

THE SORET EFFECT ON FREE CONVECTIVE UNSTEADY MHD FLOW OVER A VERTICAL PLATE WITH HEAT SOURCE

M Bhavana¹, D Chenna Kesavaiah², A Sudhakaraiiah³

Department of H&S, Mother Theresa College of Engineering & Technology, Peddapalli, Karimnagar, AP, India¹

Department of H & BS, Visvesvaraya College of Engineering & Technology, Greater Hyderabad, AP, India²

Department of Future Studies, Sri Venkateswara University, Tirupati – 517 502, AP, India³

ABSTRACT : The work is focused on free convective an unsteady MHD flow in a vertical plate with heat source, thermo diffusion (Soret effect) and the influence of the thermal radiation on hydromagnetic for a viscous fluid past a semi-infinite vertical moving porous plate embedded in a porous medium. The plate moves with constant velocity in the direction of fluid flow while the free stream velocity is assumed to follow the exponentially increasing small perturbation law. The external flow field is assumed to be uniform and the effect of the Soret parameter in the boundary layer adjacent to the vertical plate with fluid suction/injection through it is analyzed in both aiding and opposing flow situations. The dimensionless governing equations for this investigation are solved analytically by using perturbation technique. The effects of various parameters on the velocity, temperature and concentration fields as well as the skin-friction coefficient, Nusselt number and the Sherwood number are presented graphically detail.

Keywords: Heat and Mass transfer, MHD, Porous medium, Soret number, Thermal radiation.

I. INTRODUCTION

Free convection arises in the fluid when temperature changes cause density variation leading to buoyancy forces acting on the fluid elements. The most common example of free convection is the atmospheric flow which is driven by temperature differences. Sometimes along with the free convection currents caused by difference in temperature the flow is also affected by the differences in concentration or material constitution. This type of flow has applications in many branches of science and engineering. The study of such flow under the influence of magnetic field has attracted the interest of many investigators in view of its application in MHD generators, plasma studies, nuclear reactors, geothermal energy extractions and boundary layer control in the field of aerodynamics. Unsteady free convection flows in a porous medium have received much attention in recent time due to its wide applications in geothermal and oil reservoir engineering as well as other geophysical and astrophysical studies. Moreover, considerable interest has been shown in radiation interaction with convection for heat and mass transfer in fluids. This is due to the significant role of thermal radiation in the surface heat transfer when convection heat transfer is small, particularly in free convection problems involving absorbing-emitting fluids. The unsteady fluid flow past a moving plate in the presence of free convection and radiation were studied by Monsour [23], Cogley et al. [7], Raptis and Perdakis [28], Das et al. [10], Grief et al. [17], Ganeasan and Loganathan [14], Mbeledogu et al. [24], Makinde [22], and Abdus-Satter and Hamid Kalim [2]. All these studies have been confined to unsteady flow in a nonporous medium. From the previous literature survey about unsteady fluid flow, we observe that little papers were done in porous medium. The effect of radiation on MHD flow and heat transfer must be considered when high temperatures are reached. El-Hakiem [11] studied the unsteady MHD oscillatory flow on free convection-radiation through a porous medium with a vertical infinite surface that absorbs the fluid with a constant velocity. Ghaly [16] employed symbolic computation software Mathematica to study the effect of radiation on heat and mass transfer over a stretching sheet in the presence of a magnetic field. Raptis et al. [27] studied the effect of radiation on 2D steady MHD optically thin gray gas flow along an infinite vertical plate taking into account the induced magnetic field. Cookey et al. [8] researched the influence of viscous dissipation and radiation on unsteady MHD free-convection flow past on infinite heated vertical plate in a porous medium with time-dependent suction. Abd El-Naby et al. [1] employed implicit finite-difference methods to study the effect of radiation on MHD unsteady free convection flow past a semi-infinite vertical porous plate but did not take into account the viscous dissipation. Kim [21] studied unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate.

The study of heat generation or absorption effects in moving fluids is important in view of several physical problems, such as fluids undergoing exothermic or endothermic chemical reactions. Vajravelu and Hadjinicolaou [31] studied the heat transfer characteristics in the laminar boundary layer of a viscous fluid over a stretching sheet with viscous dissipation or frictional heating and internal heat generation. Hosssain et al. [20] studied the problem of natural convection flow along a vertical wavy surface with uniform surface temperature in the presence of heat generation / absorption. Alam et al. [3] studied the problem of free convection heat and mass transfer flow past an inclined semi infinite heated surface of an electrically conducting and steady viscous incompressible fluid in the presence of a magnetic field and heat generation. Chamkha [6] investigated unsteady convective heat and mass transfer past a semi-infinite porous moving plate with heat absorption. Hady et al. [18] studied the problem of free convection flow along a vertical wavy surface embedded in electrically conducting fluid saturated porous media in the presence of internal heat generation or absorption effect.

The Soret effect arises when the mass flux contains a term that depends on the temperature gradient. The major focus of our study is the effect on free convection of the addition of a second fluid. Convection in binary fluids is considerably more complicated than that in pure fluids. Both temperature and concentration gradients contribute to the initiation of convection and each may be stabilizing or destabilizing. Even when a concentration gradient is not externally imposed (the thermosolutal problem) it can be created by the applied thermal gradient via the Soret effect. Gbadeyan et.al [15] Heat and mass transfer for Soret and Dufour's effect on mixed convection boundary layer flow over a stretching vertical surface in a porous medium filled with a visco -elastic fluid in the presence of magnetic field, Alan and Rahman [4], examined Dufour and Soret effects on mixed convection flow past a vertical porous flat plate with variable suction embedded in a porous medium for a hydrogen-air mixture as the nonchemical reacting fluid pair. Gaikwad et al. [13] investigated the onset of double diffusive convection in a two component couple stress fluid layer with Soret and Dufour effects using both linear and non-linear stability analysis. Emmanuel et al. [12] studied numerically the effect of thermal-diffusion and diffusion-thermo on combined heat and mass transfer of a steady hydromagnetic convective and slip flow due to a rotating disk with viscous dissipation and Ohmic heating. Beg Anwar et al. [5] examined the combined effects of Soret and Dufour diffusion and porous impedance on laminar magneto-hydrodynamic mixed convection heat and mass transfer of an electrically-conducting, Newtonian, Boussinesq fluid from a vertical stretching surface in a Darcian porous medium under uniform transverse magnetic field. Nithyadevi and Yang [25] investigated numerically the effect of double-diffusive natural convection of water in a partially heated enclosure with Soret and Dufour coefficients around the density maximum. Olanrewaju [26] examined Dufour and Soret Effects of a transient free convective flow with radiative heat transfer past a flat plate moving through a binary mixture and Dufour effects using both linear and non-linear stability analysis.

