

Buckling Analysis of Plate Element Subjected to In Plane Loading Using ANSYS

Monica S Swamy¹, Ranjith A², Sandya D S³, Shrithi S Badami⁴

P.G. Student, Department of Civil Engineering, Adi Chunchanagiri Institute of Technology, Chikmagalur, Karnataka, India¹

Assistant Professor, Department of Civil Engineering, Adi Chunchanagiri Institute of Technology, Chikmagalur, Karnataka, India^{2,3,4}

ABSTRACT: The usage of heterogeneous materials in situations where large strength to weight ratio is required has been increased substantially over the world in all construction aspects. The behaviour of the plate under each loading is different. Whenever the delamination is located at the middle plane of laminate, the panel exhibits only global buckling, i.e. there is no buckling of delaminated region. Whenever the delamination is close to the surface, the buckling mode is predominantly local. The type of plate also plays a major role in carrying load. This article summarises the numerical study carried out using finite element software ANSYS and Timoshenko's methodology to examine the buckling behaviour of homogeneous and heterogeneous plate element with and without crack. Also the effect of aspect ratio on the buckling behaviour with varying plate thickness for different boundary conditions was also examined.

KEYWORDS: Buckling Analysis, Plate Element, Finite Element Analysis, Aspect ratio, Buckling load

I. INTRODUCTION

Buckling analysis is a type of analysis used to determine buckling loads - critical loads at which a structure becomes unstable. Buckling behaviour significantly changes with change in aspect ratio, thickness of the element as plates are subjected to different end conditions. Plate seems to work as a column of finite width at higher aspect ratio. If we decrease aspect ratio, there is also a limit below which failure does not take place by elastic buckling.

When the opening becomes inevitable for the plates under large working stress, the reduced buckling strength of the plate may be insufficient to meet the requirements of normal serviceability limits and structural safety. A design solution must be deduced to increase the structural stability of such perforated plate before it can be used to its best advantage. This always can be accomplished by selecting the thicker plate but the design solution may not be economical in terms of weight of material introduced by an adequate increase in the thickness of the plate.

The present work deals with the analysis of a rectangular element being considered as a plane stress condition under various boundary conditions and loadings. Throughout the analysis, the master element which is plane 82 are used to perform buckling analysis using ANSYS. Finally, results have been checked with exact results obtained from Timoshenko's plate buckling equation for different end conditions.

Jana et al. [1] carried out a buckling analysis of a simply supported rectangular plate without cut out subjected to various types of non-uniform compressive loads. Chai et al. [2] investigated the influence of boundary conditions, plate aspect ratios on the optimal ply angle and associated optimal buckling loads of anti-symmetrically laminated composite plates without cut out under various linearly varying in-plane loading conditions. Hu et al. [3] examined the buckling behaviour of a graphite/epoxy symmetrically laminated composite rectangular plate without cut out under parabolic variation of axial loads. Jain et al. [4] investigated the effects of the cut out shape, size and the alignment of the elliptical cut out on the buckling and the first-ply failure loads of square laminates subjected to uni-axial compression load. Aydin Komur et al. [5] investigated the effects of an elliptical/circular cut out on the buckling load of symmetric cross-ply and angle-ply laminate square composite plates.

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The work in this paper is divided in two stages: 1) Modelling of homogeneous and heterogeneous plate with and without crack 2) buckling analysis for inter-laminar crack growth in homogeneous and heterogeneous plates for different plate thickness with varying aspect ratio.

II. BUCKLING ANALYSIS

The buckling of homogeneous plates has been well researched and documented for decades. Various methods, such as energy and equilibrium methods, have been used to determine the lowest eigen value, or the actual buckling load. The results of these methods are given in this work and the reader is referred to Timoshenko's Theory of Elastic Stability [7] for a more comprehensive treatment of homogeneous plate buckling. For a homogeneous plate the following formula is used to determine the critical buckling load per unit length:

$$(N_x)_{cr} = \frac{k\pi^2 E t^3}{12(1-\nu^2) b^2}$$

Where, E is Young's Modulus, ν is Poisson's ratio, t is the plate thickness, b is the width of the plate, and k is a constant determined by the boundary condition and aspect ratio of the plate. Timoshenko [7] gives values of k for various aspect ratios under various edge boundary conditions.

