ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 2, Issue 9, September 2013

A Study of $\overline{H}$-Function of Two Variables

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Abstract: In the present paper, the author has introduced a new special function namely $\overline{H}$-function of two variables in the literature of special functions. The definition, convergence, asymptotic behaviors have been considered. Next, we obtain some properties of $\overline{H}$-function of two variables. Later on, the author establish a series representation for the $\overline{H}$-function of two variables. Some interesting special cases of $\overline{H}$-function of two variables by comparing the series representation of $\overline{H}$-function of two variables and $\overline{H}$-function of one variable is also established. In the last, the author has given two special cases of -function of two variables which are not the special cases of H-function of two variables.

Key words: $\overline{H}$-function, $\overline{H}$-function of two variables, H-function, Series representation. **(2000 Mathematics Subject Classification: 33C70)**

I. INTRODUCTION

The $\overline{H}$-function occurring in the paper will be defined and represented by Buschman and Srivastava [1] as follows:

$$\overline{H}^{M,N}_{P,Q}\left[z\right]=\overline{H}^{M,N}_{P,Q}\left[z\left|\begin{matrix}(a_j;\alpha_j;A_j)_{1,N},(a_j;\alpha_j)_{N+1,P}\\(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right.\right]=\frac{1}{2\pi i}\int_{-i\infty}^{i\infty}\overline{\phi}(\xi)z^{\xi}d\xi \qquad (1.1)$$

where
$$\overline{\phi}(\xi)=\frac{\prod_{j=1}^{M}\Gamma(b_j-\beta_j\xi)\prod_{j=1}^{N}\left\{\Gamma(1-a_j+\alpha_j\xi)\right\}^{A_j}}{\prod_{j=M+1}^{Q}\left\{\Gamma(1-b_j+\beta_j\xi)\right\}^{B_j}\prod_{j=N+1}^{P}\Gamma(a_j-\alpha_j\xi)} \qquad (1.2)$$

Which contains fractional powers of the gamma functions. Here, and throughout the paper $a_j(j=1,...,p)$ and $b_j(j=1,...,Q)$ are complex parameters, $\alpha_j\geq 0(j=1,...,P),\beta_j\geq 0(j=1,...,Q)$ (not all zero simultaneously) and exponents $A_j(j=1,...,N)$ and $B_j(j=N+1,...,Q)$ can take on non integer values.

The following sufficient condition for the absolute convergence of the defining integral for the $\overline{H}$-function given by equation (1.1) have been given by (Buschman and Srivastava).

$$\Omega \equiv \sum_{j=1}^{M}\left|\beta_j\right| + \sum_{j=1}^{N}\left|A_j\alpha_j\right| - \sum_{j=M+1}^{Q}\left|\beta_j B_j\right| - \sum_{j=N+1}^{P}\left|\alpha_j\right| > 0 \tag{1.3}$$

and $\left|\arg(z)\right| < \frac{1}{2}\pi\Omega$ (1.4)

The behavior of the $\overline{H}$-function for small values of $|z|$ follows easily from a result recently given by (Rathie [6],p.306,eq.(6.9)).

We have

$$\overline{H}_{P,Q}^{M,N}[z] = 0\left(|z|^{\gamma}\right), \gamma = \min_{1\le j\le N}\left[\operatorname{Re}\left(b_j/\beta_j\right)\right], |z| \to 0 \tag{1.5}$$

If we take $A_j = 1(j = 1,2,...,N), B_j = 1(j = M+1,...,Q)$ in (1.1), the function $\overline{H}_{P,Q}^{M,N}[.]$ reduces to the Fox's H-function [3].

