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Theory of Fuzzy Soft Matrix and its Multi Criteria in Decision Making Based on Three Basic t-Norm Operators

Md. Jalilul Islam Mondal[1] , Dr. Tapan Kumar Roy[2]

Research Scholar, Dept. of Mathematics, BESUS, Howrah-711103 , W.B., India[1]
Professor, Dept. of Mathematics, BESUS, Howrah-711103 , W.B. India[2]

ABSTRACT : The purpose of this paper is to put forward the notion of fuzzy soft matrix theory and some basic results. In this paper, we define fuzzy soft matrices and some new definitions based on t-norms with appropriate examples .Lastly we have given an application in decision making based on different operators of t-norms.

Key words: Soft sets, fuzzy soft matrices, operators of t-norms.

I. INTRODUCTION

Most of our traditional tools for formal modeling, reasoning, and computing are crisp, deterministic, and precise in character. However, in real life, there are many complicated problems in engineering, economics, environment, social sciences medical sciences etc. that involve data which are not all always crisp, precise and deterministic in character because of various uncertainties typical problems. Such uncertainties are being dealing with the help of the theories, like theory of probability, theory of fuzzy sets, theory of intuitionistic fuzzy sets, theory of interval mathematics and theory of rough sets etc. Molodtsov [1] also described the concept of "Soft Set Theory" having parameterization tools for dealing with uncertainties. Researchers on soft set theory have received much attention in recent years. Maji and Roy [3,4] first introduced soft set into decision making problems. Maji et al.[2] introduced the concept of fuzzy soft sets by combining soft sets and fuzzy sets. Cagman and Enginoglu [5] defined soft matrices which were a matrix representation of the soft sets and constructed a soft max-min decision making method. Cagman and Enginoglu [6] defined fuzzy soft matrices and constructed a decision making problem. Borah et al.[7] extended fuzzy soft matrix theory and its application. Maji and Roy [8] presented a novel method of object from an imprecise multi-observer data to deal with decision making based on fuzzy soft sets. Majumdar and Samanta[9] generalized the concept of fuzzy soft sets. In this paper, we have introduced some operators of fuzzy soft matrix on the basis of t-norms . We have also discussed their properties . Finally we have given an application in decision making problem on the basis of t-norms operators .

II. DEFINITION AND PRELIMINARIES

2.1 Soft set [1] Let U be an initial universe , P(U) be the power set of U , E be the set of all parameters and A⊆ E. A soft set (f_A,E) on the universe U is defined by the set of order pairs (f_A,E) = {(e , f_A(e)) : e∈ E , f_A(e) ∈ P(U) } where f_A : E → P(U) such that f_A(e) = ϕ if e ∉ A. Here f_A is called an approximate function of the soft set(f_A, E). The set f_A(e) is called e-approximate value set or e-approximate set which consists of related objects of the parameter e ∈ E.

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Example 1 let U = { u_1 , u_2 , u_3 , u_4} be a set of four shirts and E = { white(e_1), red (e_2) ,blue (e_3) } be a set of parameters. If A ={e_1 , e_2} ⊆ E. Let $f_A(e_1)$= { u_1 , u_2 , u_3 , u_4} and $f_A(e_2)$= { u_1 , u_2 , u_3} , then we write the soft set (f_A,E)= {(e_1,{ u_1 , u_2 , u_3 , u_4}), (e_2,{ u_1 , u_2 , u_3})} over U which describe the "colour of the shirts" which Mr. X is going to buy.

2.2 Fuzzy set [2 Let U be an initial universe , E be the set of all parameters and A⊆ E . A pair (F, A) is called a fuzzy set over U where F : A → $\tilde{P}$(U) is a mapping from A into $\tilde{P}$(U) , where $\tilde{P}$(U) denotes the collection of all subsets of U.

Example 2. Consider the example 1, here we can not express with only two real numbers 0 and 1, we can characterized it by a membership function instead of crisp number 0 and 1, which associate with each element a real number in the interval [0,1].Then (f_A,E) = { $f_A(e_1)$ = { (u_1,.7) ,(u_2,.5) ,(u_3,.4) ,(u_4,.2) }, $f_A(e_2)$ = { (u_1,.5) , (u_2,.1) , (u_3,.5)} } is the fuzzy soft set representing the "colour of the shirts" which Mr. X is going to buy.

