

# **Fuzzy Soft La-Semigroups – A New Approach**

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**ABSTRACT:** Soft set theory is one of the recent topics gaining significance in finding rational and logical solutions to various real life problems which involve uncertainty, impreciseness and vagueness. In this paper the concept of fuzzy soft LA-semigroups are introduced and some basic properties are studied.

**KEYWORDS:** Fuzzy LA-semigroup, Fuzzy soft LA-semigroup, Fuzzy soft ideal

## **I. INTRODUCTION**

Uncertainty is present in almost every sphere of our daily life. Traditional Mathematical tools are not sufficient to handle all practical problems in fields such as Medical Science, Social Science, Engineering, Economics etc. Zadeh[15], in 1965 was the first to come up with his remarkable theory of fuzzy sets for dealing these types of uncertainties. In 1999, Molodsov[9] initiated the theory of soft sets as a new mathematical tool for dealing uncertainty. Maji et al [8] initiated the concept of fuzzy soft sets. The algebraic structures of set theories dealing with uncertainties have been studied by many mathematicians. Rosenfeld [14] proposed the concept of fuzzy groups. Aktas and cagman [1] introduced the notion of soft groups. Aygunoglu and Aygun [4] introduced the concept of fuzzy soft groups. Studies of soft semigroups are carried out by several Researchers. In this paper the concept of fuzzy soft LA-semigroup is introduced and some basic properties are studied.

## **II. RELATED WORK**

In 1972, Kazim and Naseerdin[6] introduced LA-semigroups and studied their properties. The notion of Soft semigroups are introduced by Ali and Shabir [3] in 2009. Fuzzy soft semigroups are introduced by Munazza Naz et al., [11] in 2013. In 2010, Muhammad Aslam, Muhammad shabir, Asif mehmoood[10] introduced soft LA-semigroups and established some important theorems.

## **III. PRELIMINARIES**

### **Definition:3.1**

Let  $X$  be a non-empty set. A **fuzzy set**  $\mu$  of  $X$  is a mapping given by  $\mu: X \rightarrow [0,1]$ . The set of all fuzzy subsets of  $X$  is called the **fuzzy power set** of  $X$  and is denoted by  $FP(X)$ .

### **Definition:3.2**

Let  $\mu, \nu \in FP(X)$ . Then  $\mu \vee \nu$  and  $\mu \wedge \nu$  are fuzzy subsets of  $X$ , defined as follows

$$(\mu \vee \nu)(x) = \mu(x) \vee \nu(x)$$

$$(\mu \wedge \nu)(x) = \mu(x) \wedge \nu(x) \text{ for all } x \in X$$

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The fuzzy subsets  $\mu \vee \nu$  and  $\mu \wedge \nu$  are called the **fuzzy union** and **fuzzy intersection** of  $\mu$  and  $\nu$ .

### Definition:3.3

Let  $\mu$  and  $\nu$  be two fuzzy sets of a semigroup  $S$ . Then the **product**  $\mu \circ \nu$  is defined as follows,

$$(\mu \circ \nu)(x) = \begin{cases} \bigvee_{x=yz} (\mu(y) \wedge \nu(z)) & \text{if for every } y, z \in S, \exists x = yz \\ 0 & \text{otherwise} \end{cases}$$

for all  $x \in S$ . The operation  $\circ$  is associative.

### Definition:3.4

A fuzzy set  $\mu$  of a semigroup  $S$  is called a **fuzzy left (right) ideal** of  $S$  if

$$\mu(ab) \geq \mu(b) \quad (\mu(ab) \geq \mu(a)) \quad \text{for all } a, b \in S$$

A fuzzy set  $\mu$  of a semigroup  $S$  is called a **fuzzy ideal** of  $S$  if it is both a fuzzy left and a fuzzy right ideal of  $S$ .

### Definition:3.5

Let  $U$  be an initial universe and  $E$  be the set of parameters and  $A$  be the subset of  $E$  and  $P(U)$  denotes the power set of  $U$ . Then the pair  $(F, A)$  is called a **soft set** over  $U$ , where  $F$  is a mapping given by,  $F: A \rightarrow P(U)$ .

The pair  $(\tilde{F}, A)$  is called a **fuzzy soft set** over  $U$ , where  $\tilde{F}$  is a mapping given by  $\tilde{F}: A \rightarrow FP(U)$ .