The following series representation for the $\overline{H}$-function will be required in the sequel [see Rathie, pp.305-306,eq.(6.8)]:

$$\overline{H}_{P,Q}^{M,N}\left[z\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}\\(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right.\right] =$$

$$\frac{\sum_{h=1}^{M}\sum_{r=0}^{\infty}\prod_{\substack{j=1\\j\neq h}}^{M}\Gamma\left(b_j-\beta_j\xi_{h,r}\right)\prod_{j=1}^{N}\left\{\Gamma\left(1-a_j+\alpha_j\xi_{h,r}\right)\right\}^{A_j}(-1)^r z^{\xi_{h,r}}}{\prod_{j=M+1}^{Q}\left\{\Gamma\left(1-b_j+\beta_j\xi_{h,r}\right)\right\}^{B_j}\prod_{j=N+1}^{P}\Gamma\left(a_j-\alpha_j\xi_{h,r}\right)r!\beta_h} \tag{1.6}$$

Where

$$\xi_{h,r} = \frac{(b_h+r)}{\beta_h}.$$

II. THE $\overline{H}$-FUNCTION OF TWO VARIABLES

The $\overline{H}$-function of two variables will be defined and represented in the following manner:

$$\overline{H}[x,y] = \overline{H}\left[\begin{matrix}x\\y\end{matrix}\right] = \overline{H}_{p_1,q_1:p_2,q_2;p_2,q_2}^{o,n_1:\ m_2,n_2:m_3,n_2}\left[\begin{matrix}x\\y\end{matrix}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,p_1},(c_j,\gamma_j;K_j)_{1,n_2},(c_j,\gamma_j)_{n_2+1,p_2},(e_j,E_j;R_j)_{1,n_3},(e_j,E_j)_{n_3+1,p_3}\\(b_j,\beta_j;B_j)_{1,q_1},(d_j,\delta_j)_{1,m_2},(d_j,\delta_j;L_j)_{m_2+1,q_2},(f_j,F_j)_{1,m_3},(f_j,F_j;S_j)_{m_3+1,q_3}\end{matrix}\right.\right]$$

$$= -\frac{1}{4\pi^2}\int_{L_1}\int_{L_2}\phi_1(\xi,\eta)\phi_2(\xi)\phi_3(\eta)x^{\xi}y^{\eta}d\xi d\eta \tag{2.1}$$

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

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Where

$$\phi_1(\xi,\eta)=\frac{\prod_{j=1}^{n_1}\Gamma\left(1-a_j+\alpha_j\xi+A_j\eta\right)}{\prod_{j=n_1+1}^{p_1}\Gamma\left(a_j-\alpha_j\xi-A_j\eta\right)\prod_{j=1}^{q_1}\Gamma\left(1-b_j+\beta_j\xi+B_j\eta\right)} \tag{2.2}$$

$$\phi_2(\xi)=\frac{\prod_{j=1}^{n_2}\left\{\Gamma\left(1-c_j+\gamma_j\xi\right)\right\}^{K_j}\prod_{j=1}^{m_2}\Gamma\left(d_j-\delta_j\xi\right)}{\prod_{j=n_2+1}^{p_2}\Gamma\left(c_j-\gamma_j\xi\right)\prod_{j=m_2+1}^{q_2}\left\{\Gamma\left(1-d_j+\delta_j\xi\right)\right\}^{L_j}} \tag{2.3}$$

$$\phi_3(\eta)=\frac{\prod_{j=1}^{n_3}\left\{\Gamma\left(1-e_j+E_j\eta\right)\right\}^{R_j}\prod_{j=1}^{m_3}\Gamma\left(f_j-F_j\eta\right)}{\prod_{j=n_3+1}^{p_3}\Gamma\left(e_j-E_j\eta\right)\prod_{j=m_3+1}^{q_3}\left\{\Gamma\left(1-f_j+F_j\eta\right)\right\}^{S_j}} \tag{2.4}$$