2.3 Fuzzy Soft Matrices (FSM) [5] Let (f_A, E) be fuzzy soft set over U. Then a subset of U x E is uniquely defined by R_A = { (u , e) : e∈ A , u ∈ f_A(e) }, which is called relation form of (f_A, E) . The characteristic function of R_A is written by μ_{R_A} : U x E → [0 , 1] , where μ_{R_A}(u , e) ∈ [0,1] is the membership value of u∈U for each e ∈U. If $\mu_{ij} = \mu_{R_A}(u_i,e_j)$, we can define a matrix $[\mu_{ij}]_{mXn} = \begin{bmatrix} \mu_{11} & \mu_{12} & \cdots & \mu_{1n} \\ \mu_{21} & \mu_{22} & \cdots & \mu_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \mu_{m1} & \mu_{m2} & \cdots & \mu_{mn} \end{bmatrix}$, which is called an m x n soft matrix of the soft set (f_A,E) over U. Therefore we can say that a fuzzy soft set (f_A,E) is uniquely characterized by the matrix $[\mu_{ij}]_{mXn}$ and both concepts are interchangeable.

Example 3. Assume that U = { u_1 , u_2 , u_3 , u_4, u_5, u_6} is a universal set and E = { e_1 , e_2 , e_3, e_4} is a set of all parameters. If A⊆ E = { e_1 , e_2 , e_3} and $f_A(e_1)$= { (u_1 ,.3), (u_2,.4) , (u_3,.6) , (u_4,.1),(u_5,.6), (u_6,.5 }, $f_A(e_2)$= { (u_1,.2) , (u_2,.5) , (u_3,.7) , (u_4,.3) ,(u_5,.7), (u_6,.1)}, $f_A(e_3)$= { (u_1,.5) , (u_2,.2) , (u_3,.5) , (u_4,.6),(u_5,.7), (u_6,.3) } Then the fuzzy soft set (f_A,E) is a parameterized family { $f_A(e_1)$, $f_A(e_2)$, $f_A(e_3)$} of all fuzzy sets over U. Hence the fuzzy soft matrix $[\mu_{ij}]$ can be written as $[\mu_{ij}] = \begin{bmatrix} .3 & .2 & .5 & 0 \\ .4 & .5 & .2 & 0 \\ .6 & .7 & .5 & 0 \\ .1 & .3 & .6 & 0 \\ .6 & .7 & .7 & 0 \\ .5 & .1 & .3 & 0 \end{bmatrix}$

2.4. Zero Fuzzy Soft Matrix [6] Let $[a_{ij}]$ ∈ $FSM_{m\,X\,n}$. Then $[a_{ij}]$ is called a Zero Fuzzy Soft Matrix denoted by [0], if a_{ij} = 0 for all i and j.

2.5. Universal Fuzzy Soft Matrix [6] Let $[a_{ij}]$ ∈ $FSM_{m\,X\,n}$. Then $[a_{ij}]$ is called a Universal Fuzzy Soft Matrix denoted by [1], if a_{ij} = 1 for all i and j.

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2.6. Fuzzy Soft Sub Matrix [6] Let $[a_{ij}]$, $[b_{ij}]$ $\in$ FSM$_{m\,X\,n}$. Then $[a_{ij}]$ is said to be a Fuzzy Soft Sub Matrix of $[b_{ij}]$ denoted by $[a_{ij}] \tilde{\subseteq} [b_{ij}]$ if $a_{ij} \leq b_{ij}$ for all i and j .

2.7. Union of Fuzzy Soft Matrices [6] Let $[a_{ij}]$, $[b_{ij}]$ $\in$ FSM$_{m\,X\,n}$. Then Union of $[a_{ij}]$ and $[b_{ij}]$, denoted by $[a_{ij}] \tilde{\cup} [b_{ij}]$ is defined as $[a_{ij}] \tilde{\cup} [b_{ij}$ =max $\{a_{ij}, b_{ij}\}$ for all i and j.