### Definition:3.6

Let  $(F, A)$  and  $(G, B)$  be any two soft sets over a semigroup  $S$ . Then, the **restricted product** of  $(F, A)$  and  $(G, B)$  is defined as the soft set  $(H, C) = (F, A) \hat{\circ} (G, B)$ , where  $C = A \cap B$  and  $H$  is a set valued function from  $C$  to  $P(S)$  defined as  $H(e) = F(e)G(e)$  for all  $e \in C$ .

### Definition:3.7

If  $(\tilde{F}, A)$  and  $(\tilde{G}, B)$  are two fuzzy soft sets over a common universe  $U$  then  $(\tilde{F}, A)$  **AND**  $(\tilde{G}, B)$  is a fuzzy soft set denoted by  $(\tilde{F}, A) \wedge (\tilde{G}, B)$  and defined as,  $(\tilde{F}, A) \wedge (\tilde{G}, B) = (\tilde{H}, A \times B)$ , where  $\tilde{H}(a, b) = \tilde{F}(a) \wedge \tilde{G}(b)$  for all  $(a, b) \in A \times B$ .

### Definition:3.8

If  $(\tilde{F}, A)$  and  $(\tilde{G}, B)$  are two fuzzy soft sets over a common universe  $U$  then  $(\tilde{F}, A)$  **OR**  $(\tilde{G}, B)$  is a fuzzy soft set denoted by  $(\tilde{F}, A) \vee (\tilde{G}, B)$  and is defined as,  $(\tilde{F}, A) \vee (\tilde{G}, B) = (\tilde{H}, A \times B)$ , where  $\tilde{H}(a, b) = \tilde{F}(a) \vee \tilde{G}(b)$  for all  $(a, b) \in A \times B$ .

### Definition:3.9

The **extended union** of two fuzzy soft sets  $(\tilde{F}, A)$  and  $(\tilde{G}, B)$  over a common universe  $U$  is denoted as the fuzzy soft set  $(\tilde{H}, C) = (\tilde{F}, A) \cup_E (\tilde{G}, B)$ , where  $C = A \cup B$  and for all  $e \in C$ ,

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$$\tilde{H}(e) = \begin{cases} \tilde{F}(e) & \text{if } e \in A - B \\ \tilde{G}(e) & \text{if } e \in B - A \\ \tilde{F}(e) \vee \tilde{G}(e) & \text{if } e \in A \cap B \end{cases}$$

### Definition:3.10

The **extended intersection** of two fuzzy soft sets  $(\tilde{F}, A)$  and  $(\tilde{G}, B)$  over a common universe  $U$  is denoted as the fuzzy soft set  $(\tilde{H}, C) = (\tilde{F}, A) \cap_E (\tilde{G}, B)$ , where  $C = A \cup B$  and for all  $e \in C$ ,

$$\tilde{H}(e) = \begin{cases} \tilde{F}(e) & \text{if } e \in A - B \\ \tilde{G}(e) & \text{if } e \in B - A \\ \tilde{F}(e) \wedge \tilde{G}(e) & \text{if } e \in A \cap B \end{cases}$$

### Definition:3.11

If  $(\tilde{F}, A)$  and  $(\tilde{G}, B)$  are two fuzzy soft sets over a common universe  $U$  then their **restricted intersection** is a fuzzy soft set  $(\tilde{H}, C)$  denoted by  $(\tilde{F}, A) \cap_R (\tilde{G}, B) = (\tilde{H}, C)$  where  $C = A \cap B \neq \phi$  and  $\tilde{H}$  is a function from  $C$  to  $FP(U)$ , defined as  $\tilde{H}(e) = \tilde{F}(e) \wedge \tilde{G}(e)$  and for all  $e \in C$ .

### Definition:3.12

If  $(\tilde{F}, A)$  and  $(\tilde{G}, B)$  are two fuzzy soft sets over a common universe  $U$  then their **restricted union** is a fuzzy soft set  $(\tilde{H}, C)$  denoted by  $(\tilde{F}, A) \cup_R (\tilde{G}, B) = (\tilde{H}, C)$  where  $C = A \cap B \neq \phi$  and  $\tilde{H}$  is a function from  $C$  to  $FP(U)$ , defined as  $\tilde{H}(e) = \tilde{F}(e) \vee \tilde{G}(e)$  and for all  $e \in C$ .

### Definition:3.13

A groupoid  $(S, \cdot)$  is called a **left almost semigroup (LA-semigroup)**, if it satisfies left invertive law,

$$(a \cdot b) \cdot c = (c \cdot b) \cdot a \text{ for all } a, b, c \in S$$

## IV. FUZZY SOFT LA-SEMIGROUPS

Throughout this section  $S$  will denote LA-semigroup unless otherwise specified.