Where x and y are not equal to zero (real or complex), and an empty product is interpreted as unity p_i, q_i, n_i, m_j are non-negative integers such that $0 \le n_i \le p_i, o \le m_j \le q_j (i=1,2,3; j=2,3)$. All the $a_j(j=1,2,...,p_1), b_j(j=1,2,...,q_1), c_j(j=1,2,...,p_2), d_j(j=1,2,...,q_2),$ $e_j(j=1,2,...,p_3), f_j(j=1,2,...,q_3)$ are complex parameters. $\gamma_j \ge 0(j=1,2,...,p_2), \delta_j \ge 0(j=1,2,...,q_2)$ (not all zero simultaneously), similarly $E_j \ge 0(j=1,2,...,p_3), F_j \ge 0(j=1,2,...,q_3)$ (not all zero simultaneously). The exponents $K_j(j=1,2,...,n_3), L_j(j=m_2+1,...,q_2), R_j(j=1,2,...,n_3), S_j(j=m_3+1,...,q_3)$ can take on non-negative values.

The contour L_1 is in ξ-plane and runs from $-i\infty$ to $+i\infty$. The poles of $\Gamma\left(d_j-\delta_j\xi\right)(j=1,2,...,m_2)$ lie to the right and the poles of $\Gamma\left\{\left(1-c_j+\gamma_j\xi\right)\right\}^{K_j}(j=1,2,...,n_2), \Gamma\left(1-a_j+\alpha_j\xi+A_j\eta\right)(j=1,2,...,n_1)$ to the left of the contour. For $K_j(j=1,2,...,n_2)$ not an integer, the poles of gamma functions of the numerator in (2.3) are converted to the branch points.

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

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The contour L_2 is in η-plane and runs from $-i\infty$ to $+i\infty$. The poles of $\Gamma\left(f_j - F_j\eta\right)(j=1,2,...,m_3)$ lie to the right and the poles of $\Gamma\left\{\left(1-e_j+E_j\eta\right)\right\}^{R_j}(j=1,2,...,n_3),\Gamma\left(1-a_j+\alpha_j\xi+A_j\eta\right)(j=1,2,...,n_1)$ to the left of the contour. For $R_j(j=1,2,...,n_3)$ not an integer, the poles of gamma functions of the numerator in (2.4) are converted to the branch points.

The functions defined in (2.1) is an analytic function of x and y, if

$$U=\sum_{j=1}^{p_1}\alpha_j+\sum_{j=1}^{p_2}\gamma_j-\sum_{j=1}^{q_1}\beta_j-\sum_{j=1}^{q_2}\delta_j<0 \tag{2.5}$$

$$V=\sum_{j=1}^{p_1}A_j+\sum_{j=1}^{p_3}E_j-\sum_{j=1}^{q_1}B_j-\sum_{j=1}^{q_3}F_j<0 \tag{2.6}$$

The integral in (2.1) converges under the following set of conditions:

$$\Omega=\sum_{j=1}^{n_1}\alpha_j-\sum_{j=n_1+1}^{p_1}\alpha_j+\sum_{j=1}^{m_2}\delta_j-\sum_{j=m_2+1}^{q_2}\delta_jL_j+\sum_{j=1}^{n_2}\gamma_jK_j-\sum_{j=n_2+1}^{p_2}\gamma_j-\sum_{j=1}^{q_1}\beta_j>0 \tag{2.7}$$

$$\Lambda=\sum_{j=1}^{n_1}A_j-\sum_{j=n_1+1}^{p_1}A_j+\sum_{j=1}^{m_2}F_j-\sum_{j=m_2+1}^{q_2}F_jS_j+\sum_{j=1}^{n_3}E_jR_j-\sum_{j=n_2+1}^{p_3}E_j-\sum_{j=1}^{q_1}B_j>0 \tag{2.8}$$

$$|\arg x|<\frac{1}{2}\Omega\pi,\ |\arg y|<\frac{1}{2}\Lambda\pi \tag{2.9}$$

The behavior of the $\overline{H}$-function of two variables for small values of $|z|$ follows as:

$$\overline{H}[x,y]=0(|x|^{\alpha}|y|^{\beta}),\max\{|x|,|y|\}\to 0 \tag{2.10}$$

Where

$$\alpha=\min_{1\le j\le m_2}\left[\operatorname{Re}\left(\frac{d_j}{\delta_j}\right)\right]\qquad \beta=\min_{1\le j\le m_2}\left[\operatorname{Re}\left(\frac{f_j}{F_j}\right)\right] \tag{2.11}$$