2.8. Intersection of Fuzzy Soft Matrices [6] Let $[a_{ij}]$, $[b_{ij}]$ $\in$ FSM$_{m\,X\,n}$. Then Intersection of $[a_{ij}]$ and $[b_{ij}]$, denoted by $[a_{ij}] \tilde{\cap} [b_{ij}]$ is defined as $[a_{ij}] \tilde{\cap} [b_{ij}]$ =min $\{a_{ij}, b_{ij}\}$ for all i and j.

2.9. Compliment Fuzzy Soft Matrix [6] Let $[a_{ij}]$ $\in$ FSM$_{m\,X\,n}$. Then Complement of Fuzzy Soft Matrix $[a_{ij}]$, denoted by $[a_{ij}]^0$ is defined as $[a_{ij}]^0 = 1 - a_{ij}$ for all i and j.

2.10. Fuzzy Soft Equal Matrices [6] Let $[a_{ij}]$, $[b_{ij}]$ $\in$ FSM$_{m\,X\,n}$. Then $[a_{ij}]$ and $[b_{ij}]$ are said to be Fuzzy Soft Equal Matrices , denoted by $[a_{ij}]$ =$[b_{ij}]$ if a_{ij} =b_{ij} for all i and j.

Example 4. Let $[a_{ij}]$, $[b_{ij}]$ $\in$ FSM$_{3\,X4}$ where $[a_{ij}] = \begin{bmatrix} .3 & .2 & .6 & .5 \\ .1 & .7 & .5 & .4 \\ .3 & .4 & .6 & .2 \end{bmatrix}$ and $[b_{ij}] = \begin{bmatrix} .4 & .5 & .7 & .6 \\ .2 & .5 & .6 & .3 \\ .7 & .3 & .4 & .3 \end{bmatrix}$. Then $[a_{ij}] \tilde{\cup} [b_{ij}] = \begin{bmatrix} .4 & .5 & .7 & .6 \\ .2 & .7 & .6 & .4 \\ .7 & .4 & .6 & .3 \end{bmatrix}$, $[a_{ij}] \tilde{\cap} [b_{ij}] = \begin{bmatrix} .3 & .2 & .6 & .5 \\ .1 & .5 & .5 & .3 \\ .3 & .3 & .4 & .2 \end{bmatrix}$ $[a_{ij}]^0 = \begin{bmatrix} .7 & .8 & .4 & .5 \\ .9 & .3 & .5 & .6 \\ .7 & .6 & .4 & .8 \end{bmatrix}$

Proposition1. Let $[a_{ij}] \in$ FSM$_{m\,X\,n}$. Then

i) $[[a_{ij}]^0]^0 = [a_{ij}]$

ii) $[a_{ij}] \tilde{\subseteq} [a_{ij}]$

iii) $[a_{ij}] \tilde{\cup} [a_{ij}] = [a_{ij}]$

iv) $[a_{ij}] \tilde{\cap} [a_{ij}] = [a_{ij}]$

v) $a_{ij}] \tilde{\cup} [0] = [a_{ij}]$

vi) $[a_{ij}] \tilde{\cap} [0] = [0]$

Proposition2. Let $[a_{ij}]$, $[b_{ij}]$, $[c_{ij}]$ $\in$ FSM$_{m\,X\,n}$. Then

i) $[a_{ij}] = [b_{ij}]$ and $[b_{ij}] = [c_{ij}] \Rightarrow [a_{ij}] = [c_{ij}]$

ii) $a_{ij}] \tilde{\subseteq} [b_{ij}]$ and $[b_{ij}] \tilde{\subseteq} [c_{ij}] \Rightarrow [a_{ij}] = [c_{ij}]$

Proposition3. Let $[a_{ij}]$, $[b_{ij}]$ $\in$ FSM$_{m\,X\,n}$. Then De Morgan's type results are true.

i) $([a_{ij}] \tilde{\cup} [b_{ij}])^0 = [a_{ij}]^0 \tilde{\cap} [b_{ij}]^0$

ii) $([a_{ij}] \tilde{\cap} [b_{ij}])^0 = [a_{ij}]^0 \tilde{\cup} [b_{ij}]^0$

Proof: For all i and j,

i) $([a_{ij}] \tilde{\cup} [b_{ij}])^0 = [\max\{a_{ij}, b_{ij}\}]^0 = [\,1 - \max\{a_{ij}, b_{ij}\}\,] = [\,\min\{1 - a_{ij}, 1 - b_{ij}\}\,] = [a_{ij}]^0 \tilde{\cap} [b_{ij}]^0$

