{\Delta x} - \frac{\cos x \sin x * \sin \Delta x}{\Delta x}} - \frac{\frac{\sin x * \cos x}{\cos x^2} - \frac{\sin x * \sin x * \sin \Delta x}{\Delta x}}{\Delta x} \right] = \\ \frac{d}{dx}(\tan x) &= \lim_{\Delta x \rightarrow 0} \left[ \frac{\frac{\cos^2 x * \sin \Delta x}{\Delta x} - \frac{\sin^2 x * \sin \Delta x}{\Delta x} + 2 * \sin x * \cos x}{\cos^2 x} \right] \\ \frac{d}{dx}(\tan x) &= - \left[ \frac{\cos 2x - \sin 2x}{\cos^2 x} \right] \end{aligned}$$

Proof of Newton's rule

Math induction

Let η be a positive integer. It is required that

$$\frac{d(r^\eta)}{dx} = \eta r^{\eta-1}$$

Obviously, as η=1, then substitution grants us:

$$\frac{d(r^1)}{dx} = (1) \cdot r^{1-1}$$

For Newton's positive integer of η=1;

For any positive integer η=k+1

$$\frac{d(r^{k+1})}{dx} = \frac{d}{dx}(r \cdot r^k)$$

Because

$$r^k \cdot \frac{d}{dx}(r) + r \cdot \frac{d}{dx}(r^k) = r^k + k \cdot r \cdot r^{k-1} = r^k + k \cdot r^k$$

This equals

$$r^k \cdot (k + 1) = (k + 1) \cdot r^{(k+1)-1}$$

And true for all positive integers greater or at most equal to m=1.

The power rule

Let's assume

$$P(x) = r^n$$

It's first derivative definition expressed as

$$\frac{d}{dr} P(x) = \lim_{\Delta r \rightarrow 0} \left[ \frac{(r+\Delta r)^n}{\Delta r} - \frac{r^n}{\Delta r} \right]$$

The expression in brackets, expanded binomially gives me

$$\frac{dP}{dr} \sim \sum_{k=0}^{k=n} (n, k) \cdot r^{n-k} \cdot \Delta r - \frac{r^n}{\Delta r}$$

Further expansion results; thus,

$$\frac{dr^n}{dx} = \lim_{\Delta \rightarrow 0} \left[ \sum_{k=0}^{k=n} (n, k) \cdot r^{n-k} \cdot \Delta r^{k-1} - r^0 \cdot r^{-1} \right]$$

Extracting the first term gives me:

$$\frac{d(r^n)}{dx} = \lim_{\Delta r \rightarrow 0} \left[ (n, 1) \cdot r^n \cdot r^0 + \sum_{k=0}^{k=n} r^{n-k} \cdot r \right]$$

Which simplifies and equals

$$\lim_{r \rightarrow 0} \left[ \Delta r^{-1} \cdot r^n + (n, k) \sum_{k=0}^{k=n} \Delta r^{k-1} \cdot r^{n-k} - r^n \cdot \Delta r^{n-1} \right]$$

First and last terms cancel; thus,

$$\lim_{\Delta r \rightarrow 0} \left[ \sum_{k=0}^{k=n} (n, k) \cdot r^{n-k} \cdot \Delta r^{k-1} \right]$$

and

$$(n, 1) \cdot r^{n-1} \cdot \Delta r^{1-1} + \lim_{\Delta r \rightarrow 0} \left[ \sum_{k=0}^{k=n} (n, 2) \cdot r^{n-k} \cdot \Delta r^{k-1} \right]$$

Because Δr=0. However, additional analysis requires increasing our rationale of newton's logical deductions. I call this new logic-"The Logical Derivative" which increases the precision of Newton's overall results. Further analysis and assuming a convergent value of less than 1 of the decrement due to a geometric expansion with differences in the computed derivatives grants enough justification to call this new set of rules (Figures 1-7).