### Definition:4.1

If  $(\tilde{F}, A)$  be a fuzzy soft set of a LA-semigroup  $S$ . Then for any  $a \in A$ ,  $\tilde{F}(a)$  is called a **fuzzy LA-semigroup** if  $\tilde{F}(a) \circ \tilde{F}(a) \leq \tilde{F}(a)$

### Definition:4.2

Let  $(\tilde{F}, A)$  and  $(\tilde{G}, B)$  be any two fuzzy soft sets over an LA-semigroup  $S$ . Then the **restricted product** of  $(\tilde{F}, A)$  and  $(\tilde{G}, B)$  is defined as the fuzzy soft set  $(\tilde{H}, C) = (\tilde{F}, A) \odot (\tilde{G}, B)$ , where  $C = A \cap B$  and  $\tilde{H}$  is a set valued function from  $C$  to  $FP(S)$  defined as,

$$\tilde{H}(e) = \tilde{F}(e) \circ \tilde{G}(e) \text{ for all } e \in C$$

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**Definition:4.3**

A fuzzy soft set  $(\tilde{F}, A)$  over  $S$  is called a **fuzzy soft LA-semigroup** over  $S$  if

$$(\tilde{F}, A) \odot (\tilde{F}, A) \subseteq (\tilde{F}, A)$$

**Definition:4.4**

A fuzzy soft set  $(\tilde{F}, A)$  over  $S$  is said to be a **fuzzy soft left (right) ideal** over  $S$  if  $\tilde{F}(e)$  is a fuzzy left (right) ideal of  $S$  and for all  $e \in A$ . A fuzzy soft set  $(\tilde{F}, A)$  over  $S$  is said to be a **fuzzy soft ideal** if it is both a fuzzy soft left and a fuzzy soft right ideal over  $S$ .

**Theorem:4.5**

A fuzzy soft set  $(\tilde{F}, A)$  over  $S$  is a fuzzy soft LA-semigroup iff each  $\tilde{F}(a) \neq \emptyset$  is an fuzzy LA-semigroup of  $S$  for all  $a \in A$ .

**Proof:**

Assume  $(\tilde{F}, A)$  is a fuzzy soft LA-semigroup. By definition (4.3),

$$(\tilde{F}, A) \odot (\tilde{F}, A) \subseteq (\tilde{F}, A)$$

$$\Rightarrow \tilde{F}(a) \circ \tilde{F}(a) \leq \tilde{F}(a) \text{ for all } a \in A$$

Therefore  $\tilde{F}(a)$  is an fuzzy LA-semigroup of  $S$ .

Conversely, assume  $\tilde{F}(a)$  is an fuzzy LA-semigroup of  $S$ .

By definition(4.1),  $\tilde{F}(a) \circ \tilde{F}(a) \leq \tilde{F}(a)$  for all  $a \in A$

$$\Rightarrow (\tilde{F}, A) \odot (\tilde{F}, A) \subseteq (\tilde{F}, A)$$

Therefore  $(\tilde{F}, A)$  is a fuzzy soft LA-semigroup.

**Theorem:4.6**

Let  $(\tilde{F}, A)$  and  $(\tilde{G}, B)$  be two fuzzy soft LA-semigroups over  $S$ . Then,  $(\tilde{F}, A) \wedge (\tilde{G}, B)$  is also a fuzzy soft LA-semigroup over  $S$ .

**Proof:**

By definition(3.7),

$$(\tilde{F}, A) \wedge (\tilde{G}, B) = (\tilde{H}, A \times B), \text{ where } \tilde{H}(a, b) = \tilde{F}(a) \wedge \tilde{G}(b) \text{ for all } (a, b) \in A \times B.$$

Since  $\tilde{F}(a)$  and  $\tilde{G}(b)$  are fuzzy LA-semigroups over  $S$ .

Therefore, either  $\tilde{F}(a) \wedge \tilde{G}(b) = \emptyset$  (or)  $\tilde{F}(a) \wedge \tilde{G}(b)$  is an fuzzy LA-semigroup of  $S$ .

which implies,  $\tilde{H}(a, b)$  is an fuzzy LA-semigroup of  $S$ .

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$\therefore (\tilde{H}, A \times B)$  is a fuzzy soft LA-semigroup over S.