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ii) $([a_{ij}] \tilde{\cap} [b_{ij}])^0 = [\min\{a_{ij}, b_{ij}\}]^0 = [1 - \min\{a_{ij}, b_{ij}\}] = [\max\{1 - a_{ij}, 1 - b_{ij}\}] = [a_{ij}]^0 \tilde{\cup} [b_{ij}]^0$

Proposition4. Let $[a_{ij}]$, $[b_{ij}]$, $[c_{ij}] \in FSM_{m \times n}$. Then

i) $[a_{ij}] \tilde{\cup} ([b_{ij}] \tilde{\cap} [c_{ij}]) = ([a_{ij}] \tilde{\cup} [b_{ij}]) \tilde{\cap} ([a_{ij}] \tilde{\cup} [c_{ij}])$

ii) $[a_{ij}] \tilde{\cap} ([b_{ij}] \tilde{\cup} [c_{ij}]) = ([a_{ij}] \tilde{\cap} [b_{ij}]) \tilde{\cup} ([a_{ij}] \tilde{\cap} [c_{ij}])$

2.11. Fuzzy Soft Rectangular Matrix [7] Let $[a_{ij}] \in FSM_{m \times n}$. Then $[a_{ij}]$ is said to be a Fuzzy Soft Rectangular Matrix if m ≠ n .

2.12. Fuzzy Soft Square Matrix [7] Let $[a_{ij}] \in FSM_{m \times n}$. Then $[a_{ij}]$ is said to be a Fuzzy Soft Square Matrix if m = n.

2.13. Fuzzy Soft Row Matrix [7] Let $[a_{ij}] \in FSM_{m \times n}$. Then $[a_{ij}]$ is said to be a Fuzzy Soft Row Matrix if m= 1 .

2.14. Fuzzy Soft Column Matrix [7] Let $[a_{ij}] \in FSM_{m \times n}$. Then $[a_{ij}]$ is said to be a Fuzzy Soft Column Matrix if n = 1 .

2.15. Fuzzy Soft Diagonal Matrix [7] Let $[a_{ij}] \in FSM_{m \times n}$. Then $[a_{ij}]$ is said to be a Fuzzy Soft Diagonal Matrix if m = n and $a_{ij} = 0$ for all i ≠ j .

2.16. Fuzzy Soft Upper Triangular Matrix [7] Let $[a_{ij}] \in FSM_{m \times n}$. Then $[a_{ij}]$ is said to be a Fuzzy Soft Upper Triangular Matrix if m = n and $a_{ij} = 0$ for all i> j .

2.17. Fuzzy Soft Lower Triangular Matrix [7] Let $[a_{ij}] \in FSM_{m \times n}$. Then $[a_{ij}]$ is said to be a Fuzzy Soft Lower Triangular Matrix if m = n and $a_{ij} = 0$ for all i< j .

2.18. Fuzzy Soft Triangular Matrix [7] Let $[a_{ij}] \in FSM_{m \times n}$. Then $[a_{ij}]$ is said to be a Fuzzy Soft Triangular Matrix if it is either fuzzy soft lower or fuzzy soft upper triangular matrix for all i and j .

2.19. t-Norm [10]: Let T : [0,1] x [0,1] be a function satisfying the following axioms:

i) T(a , 1) = a , ∀ a ∈ [0,1] (Identity)

ii) T(a, b) = T(b ,a) , ∀ a ,b ∈ [0,1] (Commutativity)

iii) if $b_1 \le b_2$, then T(a, b_1) ≤ T(a, b_2) , ∀ a , b_1 , b_2 ∈ [0,1] (Monotonicity)

iv) T (a, T(b, c)) = T(T (a, b) , c) , ∀ a ,b ,c ∈ [0,1] (Associativity)

Then T is called t-norm. A t-norm is said to be continuous if T is continuous function in [0,1]. An example of continuous t- Norm is a b .