II. FORMULATION OF THE PROBLEM

Consider two dimensional unsteady flow of a laminar, incompressible, viscous, electrically conducting and heat generation/absorption fluid past a semi-infinite vertical moving porous plate embedded in a uniform porous medium and subjected to a uniform transverse magnetic field in the presence of a pressure gradient has been considered with free convection, thermal diffusion (Soret effect) and thermal radiation effects taking in to an account. According to the coordinate system the x^* - axis is taken along the porous plate in the upward direction and y^* - axis normal to it. The fluid is assumed to be a gray, absorbing-emitting but non-scattering medium. The radiative heat flux in the x^* - direction is considered negligible in comparison with that in the y^* - direction Sparrow and Cess [30]. It is assumed that there is no applied voltage of which implies the absence of an electric field. The transversely applied magnetic field and magnetic Reynolds number are very small and hence the induced magnetic field is negligible Cowling [9]. Viscous and Darcy resistance terms are taken into account the constant permeability porous medium. The MHD term is derived from an order-of magnitude analysis of the full Navier-stokes equation. It is assumed here that the hole size of the porous plate is significantly larger than a characteristic microscopic length scale of the porous medium. We regard the porous medium as an assemblage of small identical spherical particles fixed in space, following Yamamoto and Iwamura [32]. A homogeneous first-order chemical reaction between the fluid and the species concentration. The chemical reactions are taking place in the flow and all thermo physical properties are assumed to be constant of the linear momentum equation which is approximation. The fluid properties are assumed to be constants except that the influence of density variation with temperature and concentration has been considered in the body-force term. Due to the semi - infinite place surface assumption furthermore, the flow variable are functions of y^* and t^* only. The governing equation for this investigation is based on the balances of mass, linear momentum, energy, and concentration species. Taking into consideration the assumptions made above, these equations can be written in Cartesian frame of reference, as follows:

$$\frac{\partial v^*}{\partial y^*} = 0$$

(1)

$$\rho \left(\frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*} \right) = \frac{\partial p^*}{\partial x^*} + \mu \frac{\partial^2 u^*}{\partial y^{*2}} - g\beta - \frac{\mu}{K^*} u^* - \sigma B_0^2 u^* \quad (2)$$

$$\frac{\partial T^*}{\partial t^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{k}{\rho C_p} \frac{\partial^2 T^*}{\partial y^{*2}} - \frac{1}{\rho C_p} \left(\frac{\partial q_r^*}{\partial y^*} \right) - \frac{Q_0}{\rho C_p} (T^* - T_\infty^*) \quad (3)$$

$$\frac{\partial C^*}{\partial t^*} + v^* \frac{\partial C^*}{\partial y^*} = D_M \frac{\partial^2 C^*}{\partial y^{*2}} + D_T \frac{\partial^2 T^*}{\partial y^{*2}} \quad (4)$$

where x^* , y^* and t^* are the dimensional distances along and perpendicular to the plate and dimensional time, respectively. u^* and v^* are the components of dimensional velocities along x^* and y^* directions, ρ is the fluid density, μ is the viscosity, C_p is the specific heat at constant pressure, σ is the fluid electrical conductivity, B_0 is the magnetic induction, K^* is the permeability of the porous medium, T^* is the dimensional temperature, D_M is the coefficient of chemical molecular diffusivity, D_T is the coefficient of thermal diffusivity, C^* is the dimensional concentration, k is the thermal conductivity of the fluid, g is the acceleration due to gravity, and q_r^* , R are the local radiative heat flux, the reaction rate constant respectively. The term $Q_0(T^* - T_\infty^*)$ is assumed to be amount of heat generated or absorbed per unit volume Q_0 is a constant, which may take on either positive or negative values. When the wall temperature T^* exceeds the free stream temperature T_∞^* , the source term $Q_0 > 0$ and heat sink when $Q_0 < 0$. The magnetic and viscous dissipations are neglected in this study. It is assumed that the porous plate moves with a constant velocity u_p^* in the direction of fluid flow, and the free stream velocity U_∞^* follows the exponentially increasing small perturbation law. In addition, it is assumed that the temperature and concentration at the wall as well as the suction velocity are exponentially varying with time.

The boundary conditions for the velocity, temperature, and concentration fields are given as follows:

$$u^* = u_p^*, T^* = T_w^* + \varepsilon (T_w^* - T_\infty^*) e^{n^* t^*}, C^* = C_w^* + \varepsilon (C_w^* - C_\infty^*) e^{n^* t^*} \quad \text{at } y = 0 \quad (5)$$

$$u^* \rightarrow U_\infty^* = U_0 (1 + \varepsilon e^{n^* t^*}), \quad T^* \rightarrow T_\infty^*, \quad C^* \rightarrow C_\infty^* \quad \text{as } y \rightarrow \infty \quad (6)$$

where T_w^* and C_w^* are the wall dimensional temperature and concentration, respectively. C_∞^* is the free stream dimensional concentration. U_0 and n^* are constants.

It is clear from equation (1) that the suction velocity at the plate surface is a function of time only. Assuming that it takes the following exponential form:

$$v^* = -v_0 (1 + \varepsilon A e^{n^* t^*}) \quad (7)$$

where A is a real positive constant, ε and εA are small less than unity, and v_0 is a scale of suction velocity which has non-zero positive constant.

In the free stream, from equation (2) we get

$$\rho \frac{dU_\infty^*}{dt^*} = -\frac{\partial p^*}{\partial x^*} - \rho_\infty g - \frac{\mu}{K^*} U_\infty^* - \sigma B_0^2 U_\infty^* \quad (8)$$

Eliminating $\frac{\partial p^*}{\partial x^*}$ between equation (2) and equation (8), we obtain

$$\rho \left(\frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*} \right) = (\rho_\infty - \rho)g + \rho \frac{dU_\infty^*}{dt^*} + \mu \frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\mu}{K^*} (U_\infty^* - u^*) - \sigma B_0^2 (U_\infty^* - u^*) \quad (9)$$

by making use the equation of state Hassanien and Obied Allah [19]

$$\rho_\infty - \rho = \rho\beta(T^* - T_\infty^*) + \rho\beta^*(C^* - C_\infty^*) \quad (10)$$

where β is the volumetric coefficient of thermal expansion, β^* the volumetric coefficient of expansion with concentration, and ρ_∞ the density of the fluid far away the surface. Then substituting from equation (10) into equation (9) we obtain

$$\frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*} = \frac{dU_\infty^*}{dt^*} + \mu \frac{\partial^2 u^*}{\partial y^{*2}} + \rho\beta(T^* - T_\infty^*) + \rho\beta^*(C^* - C_\infty^*) + \frac{\nu}{K^*} (U_\infty^* - u^*) - \frac{\sigma B_0^2}{\rho} (U_\infty^* - u^*) \quad (11)$$

where $\nu = \frac{\mu}{\rho}$ is the coefficient of the kinematic viscosity. The third term on the RHS of the momentum equation (11) denote body force due to non uniform temperature, the fourth term denote body force due to non uniform concentration.