Hence  $(\tilde{F}, A) \wedge (\tilde{G}, B)$  is a fuzzy soft LA-semigroup over S.

**Theorem:4.7**

Let  $(\tilde{F}, A)$  and  $(\tilde{G}, B)$  be two fuzzy soft LA-semigroups over S. Then  $(\tilde{F}, A) \cap_E (\tilde{G}, B)$  is also a fuzzy soft LA-semigroup over S.

**Proof:**

Assume  $(\tilde{F}, A)$  and  $(\tilde{G}, B)$  be two fuzzy soft LA-semigroups over S.

By definition (3.10),

$(\tilde{F}, A) \cap_E (\tilde{G}, B) = (\tilde{H}, C)$ , where  $C = A \cup B$  and for all  $e \in C$ ,

$$\tilde{H}(e) = \begin{cases} \tilde{F}(e) & \text{if } e \in A - B \\ \tilde{G}(e) & \text{if } e \in B - A \\ \tilde{F}(e) \wedge \tilde{G}(e) & \text{if } e \in A \cap B \end{cases}$$

Then for all  $e \in C \Rightarrow$  either  $e \in A - B$  (or)  $e \in B - A$  (or)  $e \in A \cap B$ .

If  $e \in A - B \Rightarrow \tilde{H}(e) = \tilde{F}(e)$

If  $e \in B - A \Rightarrow \tilde{H}(e) = \tilde{G}(e)$

If  $e \in A \cap B \Rightarrow \tilde{H}(e) = \tilde{F}(e) \wedge \tilde{G}(e)$

From the above, we have

$\tilde{F}(e) \wedge \tilde{G}(e)$  is an fuzzy LA-semigroup of S.

$\Rightarrow \tilde{H}(e)$  is an fuzzy LA-semigroup of S. Therefore  $(\tilde{H}, C)$  is a fuzzy soft LA-semigroup over S.

Hence  $(\tilde{F}, A) \cap_E (\tilde{G}, B)$  is a fuzzy soft LA-semigroup over S.

**Theorem:4.8**

Let  $(\tilde{F}, A)$  and  $(\tilde{G}, B)$  be two fuzzy soft LA-semigroups over S such that  $A \cap B \neq \emptyset$ . Then  $(\tilde{F}, A) \cap_R (\tilde{G}, B)$  is a fuzzy soft LA-semigroup over S.

**Proof:**

Assume  $(\tilde{F}, A)$  and  $(\tilde{G}, B)$  are fuzzy soft LA-semigroups over S.

By definition (3.11),

$(\tilde{F}, A) \cap_R (\tilde{G}, B) = (\tilde{H}, C)$  where  $C = A \cap B \neq \emptyset$  and  $\tilde{H}(e) = \tilde{F}(e) \wedge \tilde{G}(e)$  for all  $e \in C$ .

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In the above,  $\tilde{F}(e)$  and  $\tilde{G}(e)$  are fuzzy LA-semigroups of S and also  $\tilde{F}(e) \wedge \tilde{G}(e)$  is a fuzzy LA-semigroup of S is either empty or a fuzzy LA-semigroup of S.

Hence,  $(\tilde{F}, A) \cap_R (\tilde{G}, B) = (\tilde{H}, C)$  is a fuzzy soft LA-semigroup over S.

**Theorem:4.9**

Let  $(\tilde{F}, A)$  and  $(\tilde{G}, B)$  be two fuzzy soft LA-semigroups over S such that  $A \cap B = \emptyset$ . Then  $(\tilde{F}, A) \cup_E (\tilde{G}, B)$  is also a fuzzy soft LA-semigroup over S.

**Proof:**

Assume  $(\tilde{F}, A)$  and  $(\tilde{G}, B)$  be two fuzzy soft LA-semigroups.

By definition (3.9),

$(\tilde{F}, A) \cup_E (\tilde{G}, B) = (\tilde{H}, C)$ , where  $C = A \cup B$  and for all  $e \in C$ ,

$$\tilde{H}(e) = \begin{cases} \tilde{F}(e) & \text{if } e \in A - B \\ \tilde{G}(e) & \text{if } e \in B - A \\ \tilde{F}(e) \vee \tilde{G}(e) & \text{if } e \in A \cap B \end{cases}$$

Then for all  $e \in C$ , either  $e \in A - B$  (or)  $e \in B - A$

If  $e \in A - B \Rightarrow \tilde{H}(e) = \tilde{F}(e)$  and

If  $e \in B - A \Rightarrow \tilde{H}(e) = \tilde{G}(e)$

From both the cases  $\tilde{H}(e)$  is a fuzzy LA-semigroup of S.