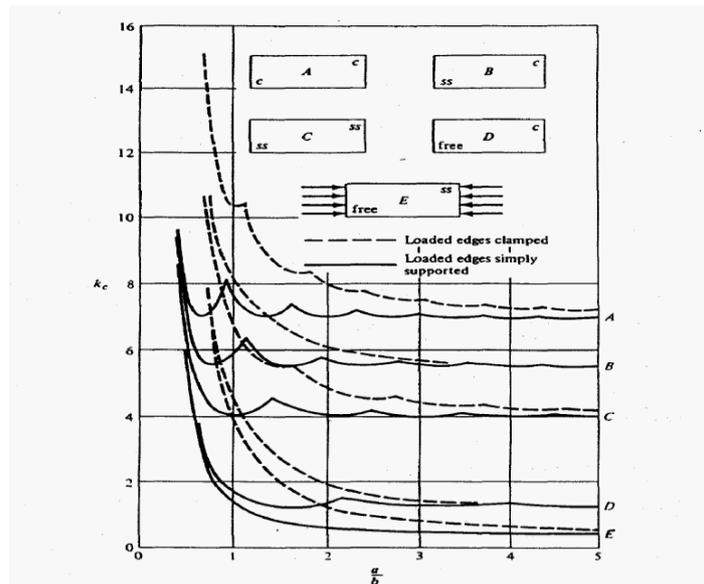


Fig. 1. Influence of boundary conditions on the buckling coefficients of plates subjected to in-plane compressive loading [7]

Figure 1 shows the influence of various support conditions in selecting the coefficients of buckling for determining critical buckling load per unit length of a plate element subjected to in plane loading as suggested by Timoshenko.

2.1 Element Type used in ANSYS

The element used in the analysis is PLANE82. PLANE82 is a higher order version of the 2-D, four node element (PLANE42). It provides more accurate results for mixed (quadrilateral-triangular) automatic meshes and can tolerate irregular shapes without as much loss of accuracy. The 8-node elements have compatible displacement shapes and are well suited to model curved boundaries. The 8-node element is defined by eight nodes having two degrees of freedom at each node: translations in the nodal x and y directions. The element may be used as a plane element or as an

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axisymmetric element. The element has plasticity, creep, swelling, stress stiffening, large deflection, and large strain capabilities.

PLANE82 Assumptions

- The area of the element must be positive.
- The element must lie in a global X-Y plane "PLANE82 Geometry" and the Y-axis must be the axis of symmetry for axisymmetric analyses. An axisymmetric structure should be modeled in the +X quadrants.
- A face with a removed mid-side node implies that the displacement varies linearly, rather than parabolic variation along that face.

PLANE82 Restrictions

- The DAMP material property is not allowed.
- Influence body loads are not applicable.
- The only special feature allowed is stress stiffening.

III. ANALYSIS

A) Homogeneous Plate

3.1.1 Analytical detail

Homogenous plate is assumed as Aluminium Alloy which in nonferrous metal with the following material properties.

Density of Aluminium = 2700 kg/m^3

Poisson's ratio of Aluminium = 0.3

Young's modulus of Aluminium = $7 \times 10^{10} \text{ pa}$

3.1.2 Modelling

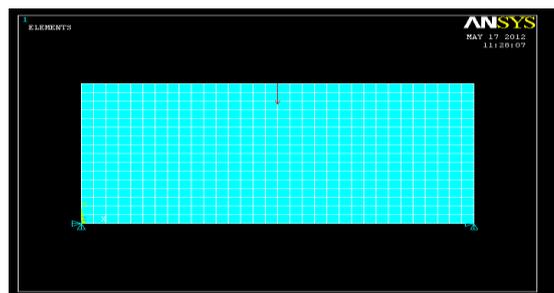


Fig. 2. Solid model of homogenous plate element

Figure 2 shows metal block of size 1m x 0.5m which was modelled as 2-D model and meshed with approximately 16 elements across the width of the metal block.