The radiative heat flux term by using the Roseland approximation is gives by

$$q_r^* = \frac{4\sigma^*}{3k_1^*} \frac{\partial T^{*4}}{\partial y^*} \quad (12)$$

Where σ^* and k_1^* are respectively the Stefan-Boltzmann constant and the mean absorption coefficient. We assume that the temperature difference within the flow are sufficiently small such that T^{*4} may be expressed as a linear function of the temperature. This is accomplished by expanding in a Taylor series about T_∞^* and neglecting higher-order terms, thus

$$T^{*4} \cong 4T_\infty^{*3} - 3T_\infty^{*4} \quad (13)$$

By using equations (12) and (13), into equation (3) is reduced to

$$\frac{\partial T^*}{\partial t^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{k}{\rho C_p} \frac{\partial^2 T^*}{\partial y^{*2}} - \frac{16\sigma^* T_\infty^{*3}}{3\rho C_p k_1^*} \frac{\partial^2 T^*}{\partial y^{*2}} - \frac{Q_0}{\rho C_p} (T^* - T_\infty^*) \quad (14)$$

Introducing the non-dimensional quantities and parameters:

$$\begin{aligned} u^* &= uU_0, v^* = vV_0, T^* = T_\infty^* + \theta(T_w^* - T_\infty^*), C^* = C_\infty^* + \phi(C_w^* - C_\infty^*) U^* = U_\infty U_0 \\ u_p^* &= U_p U_0, K^* = \frac{KV^2}{V_0^2}, y^* = \frac{vy}{V_0}, Gm = \frac{v\beta^* g (C_w^* - C_\infty^*)}{V_0^2 U_0}, Gr = \frac{v\beta g (T_w^* - T_\infty^*)}{V_0^2 U_0} \\ Pr &= \frac{v\rho C_p}{k}, M = \frac{\sigma B_0^2 v}{\rho V_0^2}, Q = \frac{Q_0 v}{\rho C_p V_0^2}, R = \frac{4\sigma^* T_\infty^{*3}}{k_1^* k}, Sc = \frac{\nu}{D_M}, t^* = \frac{t\nu}{V_0^2}, n^* = \frac{V_0^2}{\nu} \end{aligned} \quad (15)$$

Then substituting from equation (15) into equations (11), (14) and (4) and taking into account equation (7) we obtain.

$$\frac{\partial u}{\partial t} - (1 + \varepsilon Ae^m) \frac{\partial u}{\partial y} = \frac{dU_\infty}{dt} + \frac{\partial^2 u}{\partial y^2} + Gr\theta + Gm\phi + N(U_\infty - u) \quad (16)$$

$$\frac{\partial \theta}{\partial t} - (1 + \varepsilon A e^{nt}) \frac{\partial \theta}{\partial y} = \frac{1}{\text{Pr}} \left(1 + \frac{4R}{3} \right) \frac{\partial^2 \theta}{\partial y^2} - Q\theta \quad (17)$$

$$\frac{\partial \phi}{\partial t} - (1 + \varepsilon A e^{nt}) \frac{\partial \phi}{\partial y} = \frac{1}{Sc} \frac{\partial^2 \phi}{\partial y^2} + S_o \frac{\partial^2 \theta}{\partial y^2} \quad (18)$$

where Gr is the thermal Grashof number, Gm is solutal Grashof Number, Pr is Prandtl Number, M is the magnetic field parameter, Sc is Schmidt number, Q is the dimensionless heat generation parameter/absorption coefficient, S_o is the Soret number, R is the radiation parameter respectively, and $N = \left(M + \frac{1}{K} \right)$

The dimensionless form of the boundary condition (5) and (6) become

$$\begin{aligned} u = U_p, \theta = 1 + \varepsilon e^{nt}, \phi = 1 + \varepsilon e^{nt} \quad \text{at } y = 0 \\ u \rightarrow U_\infty \rightarrow 1 + \varepsilon e^{nt}, \theta \rightarrow 0, \phi \rightarrow 0 \quad \text{as } y \rightarrow \infty \end{aligned} \quad (19)$$

III. SOLUTION OF THE PROBLEM

In order to reduce the above system of partial differential equations to a system of ordinary equations in dimensionless form, we may represent the velocity, temperature and concentration as

$$\begin{aligned} u = u_0(y) + \varepsilon e^{nt} u_1(y) + O(\varepsilon^2) \\ \theta = \theta_0(y) + \varepsilon e^{nt} \theta_1(y) + O(\varepsilon^2) \\ \phi = \phi_0(y) + \varepsilon e^{nt} \phi_1(y) + O(\varepsilon^2) \end{aligned} \quad (20)$$

Where $u_0, T_0,$ and C_0 are mean velocity, mean temperature and mean concentration respectively.

By substituting the above equations (20) into equations (16)-(18), equating the harmonic and non-harmonic terms and neglecting the higher-order terms of $O(\varepsilon^2)$, we obtain the following pairs of equations for (u_0, θ_0, ϕ_0) and (u_1, θ_1, ϕ_1)

$$u_0'' + u_0' - Nu_0 = -N - Gr \theta_0 - Gm \phi_0 \quad (21)$$

$$u_1'' + u_1' - (N + n)u_1 = -(N + n) - Au_0' - Gr \theta_1 - Gm \phi_1 \quad (22)$$

$$(3 + 4R)\theta_0'' + 3\text{Pr} \theta_0' - 3Q\text{Pr} \theta_0 = 0 \quad (23)$$

$$(3 + 4R)\theta_1'' + 3\text{Pr} \theta_1' - 3(n + Q)\text{Pr} \theta_1 = -3A\text{Pr} \theta_0' \quad (24)$$

$$\phi_0'' + Sc \phi_0' = -Sc S_o \theta_0'' \quad (25)$$

$$\phi_1'' + Sc \phi_1' - nSc \phi_1 = -ASc \phi_0' - Sc S_o \theta_1'' \quad (26)$$

where the primes denote differentiation with respect to y .