The new calculus

The right side of the expression cancels; thus, we have

$$\frac{d}{dx} (r^\eta) = \eta \cdot r^{\eta-1}$$

Now, let  $F(x, \Delta x)$  represent the derivative of  $(x + \Delta x)^n$ ; that is-

$$F(x, \Delta x) = \frac{d(x+\Delta x)^n}{dx}$$

A Taylor series representation of the first derivative of  $x^n$ -

$$\sum_{k=0}^{k=\infty} \frac{a_k}{k!} * X^{n-k}$$

And Taylor series coefficients

$$a_k(0) = \frac{f^k(0)}{k!}$$

Let

$$u = (x + \Delta x)$$

And

$$u^n = (x + \Delta x)^n$$

Factoring  $\Delta x^n$  from inside the parenthesis gives me-

$$u^n = \Delta x^n * (1 + x * \Delta x^{-1})^n = \Delta x^n * (1 + x)^n$$

Differentiating left and right sides gives me-

$$\frac{d(u)^n}{dx} = \Delta x^n * \frac{d(1+x)^n}{dx} = n * \Delta x^n * (1 + x)^{n-1}$$

Setting  $X=0$  gives me

$$\frac{d}{dx} (x + \Delta x)^n = n * \Delta x^n$$

Differentiating successively gives me, generally,

$$\frac{d^k}{dx^k} (x + \Delta x)^k = n * (n - 1) * (n - 2) * \dots \dots \Delta x^k = n! * \Delta x^k$$

The expression on the right side is proportional to the  $k - th$  derivative- it's variance.

$\Delta x$  was differentiated zero times and thus must be proportional  $k=n$ . The Taylor series representation of the derivative of the  $n$  degree curve is therefore,

$$\frac{d(x+\Delta x)^n}{dx} = \sum_{k=0}^{k=\infty} \frac{n! * X^{n-k} * \Delta x^{k-1}}{k! * (n-k)!}$$

**The power rule theorem. A logical extension of Newton's calculus.**

**The derivative of the natural Log function.**

Let  $\xi(x) = Ln x$  with derivative defined as -

$$\lim_{n \rightarrow 0} \left[ Ln \left( \frac{(x+\Delta x)}{\Delta x} \right) - Ln \left( \frac{x}{\Delta x} \right) \right] = \lim_{n \rightarrow 0} \left[ Ln \left( \frac{(x+\Delta x)}{x} \right) \right]$$

Extracting the decrement of the numerator gives me-

$$= \lim_{\Delta x \rightarrow 0} \left[ \text{Ln} \left( \frac{\Delta x * (x * \Delta x^{-1} + 1)}{x} \right) \right] = \lim_{\Delta x \rightarrow 0} \left[ \text{Ln} \Delta x + \text{Ln} \left( \frac{x+1}{x} \right) \right] = \text{Ln} \left( \frac{x+1}{x} \right)$$

Because  $\text{Ln} (\Delta x^{-1})^{-1} \sim 0$  since  $\Delta x^{-1} = 0$  of above.

**Example**

Let  $X(x) = \sqrt{x}$ .

To find the tangent line at the origin of the graph. Square both sides to get:

$$X^2(x) = x$$

Differentiating both sides, implicitly for  $n=2$  at the left side between  $0 \leq k \leq 2$

$$\frac{d}{dx} (X^2(x)) = \frac{2! * X^{2-0} * \Delta x^{0-1}}{0! * (2-0)!} + \frac{2! * X^{2-1} * \Delta x^{1-1}}{1! * (2-1)!} + \frac{2! * X^{2-2} * \Delta x^{2-1}}{2! * (2-2)!} - \frac{X^2}{\Delta x^1}$$

Power rule

First and last terms cancel

Thus,

$$\frac{d}{dx} (X^2(x)) = (2x + \Delta x) * \frac{d(x(x))}{dx} = (2x + .1) * \frac{d(x(x))}{dx} = 1$$

Dividing by  $(2x + 1)$  gives me-

$$\frac{d(X(x))}{dx} = \frac{1}{2x+1}$$

To compute the tangent line at the origin, we set  $x=0$  and thus, the derivative at the origin becomes-

$$\frac{d(X(x))}{dx} = \frac{1}{.1} = 10$$

The rate of change at the origin of  $\sqrt{x}$ .