3.1.3 Buckling analysis

Buckling analysis is a technique used to determine buckling loads - critical loads at which a structure becomes unstable - and buckled mode shapes - the characteristic shape associated with a structure's buckled response. Various methods, such as energy and equilibrium methods, have been used to determine the lowest eigenvalue, or the actual buckling load.

For a homogeneous plate the following formula is used to determine the critical buckling load per unit length: Where, E is Young's Modulus, ν is Poisson's ratio, t is the plate thickness, b is the width of the plate, and k is a constant determined by the boundary condition and aspect ratio of the plate.

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The homogeneous plates analysed in ANSYS were given the following isotropic properties:

$$E = 7.0 \times 10^{10} \text{ pa}$$

$$\nu = 0.30$$

The plates were analysed under three different boundary conditions: All sides simply supported, two sides simply supported two sides clamped, two sides simply supported two sides free, and all sides clamped. Three different plate thicknesses were used: 3mm, 4mm, and 5mm. For each plate thickness, four different aspect ratios (a/b) were used: 1.0, 1.11, 1.25, 1.43, 1.67, and 2.0. The length, a, was held constant at 1000mm and the width b, was varied between 1000mm, 900mm, 800mm, 700mm, 600mm, and 500mm. The critical buckling load results are tabulated in Table 1 and table 2.

Table 1. Critical buckling loads for a) All side simply supported b) Two simply supported, two sides clamped

a (mm)	b(mm)	Aspect Ratio a/b	Plate Thickness (mm)	Calculated Critical Buckling Load, $N_x(cr)$, (N/mm)	ANSYS Critical Buckling Load, $N_x(cr)$, (N/mm): All side simply supported	Calculated Critical Buckling Load, $N_x(cr)$, (N/mm)	ANSYS Critical Buckling Load, $N_x(cr)$, (N/mm) Two side simply supported Two side clamped
1000	1000	1	3	6.8383	6.9661	13.6766	13.8999
1000	900	1.11	3	8.5364	8.6678	16.6736	16.8894
1000	800	1.25	3	11.2258	11.3852	20.5683	20.5221
1000	700	1.43	3	15.8077	15.8116	26.167	25.6639
1000	600	1.67	3	24.3984	24.6742	32.7669	32.6612
1000	500	2	3	42.7394	43.0078	41.7137	42
1000	1000	1	4	16.2093	16.3	32.4186	31.8888
1000	900	1.11	4	20.2345	19.6712	39.5227	40
1000	800	1.25	4	26.6092	25.999	48.7546	48.8996
1000	700	1.43	4	37.4071	37.641	62.0254	63.6672
1000	600	1.67	4	57.8382	58.1213	77.6696	77.8855
1000	500	2	4	101.3082	102	98.8768	98.9754
1000	1000	1	5	31.6588	30.999	63.3176	63.428
1000	900	1.11	5	39.5204	40.2313	77.1928	78.992
1000	800	1.25	5	51.9712	51.6666	95.2238	97.31
1000	700	1.43	5	73.1838	72.8101	121.1434	120
1000	600	1.67	5	112.9555	112.9999	151.6985	153.7623
1000	500	2	5	197.8676	198	193.1188	191.2083

Table 2. Critical buckling loads for a) Two side simply supported two sides free b) Two side clamped two side free c) All side clamped