For large value of $|z|$,

$$\overline{H}[x,y]=0\{|x|^{\alpha'},|y|^{\beta'}\},\min\{|x|,|y|\}\to 0 \tag{2.12}$$

Where

$$\alpha'=\max_{1\le j\le n_2}\operatorname{Re}\left(K_j\frac{c_j-1}{\gamma_j}\right),\ \beta'=\max_{1\le j\le n_3}\operatorname{Re}\left(R_j\frac{e_j-1}{E_j}\right) \tag{2.13}$$

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

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Provided that $U < 0$ and $V < 0$.

If we take

$K_j = 1(j = 1, 2, ..., n_2), L_j = 1(j = m_2 + 1, ..., q_2), R_j = 1(j = 1, 2, ..., n_3), S_j = 1(j = m_3 + 1, ..., q_3)$ in (2.1),

the $\overline{H}$ -function of two variables reduces to H -function of two variables due to [5].

III. SOME PROPERTIES OF $\overline{H}$ -FUNCTION OF TWO VARIABLES

1. If we set $n_1 = p_1 = q_1 = 0$, the $\overline{H}$ -function of two variables breaks up into a product of two $\overline{H}$ -function of one variable namely

$$\overline{H}^{0,0:m_2,n_2:m_3,n_3}_{0,0:p_2,q_2:p_3,q_3}\left[\begin{matrix} x \\ y \end{matrix}\middle| \begin{matrix} —:(c_j,\gamma_j;K_j)_{1,n_2},(c_j,\gamma_j)_{n+1,p_2}:(e_j,E_j;R_j)_{1,n_3},(e_j,E_j)_{n_3+1,p_3} \\ —:(d_j,\delta_j)_{1,m_2},(d_j,\delta_j;L_j)_{m_2+1,q_2}:(f_j,F_j)_{1,m_3},(f_j,F_j;S_j)_{m_3+1,q_3} \end{matrix}\right]$$

$$= \overline{H}^{m_2,n_2}_{p_2,q_2}\left[x\middle|\begin{matrix}(c_j,\gamma_j;K_j)_{1,n_2},(c_j,\gamma_j)_{n_2+1,p_2} \\ (d_j,\delta_j)_{1,m_2},(d_j,\delta_j;L_j)_{m_2+1,q_2}\end{matrix}\right]\overline{H}^{m_3,n_3}_{p_3,q_3}\left[y\middle|\begin{matrix}(e_j,E_j;R_j)_{1,n_3},(e_j,E_j)_{n_3+1,p_3} \\ (f_j,F_j)_{1,m_3},(f_j,F_j;S_j)_{m_3+1,q_3}\end{matrix}\right] \quad (3.1)$$

2. If $\lambda > 0$, we then obtain

$$\lambda^2\overline{H}^{0,n_1:\ m_2,n_2:m_3,n_3}_{p_1,q_1:p_2,q_2:p_3,q_3}\left[\begin{matrix} x^\lambda \\ y^\lambda \end{matrix}\middle| \begin{matrix} (a_j,\lambda\alpha_j;A_j)_{1,p_1},(c_j,\lambda\gamma_j;K_j)_{1,n_2},(c_j,\lambda\gamma_j)_{n_2+1,p_2},(e_j,\lambda E_j;R_j)_{1,n_3},(e_j,\lambda E_j)_{n_3+1,p_3} \\ (b_j,\lambda\beta_j;B_j)_{1,q_1},(d_j,\lambda\delta_j)_{1,m_2},(d_j,\lambda\delta_j;L_j)_{m_2+1,q_2},(f_j,\lambda F_j)_{1,m_3},(f_j,\lambda F_j;S_j)_{m_3+1,q_3} \end{matrix}\right]$$