The tangent line equation is expressed as of  $\sqrt{x}$

$$T(x) = 10x$$

Newton's power rule cannot explicitly compute the derivative of  $\sqrt{x}$  at its origin.

Accordingly, by the power rule theorem.

See graph below.

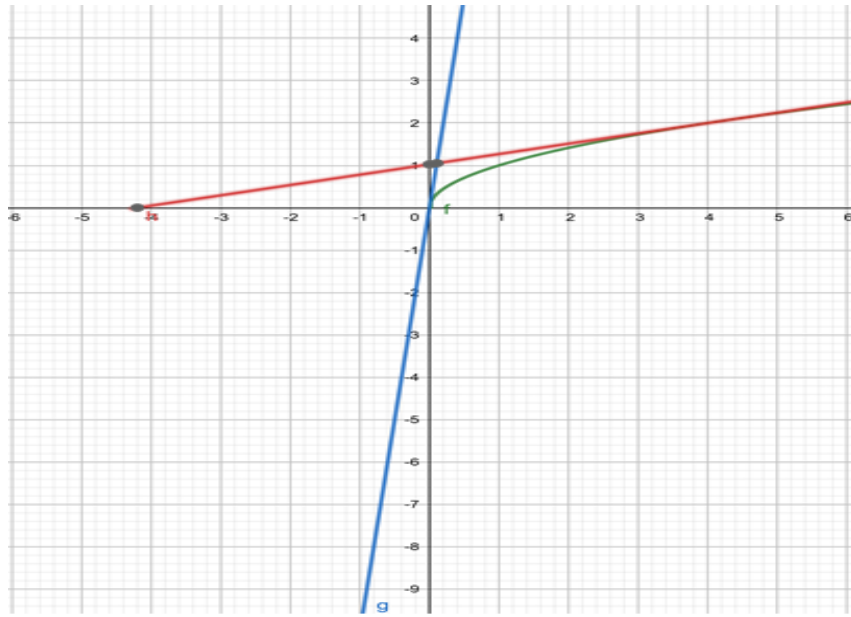
**At  $x = 4$ , the rate of change as described above is equal to  $\frac{1}{8.1} = \frac{10}{81} \sim .125$  and thus,**

The tangent line at 4 is expressed as

$$\frac{10}{81}(x - 4) + 2$$

See below with the tangent lines to the graph at  $x = 0$  and  $x = 4$ ;

Example:



**Figure 1.** The tangent lines to the graph at x=0 and x=4.

To find the tangent line at the origin of the graph I square both sides to get:

$$Y^2(x) = x^4$$

Differentiating both sides, implicitly for n=2; at the left side between  $0 \leq k \leq 2$

The derivative consists of 4 terms; thus,

$$\frac{d}{dx}(Y^2(x)) = \frac{4! \cdot X^{4-0} \cdot \Delta x^{0-1}}{0! \cdot (4-0)!} + \frac{4! \cdot X^{4-1} \cdot \Delta x^{1-1}}{1! \cdot (4-1)!} + \frac{4! \cdot X^{4-2} \cdot \Delta x^{2-1}}{2! \cdot (4-2)!} + \frac{4! \cdot X^{4-3} \cdot \Delta x^{3-1}}{3! \cdot (4-3)!} + \frac{4! \cdot X^{4-4} \cdot \Delta x^{4-1}}{4! \cdot (4-4)!} - \frac{X^4}{\Delta x^1}$$