Therefore  $(\tilde{H}, C)$  is a fuzzy soft LA-semigroup over S.

Hence  $(\tilde{F}, A) \cup_E (\tilde{G}, B)$  is a fuzzy soft LA-semigroup over S.

**Theorem:4.10**

A fuzzy soft set  $(\tilde{F}, A)$  over S is a fuzzy soft ideal over S iff  $\tilde{F}(a) \neq \emptyset$  is a fuzzy ideal of S for all  $a \in A$ .

**Proof:**

From the definition (4.4) it follows that.

**Theorem:4.11**

Let  $(\tilde{F}, A)$  and  $(\tilde{G}, A)$  are two fuzzy soft ideals over S. Then  $(\tilde{F}, A) \wedge (\tilde{G}, A)$  is also a fuzzy soft ideal over S.

**Proof:**

By definition (3.7),

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$(\tilde{F}, A) \wedge (\tilde{G}, B) = (\tilde{H}, A \times B)$ , where  $\tilde{H}(a, b) = \tilde{F}(a) \wedge \tilde{G}(b)$  for all  $(a, b) \in A \times B$ .

Since  $(\tilde{F}, A)$  and  $(\tilde{G}, B)$  are fuzzy soft ideals over  $S$ . Then  $\tilde{F}(a)$  and  $\tilde{G}(b)$  are fuzzy ideals of  $S$  for every  $a \in A$  and  $b \in B$ .

$\Rightarrow \tilde{F}(a) \wedge \tilde{G}(b)$  is a fuzzy ideal of  $S$ . Therefore  $(\tilde{H}, A \times B)$  is a fuzzy soft ideal over  $S$ .

Hence  $(\tilde{F}, A) \wedge (\tilde{G}, B)$  is a fuzzy soft ideal over  $S$ .

**Theorem:4.12**

Let  $(\tilde{F}, A)$  and  $(\tilde{G}, B)$  are two fuzzy soft ideals over  $S$ . Then  $(\tilde{F}, A) \vee (\tilde{G}, B)$  is also a fuzzy soft ideal over  $S$ .

**Proof:**

By definition(3.8),

$(\tilde{F}, A) \vee (\tilde{G}, B) = (\tilde{H}, A \times B)$ , where  $\tilde{H}(a, b) = \tilde{F}(a) \vee \tilde{G}(b)$  for all  $(a, b) \in A \times B$ .

Since  $(\tilde{F}, A)$  and  $(\tilde{G}, B)$  are fuzzy soft ideals over  $S$ . Then  $\tilde{F}(a)$  and  $\tilde{G}(b)$  are fuzzy ideals of  $S$  for every  $a \in A$  and  $b \in B$ .

$\Rightarrow \tilde{F}(a) \vee \tilde{G}(b)$  is a fuzzy ideal of  $S$ . Therefore  $(\tilde{H}, A \times B)$  is a fuzzy soft ideal over  $S$ .

Hence  $(\tilde{F}, A) \vee (\tilde{G}, B)$  is a fuzzy soft ideal over  $S$ .

**Theorem:4.13**

Let  $(\tilde{F}, A)$  and  $(\tilde{G}, B)$  are two fuzzy soft ideals over  $S$ , such that  $A \cap B \neq \emptyset$ . Then  $(\tilde{F}, A) \cap_R (\tilde{G}, B)$  is also a fuzzy soft ideal over  $S$ .

**Proof:**

By definition(3.11),

$(\tilde{F}, A) \cap_R (\tilde{G}, B) = (\tilde{H}, C)$  where  $C = A \cap B$  and  $\tilde{H}(e) = \tilde{F}(e) \wedge \tilde{G}(e)$  for all  $e \in C$ .

Then  $\tilde{F}(e) \wedge \tilde{G}(e)$  is a fuzzy ideal of  $S$ .

$\Rightarrow \tilde{H}(e)$  is a fuzzy ideal of  $S$ . Therefore  $(\tilde{H}, C)$  is a fuzzy soft ideal over  $S$ .

Hence  $(\tilde{F}, A) \cap_R (\tilde{G}, B)$  is a fuzzy soft ideal over  $S$ .