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a (mm)	b(mm)	Aspect Ratio a/b	Plate Thickness (mm)	Calculate d Critical Buckling Load, Nx(cr), (N/mm)	ANSYS Critical Buckling Load, Nx(cr), (N/mm): Two side simply supported Two side free	Calculate d Critical Buckling Load, Nx(cr), (N/mm)	ANSYS Critical Buckling Load, Nx(cr), (N/mm): Two side clamped two side free	Calculate d Critical Buckling Load, Nx(cr), (N/mm)	ANSYS Critical Buckling Load, Nx(cr), (N/mm): All side clamped
1000	1000	1	3	2.9918	3	7.6931	7.5439	17.9505	17.9999
1000	900	1.11	3	3.588	3.5978	9.2866	9.4564	21.9501	21.7254
1000	800	1.25	3	3.4726	3.3989	10.017	9.7621	24.5752	25
1000	700	1.43	3	3.4889	3.5666	11.1646	10.8881	31.2259	30.3331
1000	600	1.67	3	4.0365	4.1111	12.8218	12.9854	40.365	41
1000	500	2	3	5.1287	5.0981	13.5398	13.5444	54.7064	55.0012
1000	1000	1	4	7.0916	7.1178	18.2355	18.2222	42.5495	41.008
1000	900	1.11	4	8.5049	8.6619	22.0216	22.1111	52.0299	51.7759
1000	800	1.25	4	8.2313	8.0098	23.7441	22.9932	58.2522	59.1121
1000	700	1.43	4	8.2701	7.9912	26.4642	27.1002	74.017	75
1000	600	1.67	4	9.568	9.6666	30.3925	30.4512	95.68	96.3412
1000	500	2	4	12.157	12.26	32.0944	32.5432	129.6745	128.1228
1000	1000	1	5	13.8507	13.7891	35.6162	34.2212	83.1044	82.659
1000	900	1.11	5	16.6111	16.3218	42.9935	43.9322	101.621	100.913
1000	800	1.25	5	16.6951	16.8888	46.3752	45.765	113.7739	114.7777
1000	700	1.43	5	17.7677	18.111	51.6879	50.9974	144.5645	145
1000	600	1.67	5	18.6875	18.5	59.3603	60.881	186.875	185.8825
1000	500	2	5	23.7441	24.0001	62.6845	63.5555	253.2705	253.8421

B) Homogeneous Plate with Crack

3.2.1 Analytical detail

Homogenous plate is assumed as Aluminium Alloy. A small crack is modelled at a distance 0.25m from top and 0.5m from the side edge, it is an elliptical shaped crack having dimension 0.04m in major axis and 0.0001m in minor axis.

Density of Aluminium =2700kg/m³

Poisson's ratio of Aluminium=0.3

Young's modulus of Aluminium=7x10¹⁰pa

3.2.2 Modelling

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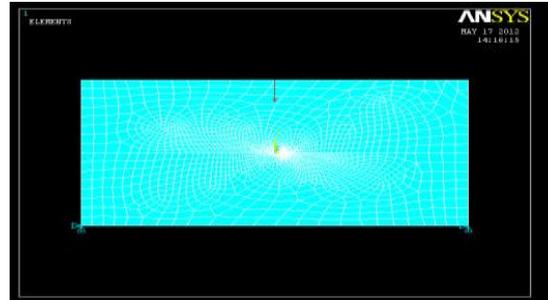


Fig. 3. Finite element model of crack within homogeneous plate

Figure 3 shows homogeneous plate element with crack, Modelling is done using solid 82 type element in ANSYS. Because of elliptical shape a fine mesh is generated near crack tips. Except at the crack tips the whole model is meshed coarsely. Bottom extreme nodes are fixed.

3.2.5 Buckling analysis

Buckling analysis for homogeneous plate with crack is carried out for different plate thickness, end condition, aspect ratios as stated in 3.1.3.