Power rule

The derivative of  $Y(x)^2$  is

$$2Y + .1$$

And therefore dividing the left term with the term on the right side gives me

$$\frac{d}{dx}(Y(x)) = \frac{4 \cdot x^3 + .6 \cdot x^2 + .04 \cdot x + .001}{2 \cdot Y + .1}$$

Therefore, the derivative of the parabola at the origin at x=0; y=0;

$$\frac{d}{dx}(Y(x)) = \frac{.001}{.1} = \frac{1}{100}$$

The flat-tangent line equation with a rate of change of 1/100 is expressed as:

$$T(x) = \frac{1}{100}x ; \text{graph 2}$$

If x= 2; then, the rate of change at that instantaneous point of the curve is:

$$\frac{34.481}{8.1}(x - 2) + 4$$

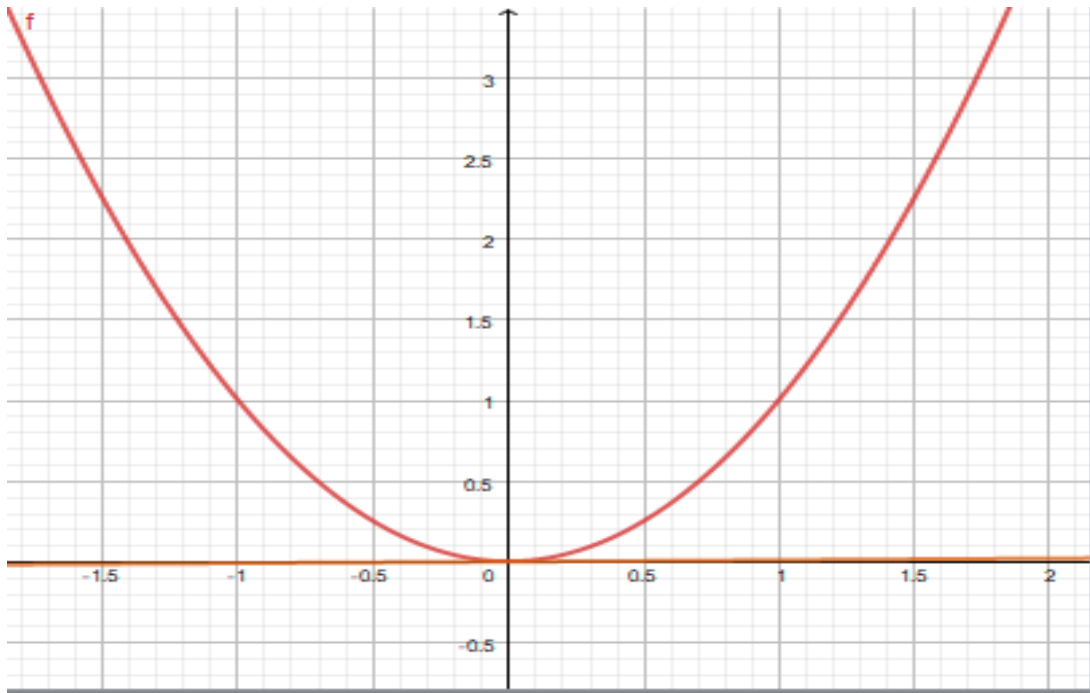


Figure 2. THE instantaneous point of the curve.