$$= \overline{H}^{0,n_1:\ m_2,n_2:m_3,n_3}_{p_1,q_1:p_2,q_2:p_3,q_3}\left[\begin{matrix} x \\ y \end{matrix}\middle| \begin{matrix} (a_j,\alpha_j;A_j)_{1,p_1},(c_j,\gamma_j;K_j)_{1,n_2},(c_j,\gamma_j)_{n_2+1,p_2},(e_j,E_j;R_j)_{1,n_3},(e_j,E_j)_{n_3+1,p_3} \\ (b_j,\beta_j;B_j)_{1,q_1},(d_j,\delta_j)_{1,m_2},(d_j,\delta_j;L_j)_{m_2+1,q_2},(f_j,F_j)_{1,m_3},(f_j,F_j;S_j)_{m_3+1,q_3} \end{matrix}\right] \quad (3.2)$$

3. $$\overline{H}^{0,n_1:\ m_2,n_2:m_3,n_3}_{p_1,q_1:p_2,q_2:p_3,q_3}\left[\begin{matrix} 1/x \\ 1/y \end{matrix}\middle| \begin{matrix} (a_j,\alpha_j;A_j)_{1,p_1},(c_j,\gamma_j;K_j)_{1,n_2},(c_j,\gamma_j)_{n_2+1,p_2},(e_j,E_j;R_j)_{1,n_3},(e_j,E_j)_{n_3+1,p_3} \\ (b_j,\beta_j;B_j)_{1,q_1},(d_j,\delta_j)_{1,m_2},(d_j,\delta_j;L_j)_{m_2+1,q_2},(f_j,F_j)_{1,m_3},(f_j,F_j;S_j)_{m_3+1,q_3} \end{matrix}\right]$$

$$= \overline{H}^{0,n_1:\ m_2,n_2:m_3,n_3}_{p_1,q_1:p_2,q_2:p_3,q_3}\left[\begin{matrix} x \\ y \end{matrix}\middle| \begin{matrix} (1-b_j,\beta_j;B_j)_{1,q_1},(1-d_j,\delta_j)_{1,m_2},(1-d_j,\delta_j;L_j)_{m_2+1,q_2},(1-f_j,F_j)_{1,m_3},(1-f_j,F_j;S_j)_{m_3+1,q_3} \\ (1-a_j,\alpha_j;A_j)_{1,p_1},(1-c_j,\gamma_j;K_j)_{1,n_2},(1-c_j,\gamma_j)_{n_2+1,p_2},(1-e_j,E_j;R_j)_{1,n_3},(1-e_j,E_j)_{n_3+1,p_3} \end{matrix}\right] \quad (3.3)$$

The series representation of the $\overline{H}$ -function of two variables defined and represented by (2.1) is as follows:

(i) $\delta_m\left(d_j + \lambda\right) \neq \delta_j\left(d_m + \mu\right)$

For $j \neq m; j, m = 1, 2, ..., m_2; \lambda, \mu = 0, 1, 2, ...$

(ii) $F_n\left(f_k + \rho\right) \neq F_k\left(f_n + v\right)$

For $n \neq k; n, k = 1, 2, ...m_3; \rho, v = 0, 1, 2, ...$

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

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(iii) $x, y \neq 0, U < 0, V < 0$ [U and V are defined by (2.5) and (2.6)], and if all the poles of (2.1) are simple, then the integral (2.1) can be evaluated as a sum of residues to give

$$\overline{H}[x,y] = \sum_{m=1}^{m_2}\sum_{n=1}^{n_3}\sum_{\mu=0}^{\infty}\sum_{\nu=0}^{\infty}\phi_1\left(\rho_{m,\mu},\sigma_{n,\nu}\right)\phi_4\left(\rho_{m,\mu}\right)\phi_5\left(\sigma_{n,\nu}\right)\frac{(-1)^{\mu+\nu}(x)^{\rho_{m,\mu}}(y)^{\sigma_{n,\nu}}}{\delta_m F_n \mu!\nu!} \quad (3.4)$$