Table. 3. Critical buckling loads for homogeneous plate with crack for a) All side simply supported b) Two simply supported two side clamped c) Two side simply supported two side free d) Two side clamped two side free e) All side clamped

a (mm)	b(mm)	Aspect Ratio a/b	Plate Thickness (mm)	ANSYS Critical Buckling Load, $N_x(cr)$, (N/mm): All side simply supported	ANSYS Critical Buckling Load, $N_x(cr)$, (N/mm) Two side simply supported Two side clamped	ANSYS Critical Buckling Load, $N_x(cr)$, (N/mm): Two side simply supported Two side free	ANSYS Critical Buckling Load, $N_x(cr)$, (N/mm): Two side clamped two side free	ANSYS Critical Buckling Load, $N_x(cr)$, (N/mm): All side clamped
1000	1000	1	3	3.6754	11.7563	1.8212	4.671	14.2218
1000	900	1.11	3	6.7231	14.996	2.332	5.9112	19.3765
1000	800	1.25	3	9.2271	19.273	2.4392	7.2315	23.8836
1000	700	1.43	3	14.2708	23.665	2.9912	9.0061	28.7685
1000	600	1.67	3	19.7685	29.6211	3.6621	10.2172	39.832
1000	500	2	3	38.9653	38.1028	3.9893	12.442	48.3217
1000	1000	1	4	10.2381	26.9987	5.2219	15.9426	37.3276
1000	900	1.11	4	14.3278	37.9325	6.9312	19.652	48.6747
1000	800	1.25	4	20.0052	45.444	7.9525	21.739	56.886
1000	700	1.43	4	31.6278	58.332	7.8704	25.2255	66.294

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1000	600	1.67	4	50.5698	66.2701	8.9822	28.721	93.3429
1000	500	2	4	96.564	97.4532	9.8524	30.2262	117.1145
1000	1000	1	5	27.8711	59.7732	10.003	29.2139	76.008
1000	900	1.11	5	36.8076	73.1842	13.0127	38.287	95.652
1000	800	1.25	5	47.238	94.6721	15.0075	41.3342	110.999
1000	700	1.43	5	67.1261	116.1172	17.2375	48.6732	136.77
1000	600	1.67	5	108.442	148.109	18.0029	53.1256	175.557
1000	500	2	5	189.5433	188.9951	21.4296	58.3469	245.5823

3.3 Laminated Heterogeneous Plate

3.3.1 Analytical detail

Laminated heterogeneous plate is assumed as the combination of Aluminium and Low Carbon Steel with following material properties.

Material properties of low carbon steel:

Density of steel= 7872 Kg/m³

Poisson's ratio=0.29

Young's modulus of steel=2x10¹¹pa

Material properties of Aluminium:

Density of Aluminium =2700kg/m³

Poisson's ratio of Aluminium=0.3

Young's modulus of Aluminium=7x10¹⁰pa

3.3.2 Modelling

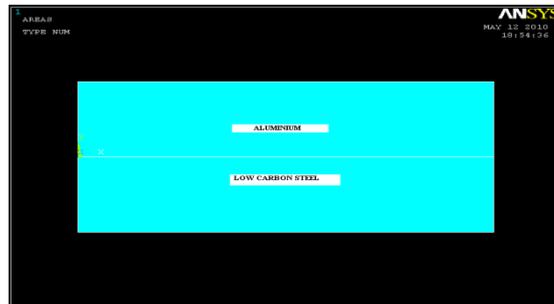


Fig. 4. Solid model of laminated heterogeneous plate

Figure 4 shows laminated heterogeneous plate element without crack. Since heterogeneous material is used, both aluminium and low carbon steel modelled with an interface in between.

3.3.5 Buckling analysis

The heterogeneous laminated plates analysed in ANSYS were given the following isotropic Properties:

For Aluminium laminate: $E = 7.0 \times 10^{10}$ pa

$\mu = 0.30$

For Low Carbon Steel laminate: $E = 2.0 \times 10^{11}$ pa

$\mu = 0.29$

The analysis is carried out for different plate thickness, end condition, aspect ratios as stated in 3.1.3. The corresponding buckling loads are tabulated in table 4.

