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Using Painleve Test on the Kinetic System for Yang-Mills Theory

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ABSTRACT*:* We study the Yang-mills theory as a kintic system, and used C. Becchi, A. Rouet and R. Stora; transformation to Renormalization of gauge theories, test the random of the system by use the painleve test.

KEY WORDS: Yang-Mills theory, Painleve Test.

I. INTRODUCTION

Yang-Mills theory is a non-Abelian gauge theory, and we used a lot in QCD calculation, the yang- mills theory is the most important in physics in the last fifty years. Yang-Mills theory was first discovered in the 1950, at this time, quantum electrodynamics was known to describe electromagnetism [1]. Yang–Mills theory seeks to describe the behavior of elementary particles. In 1954, Yang and Mills published a paper on the isotopic SU(2) invariance of the proton-neutron system [2]. Yang-Mills theory plays a central role in explaining fundamental interactions ,because both the strong and weak interactions are described by Yang-Mills theories [3].Yang-Mills theory allow one to describe both the Maxwell electromagnetic interactions and the Fermi weak interactions and to obtain the known value of the (Z^0) boson (weak boson) mass. Yang-Mills gauge theory with gauge group SU(3)×SU(2)×U(1). Here the first factor is the gauge group of QCD while (SU(2)×U(1)) gauge field is that transmitting what is called the electroweak force [4].

II. THE MODEL

We start from the lagrangian of Yang-Mills theory to SU(2) [5].

$$L = -\frac{1}{4}\,(F^a_{uv})^2 - \frac{1}{2}(\partial_u A^u)^2 - \bar{T}^a \partial_u D^u T^a \qquad (1)$$

Where $a = (1,2,3)$ and $u, v = (0,1,2,3)$

$$F^a_{uv} = \partial_u A^a_v - \partial_v A^a_u + g\epsilon^{abc} A^b_u A^c_v \qquad (2)$$

Where (A^a_u) refers to Standard fields and symbols ($T^a, \bar{T}^a$) mean the Faddeev-Popov ghosts (or ghost fields) are additional fields entered in calculation but not appear in the results for this reason it's called (ghost) for your attention all fields are operators the operations between them not commutation we used C. Becchi , A. Rouet , R. Stora and I.V. Tyutin; transformation to Renormalization of gauge theories [6].

$$\delta A^a_u = -\frac{\lambda}{g}\, D_u\, A^{au} \qquad (3)$$

$$\delta T^a = -\frac{\lambda}{2}\epsilon^{abc} T^b\, T^c \qquad (4)$$

$$\delta \bar{T}^a = -\frac{\lambda}{g\xi}\, D_u\;\; A^{au} \qquad (5)$$

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Where λ a constant now one can write the equations of motions:

$$\partial_u F^{auv} + g\epsilon^{abc} A_u^a F^{cuv} + \xi^{-1}\, \partial^u \partial_u A^{au} = g\epsilon^{abc}\, \bar{T}^b\, T^a \tag{6}$$

$$\partial^u(\partial_u T^a + \epsilon^{abc} A_u^b T^c) = 0 \tag{7}$$

$$\partial_u \partial^u \bar{T}^a + \epsilon^{abc} A_u^b \partial^u\, \bar{T}^c = 0 \tag{8}$$

with this condition $A_0^a(x,t) = 0$ C. Becchi , A. Route , REStore and I.V. Tyutin transformation in equations must to be conserved with these conditions:

$$\partial_t T^a = 0 \qquad \partial_t \bar{T}^b = 0 \tag{9}$$

we consider the fields are homogeneous in the space :

$$A_i^a(x,t) = A_i^a \quad ; \quad \partial_i A_i^a = 0 \;; \quad \partial_i T_i = 0 \tag{10}$$

the equation of motion it will be :

$$\left.\begin{array}{c} \ddot{A}_i^a + g^2 A_i^a\left(A_i^a A_j^b - A_j^a A_i^b\right) = 0 \\ \Pi T^a = 0 \\ \Pi \bar{T}^a = 0 \end{array}\right) \tag{11}$$

we use columns matrices invariant parameters :

$$A_i^a(t) = O_i^a f^a(t) \tag{12}$$

$$O_i^a O_i^b = \frac{1}{g} \tag{13}$$

we get on the new system for equation of motion:

$$\ddot{B}^a + \sum_{b\neq a} (B^b)^2 B^a = 0 \tag{14}$$

$$O_i^a = \frac{\delta_v^c}{g} \tag{15}$$

To transfers from directions of isospin to space $A_i^a = B^a$ and take in account these equations

$$\left.\begin{array}{l} \ddot{B}^1 + (B^2)^2 B^1 = 0 \\ \ddot{B}^2 + (B^1)^2 B^2 = 0 \end{array}\right\} \tag{16}$$

Where B^1 and B^2 are operators in Hilbert space and finally one can write the Heisenberg picture for equations of motion

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$$\left.\begin{array}{l} U = \langle \Psi | B^1 \, | \Psi \rangle \\ V = \langle \Psi | B^2 \, | \Psi \rangle \end{array}\right\} \qquad (17)$$

We can write equation (17) as a kinetic system

$$\left.\begin{array}{l} \ddot{U} + V^2 U = 0 \\ \ddot{V} + U^2 V = 0 \end{array}\right\} \qquad (18)$$

We can now test the random of the system by use the painleve test after we determine the leading singularity [7].

$$\left.\begin{array}{l} U(t) = a(t - t_0)^{-\alpha} \\ V(t) = b(t - t_0)^{-\beta} \end{array}\right\} \qquad (19)$$

readily one can find the constants $(a = b = \sqrt{2i})$ $(\alpha = \beta = 1)$ now searching in cases of resonance and write equations for first and second variables.

$$U(t) = a(t - t_0)^{-\alpha} + q(t - t_0)^{-\alpha + r} \qquad (20)$$

$$V(t) = b(t - t_0)^{-\beta} + q(t - t_0)^{-\beta + r} \qquad (21)$$

Inserting equations of motion with parallel linear terms for (q, p):

$$(r^2 - 3r - 4)(r^2 - 3r + 4) = 0 \qquad (22)$$

we obtained $r = -1$; $r = 4$; $r = \frac{1}{2}(3 \pm i\sqrt{7})$ we can say that Yang-mills theory is non-integrable system.

III. CONCLUSION

Using the lagrangian of Yang-Mills theory [5]. introduceing C. Becchi , A. Rouet , R. Stora and I.V. Tyutin for transformation to Renormalization of gauge theories [6]. test the random of the system by use the painleve test [7]. Finally We found that yang-mills theory is non-integrable according to Painleve test.

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