The corresponding boundary conditions can be written as

$$\begin{aligned} u_0 = u_p = u_1 = 0 = 0, \theta_0 = 1, \theta_1 = 1, \phi_0 = 1, \phi_1 = 1 \quad y = 0 \\ u_0 \rightarrow u_1 \rightarrow 1, T_{00} \rightarrow \theta_0 \rightarrow 0, \theta_1 \rightarrow 0, \phi_0 \rightarrow \phi_1 \rightarrow 0 \quad y \rightarrow \infty \end{aligned} \quad (27)$$

The analytical solutions of equations (21) - (26) with satisfying boundary conditions (27) are given by

$$u_0 = 1 + J_1 e^{m_2 y} + J_2 e^{m_6 y} + J_3 e^{m_2 y} + J_4 e^{m_{10} y}$$

$$u_1 = J_6 e^{m_{10}y} + J_7 e^{m_2y} + J_8 e^{m_6y} + J_9 e^{m_2y} + J_{10} e^{m_4y} + J_{11} e^{m_2y} + J_{12} e^{m_8y} + J_{13} e^{m_6y} \\ + J_{14} e^{m_2y} + J_{15} e^{m_{12}y}$$

$$\theta_0 = e^{m_2y}$$

$$\theta_1 = D_1 e^{m_2y} + D_2 e^{m_4y}$$

$$\phi_0 = Z_1 e^{m_2y} + Z_2 e^{m_6y}$$

$$\phi_1 = Z_3 e^{m_6y} + Z_4 e^{m_2y} + Z_5 e^{m_8y}$$

In view of the above solutions, the velocity, temperature and concentration distributions in the boundary layer become

$$u(y,t) = 1 + J_1 e^{m_2y} + J_2 e^{m_6y} + J_3 e^{m_2y} + J_4 e^{m_{10}y} + \varepsilon e^{nt} \left\{ 1 + J_6 e^{m_{10}y} + J_7 e^{m_2y} + J_8 e^{m_6y} + J_9 e^{m_2y} \right. \\ \left. + J_{10} e^{m_4y} + J_{11} e^{m_2y} + J_{12} e^{m_8y} + J_{13} e^{m_6y} + J_{14} e^{m_2y} + J_{15} e^{m_{12}y} \right\}$$

$$\theta(y,t) = e^{m_2y} + \varepsilon e^{nt} \left\{ D_1 e^{m_2y} + D_2 e^{m_4y} \right\}$$

$$\phi(y,t) = Z_1 e^{m_2y} + Z_2 e^{m_6y} + \varepsilon e^{nt} \left\{ Z_3 e^{m_6y} + Z_4 e^{m_2y} + Z_5 e^{m_8y} \right\}$$

It is now important to calculate the physical quantities of primary interest, which are the local wall shear stress, the local surface heat, and mass flux. Given the velocity field in the boundary layer, we can now calculate the local wall shear stress (i.e., skin- friction) is given by

$$\tau_w^* = \mu \left(\frac{\partial u^*}{\partial y^*} \right)_{y^*=0}$$

and in dimensionless form, we obtain

$$C_f = \frac{\tau_w^*}{\rho U_0 V_0} = \left(\frac{\partial u}{\partial y} \right)_{y=0} = u'(0)$$

$$= m_2 J_1 + m_6 J_2 + m_2 J_3 + m_{10} J_4 + m_{10} J_6 + m_2 J_7 + m_6 J_8 + m_2 J_9 + m_4 J_{10} e^{m_4y} + m_2 J_{11} + m_8 J_{12} \\ + m_6 J_{13} + m_2 J_{14} + m_{12} J_{15} + m_{12} J_{15}$$

Knowing the temperature field, it is interesting to study the effect of the free convection and radiation on the rate of heat transfer q_w^* . This is given by

$$q_w^* = -k \left(\frac{\partial T^*}{\partial y^*} \right)_{y^*=0} - \frac{4\sigma^*}{3k_1^*} \left(\frac{\partial T^*}{\partial y^*} \right)_{y^*=0}$$

by using equation (13), we can write the above equation as follow

$$q_w^* = -k \left(k + \frac{16\sigma^* T_\infty^*}{3k_1^*} \right) \left(\frac{\partial T^*}{\partial y^*} \right)_{y^*=0}$$

this is written in dimensionless form as;

$$q_w^* = - \frac{k(T_w^* - T_\infty^*) V_0}{\nu} \left(1 + \frac{4R}{3} \right) \left(\frac{\partial \theta}{\partial y} \right)_{y=0}$$

The dimensionless local surface heat flux (i.e., Nusselt number) is obtained as

$$Nu = \frac{q_w^*}{k(T_w^* - T_\infty^*)} \Rightarrow Nu Re_x^{-1} = - \left(1 + \frac{4R}{3} \right) \left(\frac{\partial \theta}{\partial y} \right)_{y=0} = - \left(1 + \frac{4R}{3} \right) [m_2 + m_2 D_1 + m_4 D_2]$$

where, $Re_x^{-1} = \frac{V_0 x}{\nu}$ is the Reynolds number.

The definition of the local mass flux and the local Sherwood number are respectively given by

$$j_w = -D \left(\frac{\partial C^*}{\partial y^*} \right)_{y^*=0}$$

$$Sh_x = \frac{j_w x}{D(C_w^* - C_\infty^*)}$$

with the help of these equations, one can write

$$Sh Re_x^{-1} = - \left(\frac{\partial \phi}{\partial y} \right)_{y=0} = m_2 Z_1 + m_6 Z_2 + m_6 Z_3 + m_2 Z_4 + m_8 Z_5$$

IV. RESULTS AND DISCUSSION

In order to get a clear insight of the physical problem, numerical results are displayed with the help of graphs. This enables us to carry out the numerical calculations for the distribution of the velocity, temperature and concentration across the boundary layer for various values of the parameters. In the present study we have chosen $A=0.5$, $t=1.0$, $n=0.1$, $=0.5 U_p$ and $\varepsilon=0.2$, while $R, Q, S_0, Gr, Gm, Sc, M, Pr$ and K are varied over a range, which are listed in the figure legends. Also, the boundary condition for $y \rightarrow \infty$ is replaced by where $\max y$ is a sufficiently large value of y where the velocity profile u approaches to the relevant free stream velocity.