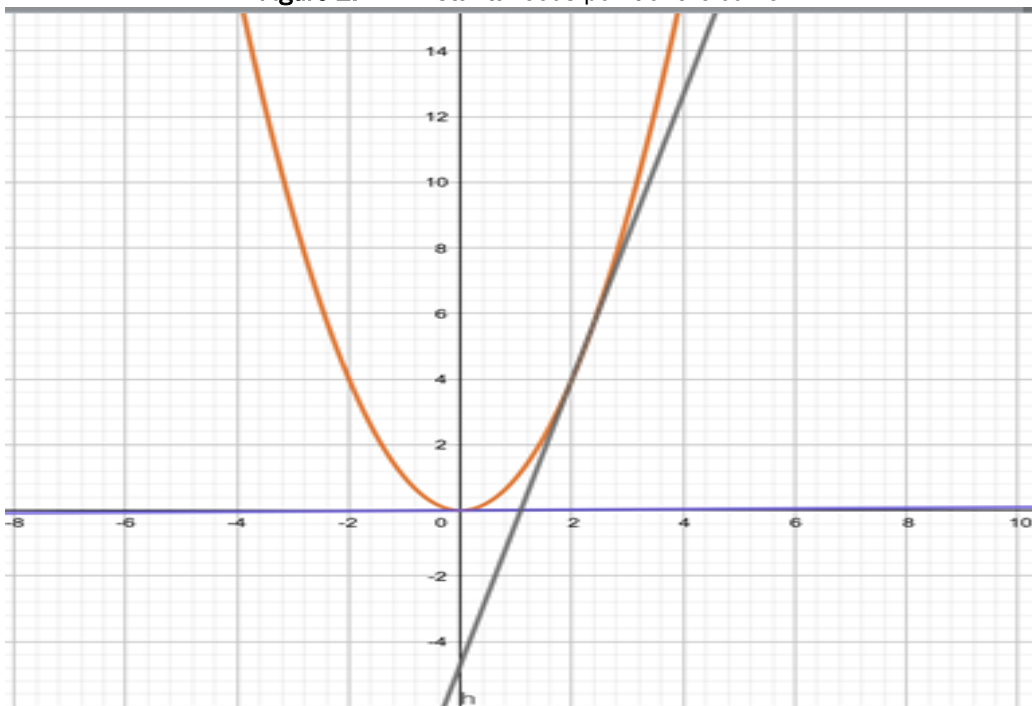


Figure 3. The tangent line at the origin.

Let  $f(x) = x^{\frac{4}{3}}$

Cubing both sides, gives me

$$f(x)^3 = x^4$$

Differentiating implicitly both sides grants a derivative of  $y$  on the left side with an explicit derivative of  $x$  on the right side of the function.

That gives me

$$(3 * f(x)^2 + .3 * f(x) + .01) * \left(\frac{dy}{dx}\right) = 4 * x^3 + .6x^2 + .04 * x^1 + .001$$

The derivative



$$\frac{d}{dx} (x^{\frac{4}{3}}) = \frac{4*x^3 + .6*x^2 + .04*x^1 + .001}{(3*f(x)^2 + .3*f(x) + .01)}$$

Power rule theorem

The tangent line at the origin is equal to

$$\frac{d}{dx} (x_0^{\frac{4}{3}}) = \frac{.001}{.01} = .1 = \frac{1}{10}$$

Tangent line equation of the irrational function is:

$$T(x) = \frac{1}{10} x$$

At  $x = 2$ , the tangent line is equal to

$$\frac{4*8 + .6*4 + .04*2 + .001}{3*2^{\frac{8}{3}} + .3*2^{\frac{4}{3}} + .01} = \frac{34.481}{19.05 + .7588 + .01} = 1.7398127$$