Where

$$\rho_{m,\mu} = \frac{(d_m+\mu)}{\delta_m}, \sigma_{n,\nu} = \frac{(f_n+\nu)}{F_n} \quad (3.5)$$

$$\phi_4(\xi) = \frac{\prod_{j=1, j\neq m}^{m_2}\Gamma\left(d_j-\delta_j\xi\right)\prod_{j=1}^{n_2}\left\{\Gamma\left(1-c_j+\gamma_j\xi\right)\right\}^{K_j}}{\prod_{j=n_2+1}^{p_2}\Gamma\left(c_j-\gamma_j\xi\right)\prod_{j=m_2+1}^{q_2}\left\{\Gamma\left(1-d_j+\delta_j\xi\right)\right\}^{L_j}} \quad (3.6)$$

ϕ_5 is defined analogously to ϕ_4 in terms of the parameter sets

$\left(e_j,E_j;R_j\right)_{1,n_3},\left(e_j,E_j\right)_{n_3+1,p_3},\left(f_j,F_j\right)_{1,m_3},\left(f_j,F_j;S_j\right)_{m_3+1,q_3}$. Also ϕ_1 is defined by means of (2.2).

By comparing the series representation of $\overline{H}[x,y]$ and $\overline{H}$ -function, we get the following interesting special case of (2.1).

$$\overline{H}^{0,n_1:\ m_2,n_2:m_3,n_3}_{p_1,q_1:p_2,q_2:p_3,q_3}\left[\begin{matrix}x\\y\end{matrix}\middle|\begin{matrix}\left(a_j,\alpha_j;A_j\right)_{1,p_1},\left(c_j,\gamma_j;K_j\right)_{1,n_2},\left(c_j,\gamma_j\right)_{n_2+1,p_2},\left(e_j,E_j;R_j\right)_{1,n_3},\left(e_j,E_j\right)_{n_3+1,p_3}\\\left(b_j,\beta_j;B_j\right)_{1,q_1},\left(d_j,\delta_j\right)_{1,m_2},\left(d_j,\delta_j;L_j\right)_{m_2+1,q_2},\left(f_j,F_j\right)_{1,m_3},\left(f_j,F_j;S_j\right)_{m_3+1,q_3}\end{matrix}\right]$$

$$= \sum_{w=0}^{\infty}\frac{(-1)^w y^{\rho_w}\prod_{j=1}^{n_3}\left\{\Gamma\left(1-e_j+E_j\rho_w\right)\right\}^{R_j}}{F\,w!\prod_{j=n_3+1}^{p_3}\Gamma\left(e_j-E_j\rho_w\right)\prod_{j=1}^{q_3}\left\{\Gamma\left(1-f_j+F_j\rho_w\right)\right\}^{S_j}}$$

$$\overline{H}^{m_2,n_1+n_2}_{p_2+q_2,q_1+q_2}\left[x\middle|\begin{matrix}\left(c_j,\gamma_j;K_j\right)_{1,n_2},\left(a_j-A_j\rho_w,\alpha_j\right)_{1,p_1},\left(c_j,\gamma_j\right)_{n_2+1,p_2}\\\left(d_j,\delta_j\right)_{1,m_2},\left(d_j,\delta_j;L_j\right)_{m_2+1,q_2},\left(b_j-\beta_j\rho_w,B_j\right)_{1,q_1}\end{matrix}\right] \quad (3.7)$$