N.B : The functions used for intersection of fuzzy sets are called t-norms.

2.20. t- Conorm [10]: Let S : [0,1] x [0,1] be a function satisfying the following axioms:

i) S(a , 0) = a , ∀ a ∈ [0,1] (Identity)

ii) S(a, b) = S(b, a) , ∀ a ,b ∈ [0,1] (Commutativity)

iii) if $b_1 \leq b_2$, then S(a, b_1) ≤ S(a, b_2) , ∀ a , b_1 , b_2 ∈ [0,1] (Monotonicity)

iv) S (a, S(b, c)) = S(S (a ,b) , c) ∀ a ,b , c ∈ [0,1] (Associativity)

Then S is called t- conorm. A t- conorm is said to be continuous if S is continuous function in [0,1].

N.B. :The functions used for union of fuzzy sets are called t-conorms.

An example of continuous t- Conorm is a + b− a .b .

2.21. Union of Fuzzy Soft Matrices on t-norm: Let $[a_{ij}]$, $[b_{ij}]$ ∈FSM$_{m\ X\ n}$.Then Union of Fuzzy Soft Matrices $[a_{ij}]$ and $[b_{ij}]$ on t-norm is defined by $[a_{ij}] \tilde{\cup} [b_{ij}] = [a_{ij} + b_{ij} - a_{ij} \,.\, b_{ij}]$ for all i and j.

2.22. Intersection of Fuzzy Soft Matrices on t-norm: Let $[a_{ij}]$, $[b_{ij}]$ ∈FSM$_{m\ X\ n}$.Then Intersection of Fuzzy Soft Matrices $[a_{ij}]$ and $[b_{ij}]$ on t-norm is defined by $[a_{ij}] \tilde{\cap} [b_{ij}] = [a_{ij} \,.\, b_{ij}]$ for all i and j.

Proposition5. Let $[a_{ij}]$, $[b_{ij}]$, $[c_{ij}]$ ∈ FSM$_{m\ X\ n}$. Then

i) $[a_{ij}] \tilde{\cup} [0] = [a_{ij}]$ iii) $[a_{ij}] \tilde{\cup} [b_{ij}] = [b_{ij}] \tilde{\cup} [a_{ij}]$

ii) $[a_{ij}] \tilde{\cup} [1] = [1]$ iv) $([a_{ij}] \tilde{\cup} [b_{ij}]) \tilde{\cup} [c_{ij}] = [a_{ij}] \tilde{\cup} ([b_{ij}] \tilde{\cup} [c_{ij}])$

Proposition6. Let $[a_{ij}]$, $[b_{ij}]$, $[c_{ij}]$ ∈ FSM$_{m\ X\ n}$. Then

i) $[a_{ij}] \tilde{\cap} [0] = [a_{ij}]$ iii) $[a_{ij}] \tilde{\cap} [b_{ij}] = [b_{ij}] \tilde{\cap} [a_{ij}]$

ii) $[a_{ij}] \tilde{\cap} [1] = [a_{ij}]$ iv) $([a_{ij}] \tilde{\cap} [b_{ij}]) \tilde{\cap} [c_{ij}] = [a_{ij}] \tilde{\cap} ([b_{ij}] \tilde{\cap} [c_{ij}])$

Proposition7. Let $[a_{ij}]$, $[b_{ij}]$ ∈ FSM$_{m\ X\ n}$. Then De Morgan's type results are true :

i) $([a_{ij}] \tilde{\cup} [b_{ij}])^0 = [a_{ij}]^0 \tilde{\cap} [b_{ij}]^0$ ii) $([a_{ij}] \tilde{\cap} [b_{ij}])^0 = [a_{ij}]^0 \tilde{\cup} [b_{ij}]^0$

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Proof: for all i and j ,

i) $([a_{ij}] \tilde{\cup} [b_{ij}])^0 = [a_{ij} + b_{ij} - a_{ij}.b_{ij}]^0 = [1 - (a_{ij} + b_{ij} - a_{ij}.b_{ij})] = [1 - a_{ij} - b_{ij} + a_{ij}.b_{ij}]$

$= [(1 - a_{ij})(1 - b_{ij}) = [1 - a_{ij}] \tilde{\cap} [1 - b_{ij}] = [a_{ij}]^0 \tilde{\cap} [b_{ij}]^0$ □

ii) The proof of (ii) is exactly similar to (i).