with tangent line equation

$$G(x) = 1.7398127(x - 2) + 2^{\frac{4}{3}}$$

See graph below for tangent lines of  $x^{\frac{4}{3}}$

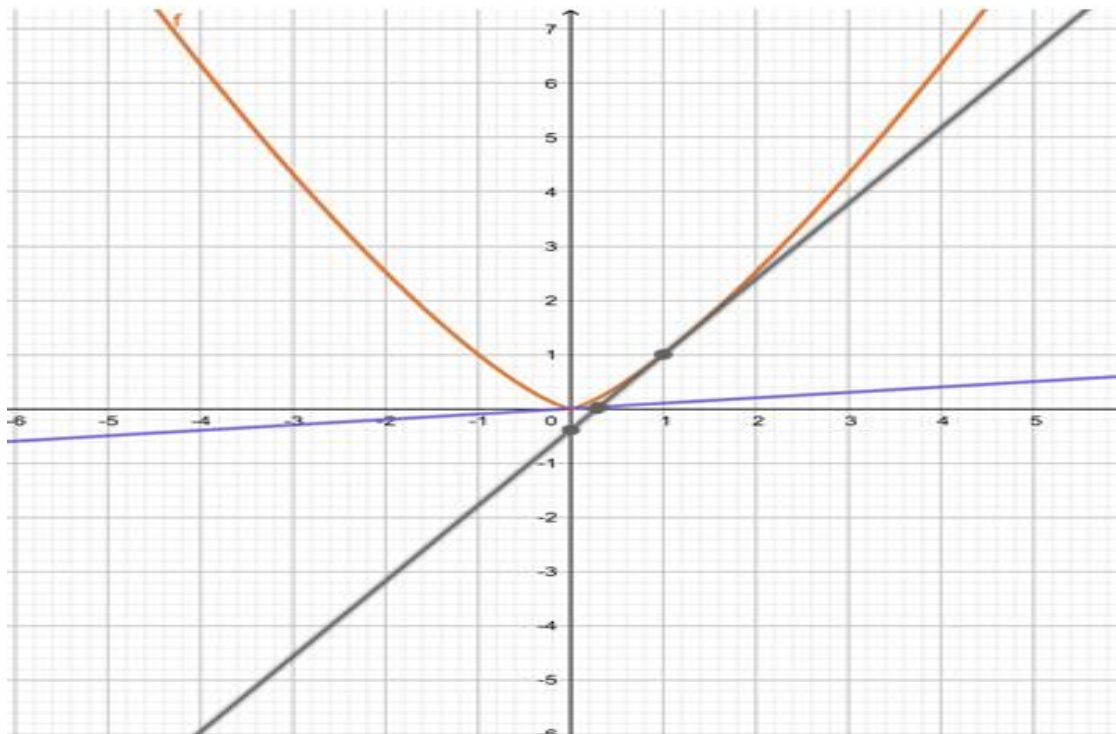


Figure 4. Tangent line equation at  $x=1$ .

The derivative of the circle

Let

$$x^2 + y^2 = 16$$

Differentiating implicitly, the left and right sides, gives me:

$$(2x + .1) + (2y + .1) * \frac{dy}{dx} = 0$$

Dividing through with  $(2x + .1)$  and  $(2y + .1)$  gives me,

$$\frac{dy}{dx} = -\frac{2x+.1}{2y+.1}$$

At  $(0,4)$ , the ratio of the perpendicular to horizontal equals  $1/8.1$  with tangent line

$$T(x) = \frac{.1}{8.1}(x) + 4 = \frac{1}{.81}x + 4$$

And that's above the origin; 90 positive degrees from the positive horizontal axis.  
At  $(0,-4)$  the horizontal measurement equals

$$2 * (-4) + .1 = -7.9$$

with a similar perpendicular measurement of 0, zero.  
Thus, the tangent line equation is

$$-\frac{.1}{7.9}(x) - 4 = -\frac{1}{.79}x - 4$$

Now, let's compute the tangent line at the right and the left edge of the circle:  $x=4$  and at  $x=-4$ .  
At  $x=4$ , the tangent line equation is a vertical asymptote line equation of

$$T(x) = \frac{8.1}{.1}(x - 4) = \frac{81}{1}(x - 4)$$

$$T(x) = -\frac{7.9}{.1}(x + 4) = -\frac{79}{1}(x + 4)$$

To explain the convergence of the tangent one to the asymptotic line, I assume the ratio of two sides of inverse tangent angle; that

$$\text{Is if } \tan \varphi = -\frac{8.1}{.1} \rightarrow \frac{\text{ratio of perpendicular}}{\text{horizontal}}$$

To compute the angle  $\varphi$ , we compute the inverse tangent of the ratio of the two sides.