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Table 4. Critical buckling loads for heterogeneous plate for a) All side simply supported b) Two simply supported two side clamped c) Two side simply supported two side free d) Two side clamped two sides free e) All side clamped

a (mm)	b(mm)	Aspect Ratio a/b	Plate Thickness (mm)	ANSYS Critical Buckling Load, Nx(cr), (N/mm): All side simply supported	ANSYS Critical Buckling Load, Nx(cr), (N/mm) Two side simply supported Two side clamped	ANSYS Critical Buckling Load, Nx(cr), (N/mm): Two side simply supported Two side free	ANSYS Critical Buckling Load, Nx(cr), (N/mm): Two side clamped two side free	ANSYS Critical Buckling Load, Nx(cr), (N/mm): All side clamped
1000	1000	1	3	4.8263	7.3761	1.976	4.9986	9.9986
1000	900	1.11	3	7.1082	11.7951	2.637	6.3275	16.9852
1000	800	1.25	3	9.111	16.2362	2.9491	7.3791	18.239
1000	700	1.43	3	13.0723	20.7763	3.1123	9.672	21.5677
1000	600	1.67	3	17.7768	29.6622	3.8214	11.0027	29.8745
1000	500	2	3	33.3261	37.9227	4.0812	12.001	34.2255
1000	1000	1	4	10.9723	26.7112	4.6731	13.0217	27.6238
1000	900	1.11	4	14.2638	33.1111	5.2361	16.6808	39.4271
1000	800	1.25	4	19.999	42.4295	6.0021	21.9924	43.2276
1000	700	1.43	4	29.6278	54.8831	7.8913	25.1992	59.7823
1000	600	1.67	4	47.5698	68.5538	9.0015	27.0082	75.7728
1000	500	2	4	81.8234	81.026	10.9241	30.1188	99.4562
1000	1000	1	5	19.289	44.4328	8.4681	26.5498	56.8892
1000	900	1.11	5	31.5712	65.3327	12.003	37.3421	89.2251
1000	800	1.25	5	43.752	79.5538	14.0127	39.6621	97.2281
1000	700	1.43	5	64.912	104.3271	16.0075	46.3391	128.4
1000	600	1.67	5	87.2213	139.2269	17.2375	54.1234	165.1172
1000	500	2	5	153.2265	191.2083	20.0029	57.9932	190.2521

3.4 Laminated Heterogeneous Plate with Crack

3.4.1 Modelling

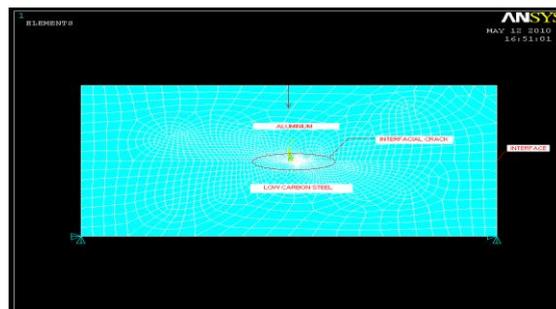


Fig. 5. Solid model of laminated heterogeneous plate with interfacial crack

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Figure 5 shows laminated heterogeneous plate with interfacial crack. Heterogeneous material is assumed as the combination of Aluminium and Low Carbon Steel, the crack is situated exactly at the interface between steel and aluminium as shown in figure which is often called as interfacial crack occurs in the heterogeneous laminates, material properties of aluminium and low carbon steel are assumed as in the previous analysis.

The analysis is carried out same as laminated heterogeneous plate but here the interfacial crack is introduced then the analysis is carried out for different thickness, aspect ratios, and end conditions. The critical buckling loads for different boundary conditions are tabulated in table 5.