2.23. Scalar Multiplication of Fuzzy Soft Matrix : Let $[a_{ij}] \in FSM_{m\,X\,n}$. Then Scalar Multiplication of Fuzzy Soft Matrix $[a_{ij}]$ by a scalar k denoted by k $[a_{ij}]$ is defined as k $[a_{ij}]$ = [ka_{ij}] , $0 \leq k \leq 1$.

Proposition8. Let $[a_{ij}] \in FSM_{m\,X\,n}$ and s and t are two scalars such that $0 \leq s, t \leq 1$. Then

i) $s(t[a_{ij}]) = (st)[a_{ij}]$ ii) $s \leq t \Rightarrow s[a_{ij}] \tilde{\subseteq} t[a_{ij}]$ iii) $[a_{ij}] \tilde{\subseteq} [b_{ij}] \Rightarrow s[a_{ij}] \tilde{\subseteq} s[b_{ij}]$

2.24. Three Important Operators of t- Norm :

i) **Minimum Operator :** $T_M(\mu_1, \mu_2, \ldots, \mu_n) = \min(\mu_1, \mu_2, \ldots, \mu_n)$

ii) **Product Operator :** $T_P(\mu_1, \mu_2, \ldots, \mu_n) = \prod_{i=1}^{n} \mu_i$

iii) **Operator Lukasiewicz t-norm (Bounded t-norm) :** $T_L(\mu_1, \mu_2, \ldots, \mu_n) = \max(\sum_{i=1}^{n} \mu_i - n + 1, 0)$

Proposition9. Let $[a_{ij}], [b_{ij}], [c_{ij}] \in FSM_{m\,X\,n}$. Then

i) $[a_{ij}] \tilde{\cap}_{T_M} [0] = [a_{ij}]$ ii) $[a_{ij}] \tilde{\cap}_{T_M} [1] = [a_{ij}]$

iii) $[a_{ij}] \tilde{\cap}_{T_M} [b_{ij}] = [b_{ij}] \tilde{\cap}_{T_M} [a_{ij}]$ iv) $([a_{ij}] \tilde{\cap}_{T_M} [b_{ij}]) \tilde{\cap}_{T_M} [c_{ij}] = [a_{ij}] \tilde{\cap}_{T_M} ([b_{ij}] \tilde{\cap}_{T_M} [c_{ij}])$

Proposition10. Let $[a_{ij}], [b_{ij}], [c_{ij}] \in FSM_{m\,X\,n}$. Then

i) $[a_{ij}] \tilde{\cap}_{T_P} [0] = [0]$ ii) $[a_{ij}] \tilde{\cap}_{T_P} [1] = [a_{ij}]$

iii) $[a_{ij}] \tilde{\cap}_{T_P} [b_{ij}] = [b_{ij}] \tilde{\cap}_{T_P} [a_{ij}]$ iv) $([a_{ij}] \tilde{\cap}_{T_P} [b_{ij}]) \tilde{\cap}_{T_P} [c_{ij}] = [a_{ij}] \tilde{\cap}_{T_P} ([b_{ij}] \tilde{\cap}_{T_P} [c_{ij}])$

Proposition11. Let $[a_{ij}], [b_{ij}], [c_{ij}] \in FSM_{m\,X\,n}$. Then

i) $[a_{ij}] \tilde{\cap}_{T_L} [0] = [0]$ ii) $[a_{ij}] \tilde{\cap}_{T_L} [1] = [a_{ij}]$

iii) $[a_{ij}] \tilde{\cap}_{T_L} [b_{ij}] = [b_{ij}] \tilde{\cap}_{T_L} [a_{ij}]$ iv) $([a_{ij}] \tilde{\cap}_{T_L} [b_{ij}]) \tilde{\cap}_{T_L} [c_{ij}] = [a_{ij}] \tilde{\cap}_{T_L} ([b_{ij}] \tilde{\cap}_{T_L} [c_{ij}])$