$$\tan^{-1}(\tan \varphi) = \varphi = \tan^{-1}\left(-\frac{8.1}{.1}\right) = |-1.56^\circ| = 1.56^\circ$$

Now, we compute the hypotenuse of our right triangle because of the perpendicularity of the vertical asymptote line:

$$\sqrt{(-8.1)^2 + (.1)^2} = 8.1006172$$

Which is the measurement of the hypotenuse opposite to our perpendicular asymptote line which differs in measurement equal to .0006172. Thus, the perpendicular of our right triangle equal in measurement to the hypotenuse by less than .0006172 is equal to the vertical asymptote line with horizontal line equal to .1 units. Unit-wise the ratio of the two sides is 1:.0125.

$$\text{At } x = 2, T(x) = \frac{4.1}{4*\sqrt{3}+.1}(x - 2) + 2 * \sqrt{3} \text{ and at } x=-2, T(x) = \frac{-3.9}{4*\sqrt{3}+.1}(x + 2) - 2 * \sqrt{3};$$

The corresponding tangent lines are graphed on the subsequent graphs following the vertical asymptote lines of Newton's Circle.



Figure 5. The vertical asymptote lines of Newton's circle.

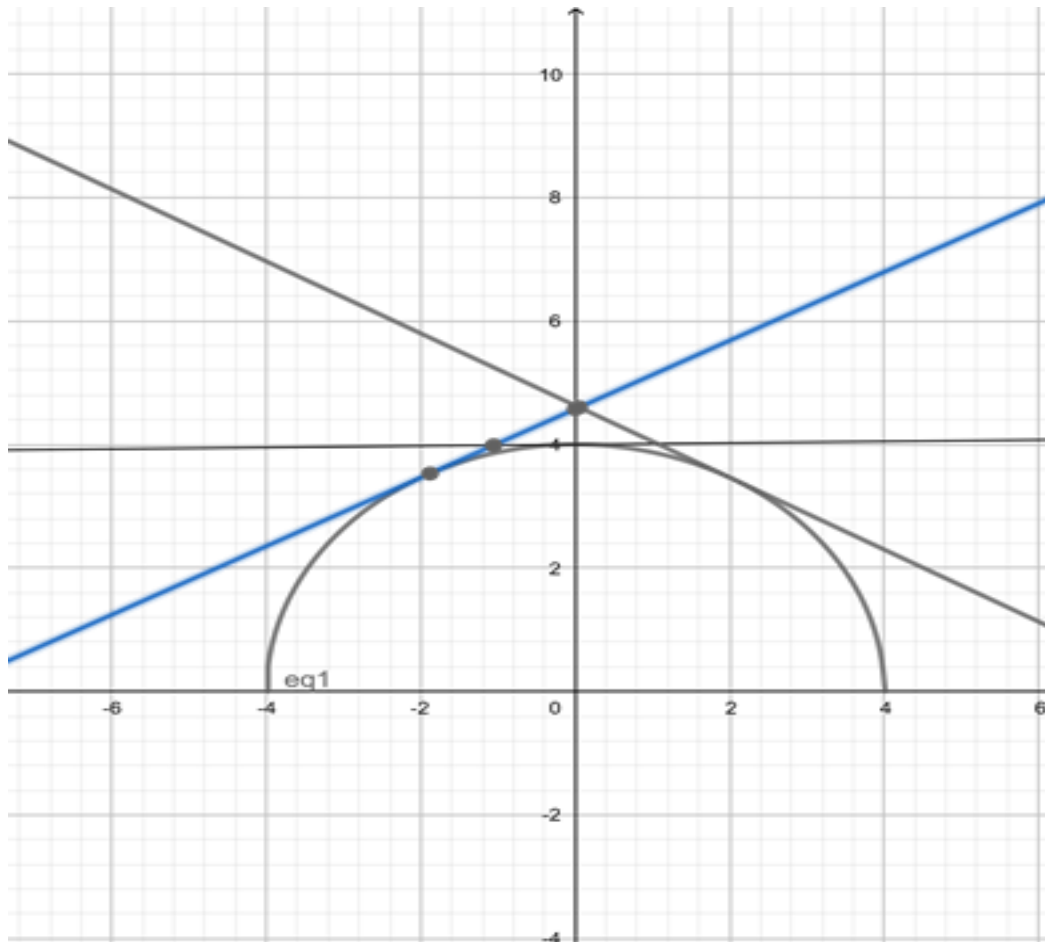


Figure 6. Horizontal tangent lines.