In (3.7), when we put $f=0, F=1$ and $y\to 0$, we get the following useful confluent of the $\overline{H}$ -function of two variables:

$$\lim_{y\to 0}\overline{H}^{0,n_1:\ m_2,n_2:1,n_3}_{p_1,q_1:p_2,q_2:p_3,q_3+1}\left[\begin{matrix}x\\y\end{matrix}\middle|\begin{matrix}\left(a_j,\alpha_j;A_j\right)_{1,p_1},\left(c_j,\gamma_j;K_j\right)_{1,n_2},\left(c_j,\gamma_j\right)_{n_2+1,p_2},\left(e_j,E_j;R_j\right)_{1,n_3},\left(e_j,E_j\right)_{n_3+1,p_3}\\\left(b_j,\beta_j;B_j\right)_{1,q_1},\left(d_j,\delta_j\right)_{1,m_2},\left(d_j,\delta_j;L_j\right)_{m_2+1,q_2},(0,1),\left(f_j,F_j;S_j\right)_{1,q_3}\end{matrix}\right]$$

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

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$$= \frac{\prod_{j=1}^{n_3}\left\{\Gamma\left(1-e_j\right)\right\}^{R_j}}{F\,w!\prod_{j=n_3+1}^{p_3}\Gamma\left(e_j\right)\prod_{j=1}^{q_3}\left\{\Gamma\left(1-f_j\right)\right\}^{S_j}}\,\overline{H}^{m_2,n_1+n_2}_{p_2+q_2,q_1+q_2}\left[x\left|\begin{matrix}\left(a_j,\alpha_j;A_j\right)_{1,p_1},\left(c_j,\gamma_j;K_j\right)_{1,n_2},\left(c_j,\gamma_j\right)_{n_2+1,p_2}\\ \left(b_j,\beta_j;B_j\right)_{1,q_1},\left(d_j,\delta_j\right)_{1,m_2},\left(d_j,\delta_j;L_j\right)_{m_2+1,q_2}\end{matrix}\right.\right] \quad (3.8)$$

Also making use of a result [2, vol.1, p.104, eq. (46)] and (2.1) in (3.7), we obtain another interesting result:

$$\overline{H}^{0,1:m_2,n_2:1,0}_{2,0:p_2,q_2:0,2}\left[\begin{matrix}x\\1\end{matrix}\left|\begin{matrix}(a,\rho;1),(b,\sigma;1),\left(c_j,\gamma_j;K_j\right)_{1,n_2},\left(c_j,\gamma_j\right)_{n_2+1,p_2},--\\ --,\left(d_j,\delta_j\right)_{1,m_2},\left(d_j,\delta_j;L_j\right)_{m_2+1,q_2},(f,1),(h,1;1)\end{matrix}\right.\right]$$

$$=\overline{H}^{m_2+1,n_2+1}_{p_2+4,q_2+1}\left[x\left|\begin{matrix}\left(a_j-f,\rho;1\right),\left(c_j,\gamma_j;K_j\right)_{1,n_2},,\left(c_j,\gamma_j\right)_{n_2+1,p_2},(b-f,\sigma),(b-h,\sigma),(a-h,\rho)\\ (a+b-f-h-1,\rho+\sigma),\left(d_j,\delta_j\right)_{1,m_2},\left(d_j,\delta_j;L_j\right)_{m_2+1,q_2}\end{matrix}\right.\right] \quad (3.9)$$

Where $\mathrm{Re}[a+b-f-h-1]>0, \rho>0, \sigma>0$.

IV. SPECIAL CASES

The functions $\overline{J}^{v}_{\lambda}\left[\begin{smallmatrix}x\\y\end{smallmatrix}\right]$ and ${}_p\overline{\psi}_q\left[\begin{smallmatrix}x\\y\end{smallmatrix}\right]$ are the Wright's generalized Bessel functions of two variables and generalized hypergeometric function of two variables respectively. The study of the $\overline{H}$-function of two variables will cover wider range then the H-function of two variables and gives deeper , more general and more useful results directly applicable in various problems of physical and biological sciences.