2.25. Three Important Operators of t-Conorm :

i) $S_M\{\mu_1, \mu_2, \ldots, \mu_n\} = \max\{\mu_1, \mu_2, \ldots, \mu_n\}$

ii) $S_P\{ \mu_1 , \mu_2 ,\ldots, \mu_n \} = 1 - \prod_{i=1}^{n}(1 - \mu_{\boldsymbol{i}})$

iii) $S_L\{ \mu_1 , \mu_2 ,\ldots, \mu_n \} = \min \{ \sum_{i=1}^{n} \mu_{\boldsymbol{i}} , 1 \}$

Proposition12. Let $[a_{ij}] , [b_{ij}] \in FSM_{m\ X\ n}$. Then

i) $[a_{ij}] \tilde{\cup}_{S_M} [0] = [a_{ij}]$ ii) $[a_{ij}] \tilde{\cup}_{S_M} [1] = [1]$

iii) $[a_{ij}] \tilde{\cup}_{S_M} [b_{ij}] = [b_{ij}] \tilde{\cup}_{S_M} [a_{ij}]$ iv) $([a_{ij}] \tilde{\cup}_{S_M} [b_{ij}]) \tilde{\cup}_{S_M} [c_{ij}] = [a_{ij}] \tilde{\cup}_{S_M} ([b_{ij}] \tilde{\cup}_{S_M} [c_{ij}])$

Proposition13. Let $[a_{ij}] , [b_{ij}] \in FSM_{m\ X\ n}$. Then

i) $[a_{ij}] \tilde{\cup}_{S_P} [0] = [a_{ij}]$ ii) $[a_{ij}] \tilde{\cup}_{S_P} [1] = [1]$

iii) $[a_{ij}] \tilde{\cup}_{S_P} [b_{ij}] = [b_{ij}] \tilde{\cup}_{S_P} [a_{ij}]$ iv) $([a_{ij}] \tilde{\cup}_{S_P} [b_{ij}]) \tilde{\cup}_{S_P} [c_{ij}] = [a_{ij}] \tilde{\cup}_{S_P} ([b_{ij}] \tilde{\cup}_{S_P} [c_{ij}])$

Proposition14. Let $[a_{ij}] , [b_{ij}] \in FSM_{m\ X\ n}$. Then

i) $[a_{ij}] \tilde{\cup}_{S_L} [0] = [a_{ij}]$ ii) $[a_{ij}] \tilde{\cup}_{S_L} [1] = [1]$

iii) $[a_{ij}] \tilde{\cup}_{S_L} [b_{ij}] = [b_{ij}] \tilde{\cup}_{S_L} [a_{ij}]$ iv) $([a_{ij}] \tilde{\cup}_{S_L} [b_{ij}]) \tilde{\cup}_{S_L} [c_{ij}] = [a_{ij}] \tilde{\cup}_{S_L} ([b_{ij}] \tilde{\cup}_{S_L} [c_{ij}])$

Example 5. From the above **Example 4**, $[a_{ij}] , [b_{ij}] \in FSM_{3\ X4}$ where

$[a_{ij}] = \begin{bmatrix} .3 & .2 & .6 & .5 \\ .1 & .7 & .5 & .4 \\ .3 & .4 & .6 & .2 \end{bmatrix}$ and $[b_{ij}] = \begin{bmatrix} .4 & .5 & .7 & .6 \\ .2 & .5 & .6 & .3 \\ .7 & .3 & .4 & .3 \end{bmatrix}$. Then $T_M([a_{ij}] , [b_{ij}]) = \begin{bmatrix} .3 & .2 & .6 & .5 \\ .1 & .5 & .5 & .3 \\ .3 & .3 & .4 & .2 \end{bmatrix}$,

$T_P([a_{ij}] , [b_{ij}]) = \begin{bmatrix} .12 & .1 & .42 & .3 \\ .2 & .35 & .3 & .12 \\ .21 & .12 & .24 & .06 \end{bmatrix}$ and $T_L([a_{ij}] , [b_{ij}]) = \begin{bmatrix} 0 & 0 & .3 & .1 \\ 0 & .2 & .1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

2.25. Arithmetic Mean (A.M.) of Fuzzy Soft Matrix : Let $\tilde{A} = [a_{ij}] \in FSM_{m\ X\ n}$. Then Arithmetic Mean of Fuzzy Soft Matrix of membership value denoted by $\tilde{A}_{AM}$ is defined as $\tilde{A}_{AM} = \frac{\sum_{j=1}^{n} \mu_{ij}^{\tilde{A}}}{n}$.