Table. 5. Buckling analysis for heterogeneous plate with crack for a) All side simply supported b) Two simply supported two side clamped c) Two side simply supported two side free d) Two side clamped two side free e) All side clamped

a (mm)	b (mm)	Aspect Ratio a/b	Plate Thickness (mm)	ANSYS Critical Buckling Load, $N_x(cr)$, (N/mm): All side simply supported	ANSYS Critical Buckling Load, $N_x(cr)$, (N/mm) Two side simply supported Two side clamped	ANSYS Critical Buckling Load, $N_x(cr)$, (N/mm): Two side simply supported Two side free	ANSYS Critical Buckling Load, $N_x(cr)$, (N/mm): Two side clamped two side free	ANSYS Critical Buckling Load, $N_x(cr)$, (N/mm): All side clamped
1000	1000	1	3	2.2341	5.9833	1.6732	3.3286	6.9782
1000	900	1.11	3	5.1582	7.5548	2.118	4.3875	13.5052
1000	800	1.25	3	6.1621	13.1333	2.3856	5.6691	15.2621
1000	700	1.43	3	9.1321	17.8715	2.98	8.3422	19.1174
1000	600	1.67	3	14.732	25.1628	3.19	9.0625	22
1000	500	2	3	27	29.5568	3.9579	11.7004	30.9046
1000	1000	1	4	7.9173	23.7289	2.0829	9.034	25.76
1000	900	1.11	4	9.9478	28.4438	3.8845	14.6026	33.9371
1000	800	1.25	4	17.1111	35.3826	4.3073	19.4328	40.2239
1000	700	1.43	4	24.4278	39	4.9117	22.1912	47.999
1000	600	1.67	4	38.5848	58.665	6	25.0582	66
1000	500	2	4	78.8204	75.3276	8.9241	27.1178	94.1009
1000	1000	1	5	14.7089	37.876	5.3218	23.5091	48
1000	900	1.11	5	25.2212	57.2369	8.9003	27.7021	77.5672
1000	800	1.25	5	38.2962	66.359	10.5137	33.6621	89.332
1000	700	1.43	5	57.2042	97.6381	13.3675	39.3	120.538
1000	600	1.67	5	77.2513	124.201	15.4775	44.1934	163.1348
1000	500	2	5	146.2165	187.3368	18.7829	52.3532	185.556

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III. CONCLUSION

- Critical Buckling load for homogeneous plate is more compare to laminated heterogeneous plate
- Buckling load per unit length of the plate decreases with increase in aspect ratio
- It is noted that the presence of crack lowers the buckling load
- The study is useful in selecting the above said parameters in designing the laminates in buckling point of view

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REFERENCES

- [1] Jana P and Bhaskar K (2006), "Stability analysis of simply-supported rectangular plates under non-uniform uniaxial compression using rigorous and approximate plane stress solutions". *Thin-Walled Struct.* 44: 507–516
- [2] Chai G B, Ooi K T and Khong P W (1993), "Buckling strength optimization of laminated composite plates". *Comput. Struct.* 46: 77–82
- [3] Hu H, Badir A and Abatan A (2003), "Buckling behavior of graphite/epoxy composite plate under parabolic variation of axial loads". *Int. J. Mech. Sci.* 45: 1135–1147
- [4] Jain P and Ashwin K (2004), "Post buckling response of square laminates with a central circular/elliptical cutout". *Compos. Struct.* 65: 179–185
- [5] Aydin Komur M, Sen F, Atas A and Arslan N (2010), "Buckling analysis of laminated composite plates with an elliptical/circular cutout using FEM". *Adv. Eng. Soft.* 41: 161–164
- [6] ANSYS Documentation. Timoshenko S. P., (1961), *Theory of Elastic Stability*, McGraw-Hill, New York. Choudary R. B. "Introduction to ANSYS 10.0" *I. K. International Publishers Pvt .Ltd.*
- [7] Dr. A. V. Phan "ANSYS tutorial- 2-D Fracture analysis" *University of SouthAlabama.* Ugural, A. C. "Stresses in Plates and Shells", U.S.A.: McGraw-Hill Book Company, 1981