Find the horizontal tangent line equation of

$$\sqrt{x - 1} \text{ at } (1, 0)$$

Squaring both sides gives me;

$$y^2 = \sqrt{x - 1}$$

Differentiating both sides gives me;

$$(2y + .1) \cdot \frac{dy}{dx} = 1$$

Dividing through by the term on the left, I get;

$$dy/dx = 1/2y + .1$$

At the origin, (1, 0), the tangent line equation is and becomes equal to-

$$10(x - 1)$$

And if x=10, the tangent line equation, according to the above rule is and becomes equal to

$$\frac{1}{6.1} * (x - 3) + 3$$

Below are GeoGebra graphs of the computations.

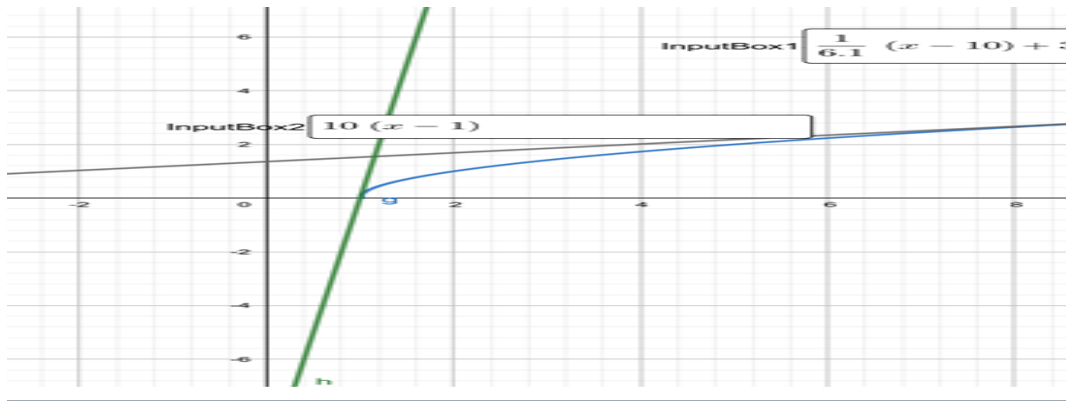


Figure 7. The horizontal tangent lines.

Method of solution of n-degree polynomial

Step 1. Square both sides.

Step 2. Implicitly differentiate the left and right side of the equation.

Step 3. Set the left side or the dependent side of the function equal to zero.

Step 4. Determine the solutions of the  $2n-1$  degree polynomial or the zeroes of the right side of the function of  $-x$ .

Step 5. Step 4 determines the minimums and maximums of the n-degree polynomial.

Step 6. Determine the logical derivative of the n-degree function.

Step 7. Plot tangent lines of the minimums and maximums given I step 5.

**Horizontal tangent line of irrational functions with no horizontal intercept.**

Given a function  $I(x) = X^{\frac{m}{n}}$ , its derivative is computed using the following procedure.

Step 1. Multiply the left and right sides by the lower degree of  $I(x)$ ;  $n$ .

Step 2. Implicitly differentiate  $I(x)$  on both sides with the Power Rule Theorem.

Step 3. Isolate  $\frac{dy}{dx}$  on the left side of the differential equation.

Step 4. Set  $x = 0$  and simplify the value of the quotient.

Step 5. Plot tangent line at  $x=0$ .

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