III. FUZZY SOFT MATRICES IN DECISION MAKING BASED ON T–NORM OPERATORS

In this section, we put forward fuzzy soft matrices in decision making by using different operators of t- norm.

Input: Fuzzy soft set of m objects, each of which has n parameters. **Output:** An optimum result.

ALGORITHM

Step- 1: Choose the set of parameters.

Step -2: Construct the fuzzy soft matrix for the set of parameters.

Step -3: Compute T_M.

Step- 4: Compute the arithmetic mean of membership value of fuzzy soft matrix as $A_{AM}(T_M)$.

Step-5: Find the decision with highest membership value.

Example 6: Suppose a company produces five types of water purifier p_1, p_2, p_3, p_4, p_5 such that U = { p_1, p_2, p_3, p_4, p_5 } and E = { e_1(only filter available) , e_2(only UV available) , e_3(UV+ RO available) } be the set of parameters . Suppose Mr. X is going to buy a purifier.

On the basis of the parameters, three experts Mr. A, Mr. B and Mr. C give their valuable comments on the water purifier and the following fuzzy soft matrices are constructed as follows:

$$A = \begin{bmatrix} .8 & .9 & .8 \\ .65 & .8 & .4 \\ .9 & .9 & .6 \\ 1 & .6 & .9 \\ .8 & .8 & .7 \end{bmatrix}, \quad B = \begin{bmatrix} .6 & .8 & .6 \\ .4 & .4 & .5 \\ .8 & .8 & .7 \\ .6 & .81 & .5 \\ .7 & .4 & .7 \end{bmatrix} \text{ and } C = \begin{bmatrix} .9 & .5 & .8 \\ .4 & .56 & .6 \\ .5 & .5 & .8 \\ .7 & .8 & .8 \\ .7 & .83 & .8 \end{bmatrix}.$$

$$\text{Then} \quad T_M = \begin{bmatrix} .6 & .5 & .6 \\ .4 & .4 & .4 \\ .5 & .5 & .6 \\ .6 & .6 & .5 \\ .7 & .4 & .7 \end{bmatrix} \quad \text{and} \quad A_{AM}(T_M) = \begin{bmatrix} .567 \\ .4 \\ .533 \\ .567 \\ .6 \end{bmatrix} \text{..........(3.1)}$$

From the above result (3.1) ,it is obvious that p_5 water purifier will be preferred .

Note: If T_P and T_L are used instead of T_M,then we have $T_P = \begin{bmatrix} .432 & .36 & .384 \\ .104 & .1792 & .12 \\ .36 & .36 & .336 \\ .42 & .3888 & .36 \\ .392 & .2656 & .392 \end{bmatrix}$ and $A_{AM}(T_P) = \begin{bmatrix} .392 \\ .1344 \\ .352 \\ .3896 \\ .3499 \end{bmatrix}$ ..(3.2)

$$T_L = \begin{bmatrix} .3 & .2 & .2 \\ 0 & 0 & 0 \\ .2 & .2 & .1 \\ .3 & .21 & .2 \\ .2 & .03 & .2 \end{bmatrix} \quad \text{and} \quad A_{AM}(T_L) = \begin{bmatrix} .233 \\ 0 \\ .167 \\ .236 \\ .143 \end{bmatrix} \text{............(3.3)}$$

From the above results (3.2) and (3.3), it is clear that p_1 water purifier and p_3 water purifier will be selected respectively by Mr. X .

IV. CONCLUSION

In this paper, we proposed fuzzy soft matrices and defined different types of fuzzy soft matrices. We have also given some definitions based on t-norm with examples and some properties with proof . Some operators on t-norm are also

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given. Finally, we extend our approach on t-norms in application of decision making problems. We have shown that decisions are different for different methods on same application. Our future work in this regard is to find other methods whether the notions in this paper yield fruitful result.

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