The velocity profiles for different values of Grashof number (Gr) are described in figure (1). It is observed that an increasing in Gr leads to a rise in the values of velocity. Here the Grashof number represents the effects of the free convection currents. Physically, $Gr > 0$ means heating of the fluid of cooling of the boundary surface $Gr < 0$ means cooling of the fluid of heating of the boundary surface and $Gr = 0$ corresponds to the absence of free convection current. In addition, the curves show that the peak value of velocity increases rapidly near the wall of the porous plate as Grashof number increases, and then decays to the relevant free stream velocity. The velocity profiles for different values of solutal Grashof number (Gm) are described in figure (2). It is observed that an increasing in Gm leads to a rise in the values of velocity. In addition, the curves show that the peak value of velocity increases rapidly near the wall of the porous plate as solutal Grashof number increases, and then decays to the relevant free stream velocity. For different values of the magnetic field parameter M , the velocity profiles are plotted in figure (3). It is obvious that the effect of increasing values of M parameter results in decreasing velocity distribution across the boundary layer because of the application of transfer magnetic field will result a restrictive type force (Lorentz force) similar to drag force which tends to resist the fluid flow and thus reducing its velocity. Figure (4) shows the velocity profiles for different values of the permeability (K). Clearly as K increases the peak value of velocity tends to increase. These results could be very useful in deciding the applicability of enhanced oil recovery in reservoir engineering. For different values of radiation parameter (R) the velocity and concentration profiles are plotted in figure 5(a) and 5(c). Here we find that, as the value of R increases the velocity and concentration increases, with an increasing in the flow boundary layer thickness. Thus, thermal radiation enhances convective flow. The effects of radiation parameter (R) on the temperature profiles are presented in figure 5(b). From this figure we observe that, as the value of R increases the temperature profiles decreases, with an increasing in the thermal boundary layer thickness. The effect of heat source (Q) on the velocity and concentration profiles is shown in figure 6 (a) and 6(c). From this figure we see that the heat is generated the buoyancy force increases which induces the flow rate to increase giving rise to the increase in the velocity and concentration profiles. Figure 6(b) shows the variation of temperature profiles for different values of Q . It is seen from this figure that temperature profiles decrease with an increasing of heat source parameter (Q). Figure 7(a) shows the velocity profiles across the boundary layer for different values of Prandtl number (Pr). The results show that the effect of increasing values of Pr results in a decreasing the velocity. Typical variations of the velocity and temperature profiles along the span wise coordinate y are shown in figure 7(b) for different values of Prandtl number (Pr). The results show that an increase of Prandtl number results in a increasing the temperature profiles of thermal boundary layer thickness and more uniform temperature distribution across the boundary layer. Figure 7(c) illustrates the concentration profiles for various values of Pr . We see that the concentration profiles decreases near (far) from vertical porous plate as parameter Pr increases. For different values of the Schmidt number (Sc), the velocity

profiles are plotted in figure 8(a). It is obvious that the effect of increasing values of Sc results in a decreasing velocity distribution across the boundary layer. Figure 8(b) shows the concentration profiles across the boundary layer for various values of Schmidt number Sc . The figure shows that an increasing in Sc results in a decreasing the concentration distribution, because the smaller values of Sc are equivalent to increasing the chemical molecular diffusivity. The effects of Soret number (S_0) on the velocity profiles is shown in figure 9(a). From this figure we see that velocity profiles increase with an increasing of S_0 from which we conclude that the fluid velocity rises due to greater thermal diffusion. Figure 9(b) represents the concentration profiles for different values of Soret number (S_0). From this figure we observe that the concentration profiles increase significantly with an increasing of Soret number.

Figure (10) depict the effects of the radiation parameter (R) on the skin friction coefficients C_f versus Gr . It is observed from this figure that as R increases, the skin- friction coefficients increases. Figure (11) show that the effect of radiation parameter (R) on Nusselt number (Nu) versus t . It is observed that an increasing R the Nusselt number increases. The effects of the Soret number (S_0) on the Sherwood number (Sh) shown in figure (12). It is seen from this table that as S_0 increases, the Sherwood number increases and Sherwood number increases whereas the Nusselt number remains unchanged.

APPENDIX

$$\begin{aligned}
 m_2 &= -\left(\frac{1 + \sqrt{1 + 4Q\beta_1}}{2}\right), m_4 = -\left(\frac{1 + \sqrt{1 + 4(n-Q)\beta_1}}{2}\right), m_6 = -Sc, m_8 = -\left(\frac{Sc + \sqrt{Sc^2 + 4nSc}}{2}\right) \\
 m_{10} &= -\left(\frac{1 + \sqrt{1 + 4N}}{2}\right), m_{12} = -\left(\frac{1 + \sqrt{1 + 4(n-N)}}{2}\right), \beta_1 = \left(\frac{3 + 4R}{3Pr}\right), D_1 = -\frac{Am_2}{\beta_1 m_2^2 + m_2 - (n-Q)} \\
 D_2 &= (1 - D_1), Z_1 = -\frac{S_0 Sc}{m_2^2 + Scm_2}, Z_2 = (1 - Z_1), Z_3 = -\frac{AZ_2 Scm_6}{m_6^2 + Scm_6 - nSc}, Z_4 = -\frac{AZ_1 Scm_2}{m_2^2 + Scm_2 - nSc} \\
 Z_5 &= (1 - Z_3 - Z_4), J_1 = -\frac{Gr}{m_2^2 + m_2 - N}, J_2 = -\frac{GmZ_2}{m_6^2 + m_6 - N}, J_3 = -\frac{GmZ_1}{m_2^2 + m_2 - N} \\
 J_4 &= (U_p - 1 - J_1 - J_2 - J_3), J_6 = -\frac{AJ_4 m_{10}}{m_{10}^2 + m_{10} - (n + N)}, J_7 = -\frac{AJ_1 m_2}{m_2^2 + m_2 - (n + N)} \\
 J_8 &= -\frac{AJ_2 m_6}{m_6^2 + m_6 - (n + N)}, J_9 = -\frac{AJ_3 m_2}{m_2^2 + m_2 - (n + N)}, J_{10} = -\frac{GrD_2}{m_4^2 + m_4 - (n + N)} \\
 J_{11} &= -\frac{GrD_1}{m_2^2 + m_2 - (n + N)}, J_{12} = -\frac{GmZ_5}{m_8^2 + m_8 - (n + N)}, J_{13} = -\frac{GmZ_3}{m_6^2 + m_6 - (n + N)} \\
 J_{14} &= -\frac{GmZ_4}{m_2^2 + m_2 - (n + N)}, J_{15} = -(1 + J_6 + J_7 + J_8 + J_9 + J_{10} + J_{11} + J_{12} + J_{13} + J_{14})
 \end{aligned}$$

REFERENCES

- [1] Abd EL-Naby M. A, El-Barbary E M E and N. Y. Abdelazem "Finite difference solution of radiation effects on MHD unsteady free-convection flow on vertical porous plate", Appl. Math. Comput.151 (2), pp. 327 – 346, 2004
- [2] Abdus – Sattar M. D, and Hamid Kalim M. D "Unsteady free-convection interaction with thermal radiation in a boundary layer flow past a vertical porous plate", J. Math. Phys. Sci. 30, pp. 25 – 37, 1996
- [3] Alam M. S. , M. Rahman and M. A. Sattar "MHD Free convection heat and mass transfer flow past an inclined surface with heat generation", Thamasat. Int. J. Sci. Tech. 11 (4), pp. 1 – 8, 2006

[4] Alam, M.M. Rahman M. S “Dufour and Soret effects on mixed convection flow past a vertical porous flat plate with variable suction”, *Nonlinear Analysis: Modelling and Control*, Vol. 11, No. 1, pp. 3-12,2006

[5] Beg Anwa. O, Bakier. A. Y and Prasad. V. R “Numerical study of free convection magnetohydrodynamic heat and mass transfer from a stretching surface to a saturated porous medium with Soret and Dufour effects”, *Computational Material Science*, 46, pp. 57-65, 2009