**Theorem:4.14**

Let  $(\tilde{F}, A)$  and  $(\tilde{G}, B)$  are two fuzzy soft ideals over  $S$ . Then  $(\tilde{F}, A) \cap_E (\tilde{G}, B)$  is also a fuzzy soft ideal over  $S$ .

**Proof:**

By definition(3.10),

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$(\tilde{F}, A) \cap_E (\tilde{G}, B) = (\tilde{H}, C)$ , where  $C = A \cup B$  and for all  $e \in C$ ,

$$\tilde{H}(e) = \begin{cases} \tilde{F}(e) & \text{if } e \in A - B \\ \tilde{G}(e) & \text{if } e \in B - A \\ \tilde{F}(e) \wedge \tilde{G}(e) & \text{if } e \in A \cap B \end{cases}$$

Then for all  $e \in C \Rightarrow$  either  $e \in A - B$  (or)  $e \in B - A$  (or)  $e \in A \cap B$ .

If  $e \in A - B \Rightarrow \tilde{H}(e) = \tilde{F}(e)$

If  $e \in B - A \Rightarrow \tilde{H}(e) = \tilde{G}(e)$

If  $e \in A \cap B \Rightarrow \tilde{H}(e) = \tilde{F}(e) \wedge \tilde{G}(e)$

From the above, we have

$\tilde{F}(e) \wedge \tilde{G}(e)$  is a fuzzy ideal of S.

$\Rightarrow \tilde{H}(e)$  is a fuzzy ideal of S. Therefore  $(\tilde{H}, C)$  is a fuzzy soft ideal over S.

Hence  $(\tilde{F}, A) \cap_E (\tilde{G}, B)$  is a fuzzy soft ideal over S.

### Theorem:4.15

Let  $(\tilde{F}, A)$  and  $(\tilde{G}, B)$  are two fuzzy soft ideals over S. Then  $(\tilde{F}, A) \cup_E (\tilde{G}, B)$  is also a fuzzy soft ideal over S.

#### Proof:

By definition(3.9),

$(\tilde{F}, A) \cup_E (\tilde{G}, B) = (\tilde{H}, C)$ , where  $C = A \cup B$  and for all  $e \in C$ ,

$$\tilde{H}(e) = \begin{cases} \tilde{F}(e) & \text{if } e \in A - B \\ \tilde{G}(e) & \text{if } e \in B - A \\ \tilde{F}(e) \vee \tilde{G}(e) & \text{if } e \in A \cap B \end{cases}$$

Then for all  $e \in C$ , either  $e \in A - B$  (or)  $e \in B - A$  (or)  $e \in A \cap B$ .

If  $e \in A - B \Rightarrow \tilde{H}(e) = \tilde{F}(e)$

If  $e \in B - A \Rightarrow \tilde{H}(e) = \tilde{G}(e)$

If  $e \in A \cap B \Rightarrow \tilde{H}(e) = \tilde{F}(e) \vee \tilde{G}(e)$

From the above, we have

$\tilde{F}(e) \vee \tilde{G}(e)$  is a fuzzy ideal of S.

$\Rightarrow \tilde{H}(e)$  is a fuzzy ideal of S. Therefore  $(\tilde{H}, C)$  is a fuzzy soft ideal over S.

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Hence  $(\tilde{F}, A) \cup_E (\tilde{G}, B)$  is a fuzzy soft ideal over S.

### Theorem:4.16

Let  $(\tilde{F}, A)$  and  $(\tilde{G}, A)$  are two fuzzy soft ideals over S. Then  $(\tilde{F}, A) \cup_R (\tilde{G}, A)$  is also a fuzzy soft ideal over S.

### Proof:

By definition(3.12),

$(\tilde{F}, A) \cup_R (\tilde{G}, B) = (\tilde{H}, C)$  where  $C = A \cap B \neq \emptyset$  and  $\tilde{H}(e) = \tilde{F}(e) \vee \tilde{G}(e)$  for all  $e \in C$

Then  $\tilde{F}(a) \vee \tilde{G}(b)$  is an fuzzy ideal of S.

$\Rightarrow \tilde{H}(e)$  is an fuzzy ideal of S. Therefore  $(\tilde{H}, C)$  is a fuzzy soft ideal over S.

Hence  $(\tilde{F}, A) \cup_R (\tilde{G}, B)$  is a fuzzy soft ideal over S.

## V. CONCLUSION

The algebraic structure of set theories deal with uncertainties have been studied by many mathematicians. In this paper the concept of fuzzy soft LA-semigroup is introduced and some basic properties are studied.

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