1. $$\overline{H}^{0,0:1,0:1,0}_{0,0:0,2:0,2}\left[\begin{matrix}x\\y\end{matrix}\left|\begin{matrix}---:---:---\\ --:(0,1),(-v,\mu;K),(0,1),(-v',\mu';S)\end{matrix}\right.\right]=\overline{J}^{\mu}_{v}\left[\begin{matrix}x\\y\end{matrix}\right]\overline{J}^{\mu'}_{v'}\left[\begin{matrix}x\\y\end{matrix}\right] \quad (4.1)$$

2. $$\overline{H}^{0,0:1,p_2:1,p_3}_{0,0:p_2,q_2+1:p_3,q_3+1}\left[\begin{matrix}-x\\-y\end{matrix}\left|\begin{matrix}--:\left(1-c_j,\gamma_j;K_j\right)_{1,p_2},\left(1-e_j,E_j;R_j\right)_{1,p_3}\\ --:(0,1),\left(1-d_j,\delta_j;L_j\right)_{1,q_2},(0,1),\left(1-f_j,F_j;S_j\right)_{1,q_3}\end{matrix}\right.\right]$$

$$={}_{p_2}\overline{\psi}_{q_2}\left[x\left|\begin{matrix}\left(c_j,\gamma_j;K_j\right)_{1,p_2}\\ \left(d_j,\delta_j;L_j\right)_{1,q_2}\end{matrix}\right.\right]{}_{p_3}\overline{\psi}_{q_3}\left[y\left|\begin{matrix}\left(e_j,E_j;R_j\right)_{1,p_3}\\ \left(f_j,F_j;S_j\right)_{1,q_3}\end{matrix}\right.\right] \quad (4.2)$$

3. $$\overline{H}^{0,0:1,0:1,0}_{0,0:0,2:0,2}\left[\begin{matrix}x^2/4\\y^2/4\end{matrix}\left|\begin{matrix}--:--:--\\ --:\left(\frac{a+v}{2},1\right),\left(\frac{a-v}{2},1;L_j\right),\left(\frac{a'+v'}{2},1\right),\left(\frac{a'-v'}{2},1;S_j\right)\end{matrix}\right.\right]=\left(\frac{x}{2}\right)^{a}\left(\frac{y}{2}\right)^{a'}\overline{J}_{v}(x)\overline{J}_{v'}(y) \quad (4.3)$$

Where $\overline{J}^{v}_{\lambda}(.)$ and ${}_p\overline{\psi}_q(.)$ are known as Wright's generalized Bessel function and Wright's generalized hypergeometric function respectively and defined as [4]:

$$\overline{H}^{1,0}_{0,2}\left[x\left|\begin{matrix}--\\(0,1),(-\lambda,v;A)\end{matrix}\right.\right]=\overline{J}^{v}_{\lambda}(x)=\sum_{r=0}^{\infty}\frac{(-x)^r}{r!\left\{\Gamma\left(1+\lambda+vr\right)\right\}^{A}} \quad (4.4)$$

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 2, Issue 9, September 2013

And $\overline{H}_{p,q+1}^{1,p}\left[-x\left|\begin{matrix}(1-a_j,\alpha_j;A_j)_{1,p}\\(0,1),(1-b_j,\beta_j;B_j)_{1,q}\end{matrix}\right.\right]={}_p\overline{\psi}_q\left[x\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,p}\\(b_j,\beta_j;B_j)_{1,q}\end{matrix}\right.\right]$

$$=\sum_{r=0}^{\infty}\frac{\prod_{j=1}^{p}\left\{\Gamma\left(a_j+\alpha_j r\right)\right\}^{A_j}}{\prod_{j=1}^{q}\left\{\Gamma\left(b_j+\beta_j r\right)\right\}^{B_j}}\frac{x^r}{r!} \qquad (4.5)$$

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