[6] Chamkha A. J “Unsteady MHD convective heat and mass transfer past a semi- infinite vertical permeable moving plate with heat absorption”, *Int. J. Eng. Sci.* 24, pp. 217 – 230, 2004

[7] Cogley A. C., Vincenti. W. C and Gilles. S. E “Differential approximation for radiation transfer in a non-gray gas near equilibrium”, *Am. Inst. Aeronaut. Astronaut. J.* 6, pp. 551 – 555, 1968

[8] Cookey C. I., Oglu. A and Omubo-Pepple. V. M “Influence of viscous dissipation and radiation on unsteady MHD free-convection flow past an infinite heated vertical plate in a porous medium with time-dependent suction”, *Int. J. Heat Mass Transfer*, 46, pp. 2305 – 2311, 2003

[9] Cowling. T. G “Magnetohydrodynamics”, Inter science Publishers, New York, 1957

[10] Das U. N., Deka. R and Soundalgekar. V. M. “Radiation effect on flow past an impulsively started vertical plate- an exact solutions”, *J. Theo. Appl. Fluid Mech.* 1 (2), pp. 111 – 115, 1996

[11] El-Hakim. M.A “MHD oscillatory flow on free-convection radiation through a porous medium with constant velocity”, *J. Magn. and Magnetic Mater.* 220 (2, 3), pp. 271 – 276, 2000

[12] Emmanuel Osalusi, Jonathan Side and Robert Harris “Thermal-diffusion and diffusion-thermo effects on combined heat and mass transfer of a steady MHD convective and slip flow due to a rotating disk with viscous dissipation and Ohmic heating”, *International Communications in Heat and Mass Transfer*, 35, pp. 908-915, 2008

[13] Gaikwad, S. N, Malashetty, M. S and Prasad K Rama “An analytical study of linear and nonlinear double diffusive convection with Soret and Dufour effects in couple stress fluid”, *International Journal of Non- Linear Mechanics*, 42, pp. 903-913, 2007

[14] Ganesan, P. and Loganathan, P “Radiation and mass transfer effects on flow of an incompressible viscous fluid past a moving vertical cylinder”, *Int. J. of Heat and Mass Transfer* 45, pp. 4281– 4288, 2002

[15] Gbadeyan, J. A, Idowu A. S, Ogunsola, A. W, O. O. Agboola and P.O. Olanrewaju “Heat and mass transfer for Soret and Dufour’s effect on mixed convection boundary layer flow over a stretching vertical surface in a porous medium filled with a viscoelastic fluid in the presence of magnetic field”, *Global Journal of Science Frontier Research*, 11(8), 1, pp. 97-114 , 2011

[16] Ghaly, A. Y “Radiation effects on a certain MHD free-convection flow”, *chaos, solutions & Fractals*, 13 (9), pp. 1843 – 1850, 2002

[17] Grief, R., Habib, I. S and Lin, L. C “Laminar convection of radiating gas in a vertical channel”, *J. Fluid Mech.* 46, pp. 513 – 520, 1971

[18] Hady, F. M, Mohamed, R. A and A. Mahdy A “MHD Free convection flow along a vertical wavy surface with heat generation or absorption effect”, *Int. Comm. Heat Mass Transfer*, 33, pp. 1253 – 1263, 2006

[19] Hassani, I. A and Obied Allah, M. H “Oscillatory hydromagnetic flow through a porous medium with variable permeability in the presence of free convection and mass transfer flow”, *Int. Comm. Heat mass transfer*, 29 (4) pp. 567–575, 2002

[20] Hossain, M. A., Molla. M. M and Yaa, L. S “Natural convection flow along a vertical wavy surface temperature in the presence of heat generation/absorption”, *Int. J. Thermal Science*, 43, pp. 157 – 163, 2004

[21] Kim Y. J “Unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction”, *Int. J. Eng. Sci.* 38, pp.833– 845, 2000

[22] Makinde, O. D “Free-convection flow with thermal radiation and mass transfer past a moving vertical porous plate”, *Int. Comm. Heat Mass Transfer*, 32, pp. 1411 – 1419, 2005

[23] Mansour, M. A “Radiation and free convection effects on the oscillating flow past a vertical plate”, *Astrophys. Space Sci.* 166, pp. 269 – 275, 1990

[24] Mbeledogu, I. U, Amakiri, A.R.C and Ogulu, A “Unsteady MHD free convection flow of a compressible fluid past a moving vertical plate in the presence of radiative heat transfer”, *Int. J. of Heat and Mass Transfer*, 50, pp. 1668 – 1674, 2007

[25] Nithyadevi, N and Yang Ruey-Jen “Double diffusive natural convection in a partially heated enclosure with Soret and Dufour effects”, *International Journal of Heat and Fluid Flow*, 2009

[26] Olanrewaju Philip Oladapo “Dufour and Soret Effects of a Transient Free Convective Flow With Radiative Heat Transfer Past a Flat Plate Moving Through a Binary Mixture”, *Pacific Journal of Science and Technology*, Vol. 11(1), 2010

[27] Raptis, A., Perdikis, C and A. Leontitsis “Effects of radiation in an optically thin gray gas flowing past a vertical infinite plate in the presence of a magnetic field”, *Heat Mass Transfer* 39, pp. 771 – 773, 2003

[28] Raptis, A., and Perdikis, C “Radiation and free convection flow past a moving plate”, *Appl. Mech. Eng.* 4 (4), pp. 817 – 821, 1999

[29] Seddeek, M.A., Darwish, A. A and M. S. Abdelmeguid “Effects of chemical reaction and variable viscosity on hydromagnetic mixed convection heat and mass transfer for Hiemenz flow through porous media with radiation”, *Communications in Non-linear Science and Numerical Simulation*, 12, pp. 195 – 213, 2007

[30] Sparrow, E. M, Cess, R.D, “Hemisphere Publ. Stuart”, J. T., *Proc. Soc. London A23*, pp. 116-121, 1995

[31] Vajravelu., K and A. Hadjinicolaou “Heat transfer in a viscous fluid over a stretching sheet with viscous dissipation and internal heat generation”, *Int. Comm. Heat Mass Transfer*, 20, pp. 417-430, 1993

[32] Yamamoto.K and Iwamura., N “Flow with convective acceleration through a porous medium”, *J. Eng. Math.* 10, pp. 41 – 54, 1976

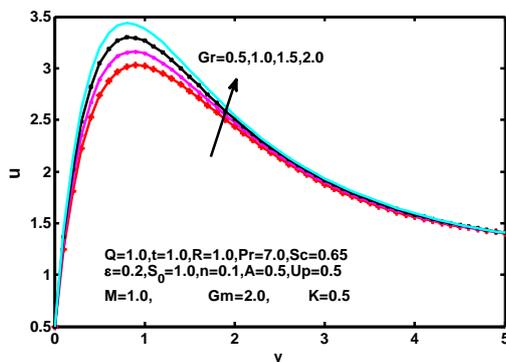


Figure 1: Velocity profiles for different values of Gr

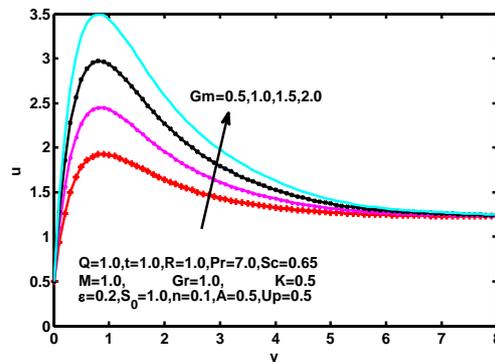


Figure 2: Velocity profiles for different values of Gm

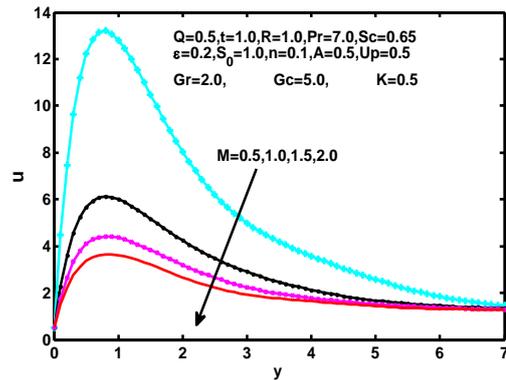


Figure 3: Velocity profiles for different values of M

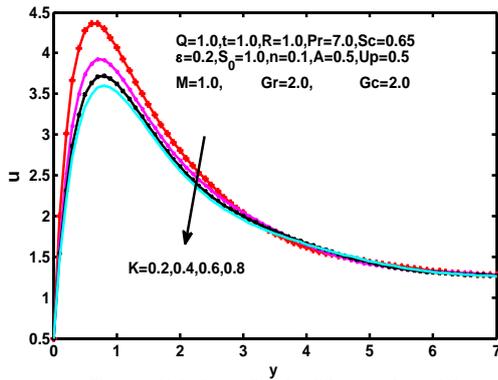


Figure 4: Velocity profiles for different values of K

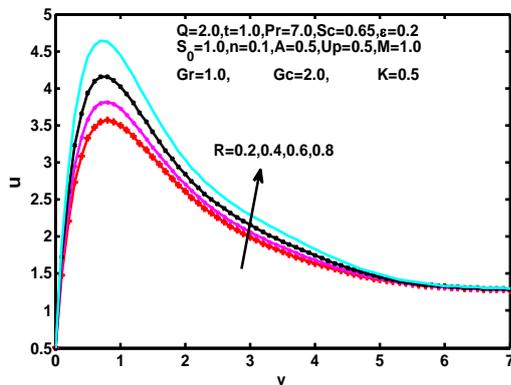


Figure 5(a): Velocity profiles for different values of R

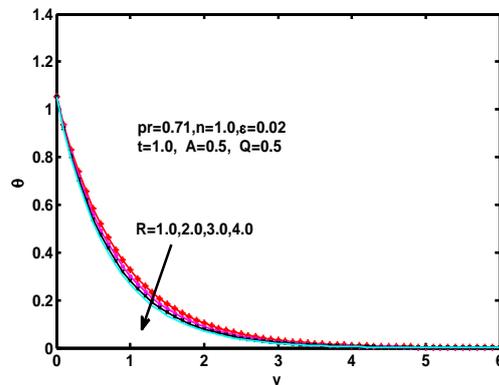


Figure 5(b): Temperature profiles for different values of R

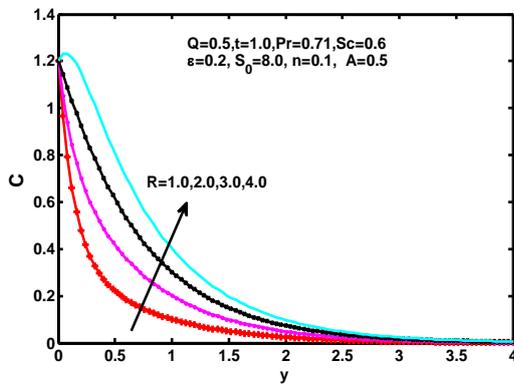


Figure (c): Concentration profiles for different values of R

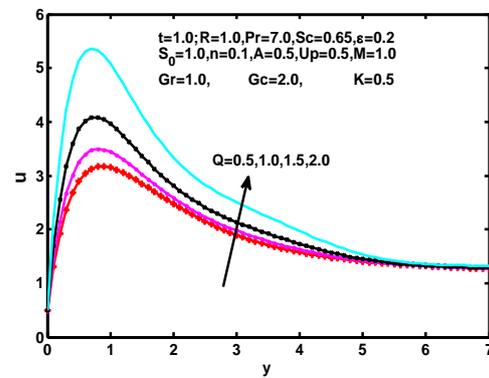


Figure 6(a): Velocity profiles for different values of Q

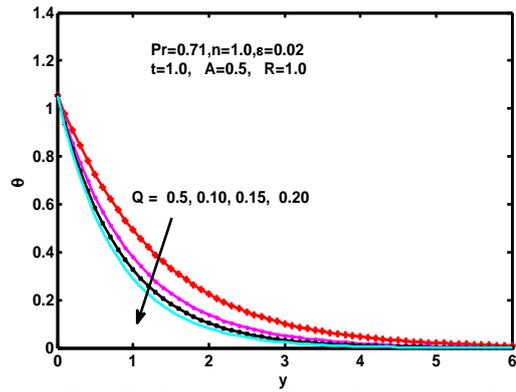


Figure 6(b): Temperature profiles for different values of Q

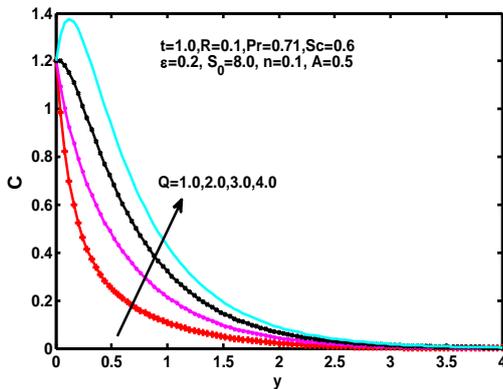


Figure 6(c): Concentration profiles for different values of Q

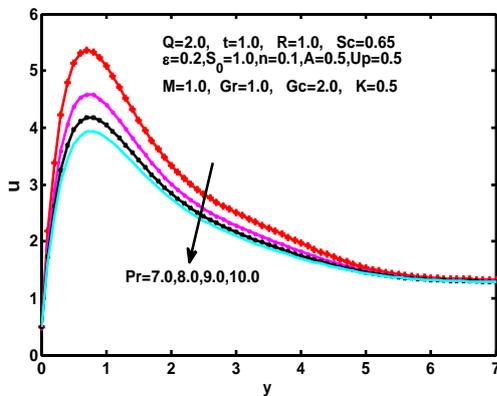


Figure 7(a): Velocity profiles for different values of Pr

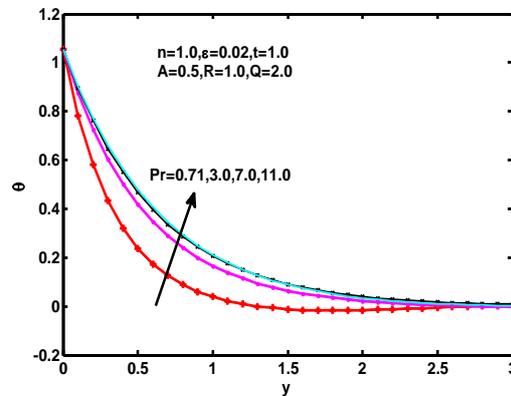


Figure 7(b): temperature profiles for different values of Pr

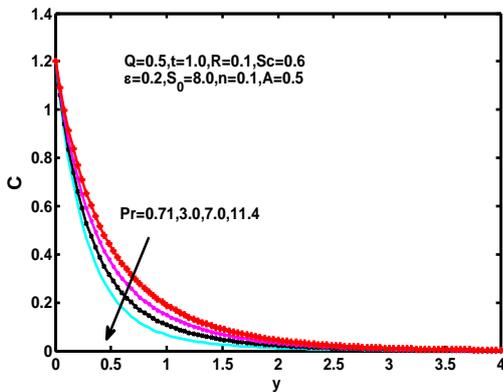


Figure 7(c): Concentration profiles for different values of Pr

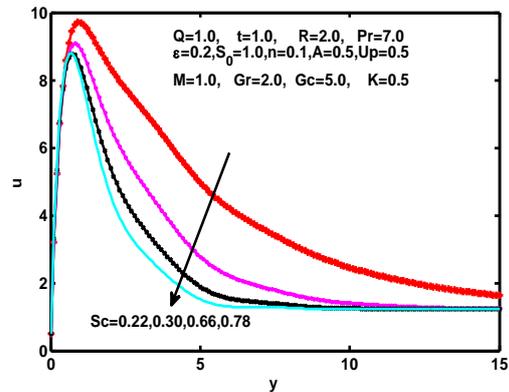


Figure 8(a): Velocity profiles for different values Sc

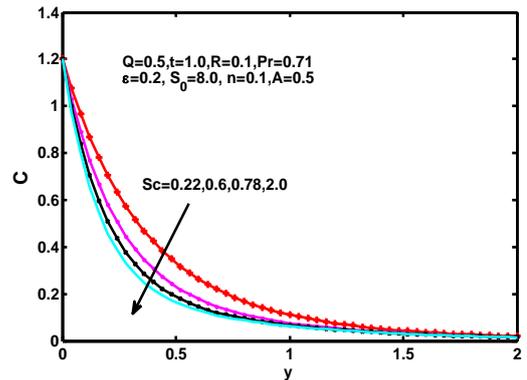


Figure 8(b): Concentration profiles for different values of Sc

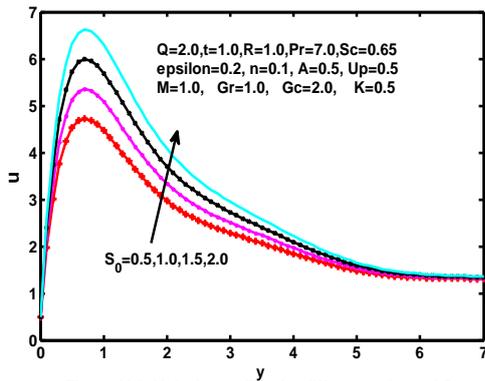


Figure 9(a): Velocity profiles for different values of S_0

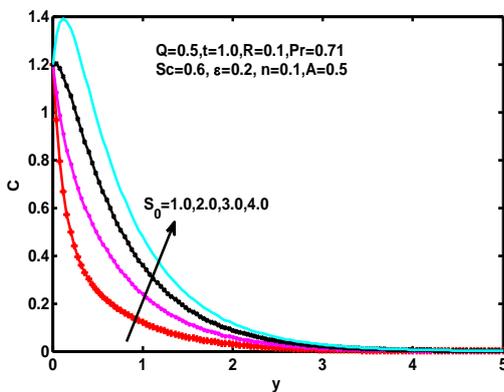


Figure 9(b): Concentration profiles for different values of S_0

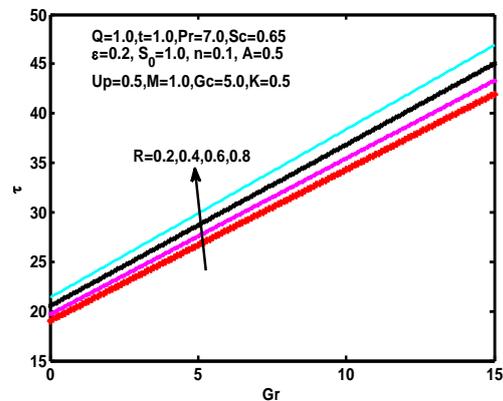


Figure 10: Skin friction for different values of R versus Gr

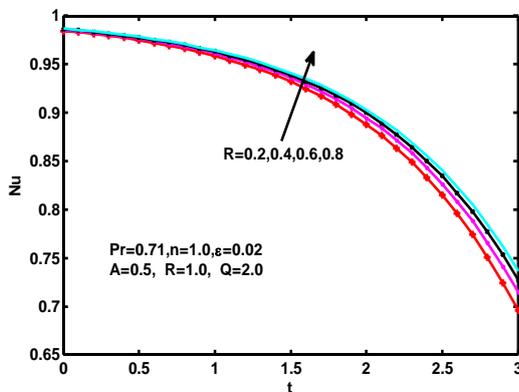


Figure 11: Nusselt number for different values of R versus t

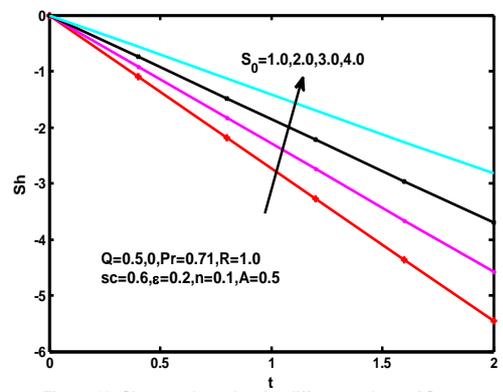


Figure 12: Sherwood number for different values of S_0 versus t