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πgb*-Continuity in Topological Spaces

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ABSTRACT : In this paper using πgb*-closed set in topological spaces due to Dhanya R and A Parvathi [22] we introduced a new class of functions in a topological spaces called π generalized b*-continuous functions (briefly πgb*-continuous functions). Further the concept of almost πgb*-continuous function and πgb*-irresolute function are discussed.

KEYWORDS: πgb*-continuous function, πgb*- irresolute function, almost πgb*-continuous function.

I. INTRODUCTION

Generalized open sets play a very important role in general topology and they are now the research topics of many researchers worldwide. Indeed a significant topic in general topology and real analysis concerns the variously modified forms of continuity, separation axioms etc., by utilizing generalized open sets. Levine [4] introduced the concept of generalized closed sets in topological spaces. Since then many authors have contributed to the study of the various concepts using the notion of generalized b-closed sets. New and interesting applications have been found in the field of Economics, Biology and Robotics etc. Generalized closed sets remains as an active and fascinating field within mathematicians.

II. RELATED WORK

Levine [4] and Andrijevic [1] introduced the concept of generalized open sets and b-open sets respectively in topological spaces. The class of b-open sets is contained in the class of semipre-open sets and contains the class of semi-open and the class of pre-open sets. Since then several researches were done and the notion of generalized semi-closed, generalized pre-closed and generalized semipre-open sets were investigated in [2, 5, 10]. The notion of π-closed sets was introduced by Zaitsev [12]. Later Dontchev and Noiri [9] introduced the notion of πg-closed sets. Park [11] defined πgp-closed sets. Then Aslim, Caksu and Noiri [3] introduced the notion of πgs-closed sets. D. Sreeja and S. Janaki [7] studied the idea of πgb-closed sets and introduced the concept of πgb-continuity. Later the properties and characteristics of πgb-closed sets and πgb-continuity were introduced by Sinem Caglar and Gulhan Ashim [6]. Dhanya. R and A. Parvathi[22] introduced the concept of πgb*-closed sets in topological spaces.

III. PRELIMINARIES

Throughout this paper (X, τ) represent non-empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ), cl(A) and int(A) denote the closure of A and the interior of A respectively. (X, τ) will be replaced by X if there is no chance of confusion.

Definition 2.1 Let (X, τ) be a topological space. A subset A of (X, τ) is called

(1) **a semi-closed set** [18] if $int(cl(A)) \subseteq A$
(2) **a α-closed set** [19] if $cl(int(cl(A))) \subseteq A$
(3) **a pre-closed set** [16] if $cl(int(A)) \subseteq A$
(4) **a semipre-closed set** [20] if $int(cl(int(A))) \subseteq A$
(5) **a regular closed set** [21] if $A = cl(int(A))$
(6) **a b-closed set** [1] if $cl(int(A)) \cap int(cl(A)) \subseteq A$.
(7) **a b*-closed** [13]set if $int(cl(A)) \subset U$, whenever $A \subset U$ and U is b-open.

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The complements of the above mentioned sets are called semi open, α-open, pre-open, semipre-open, regular open, b-open and b*-open sets respectively. The intersection of all semi closed (resp. α-closed, pre-closed, semipre-closed, regular closed and b- closed) subsets of (X, τ) containing A is called the semi closure (resp. α-closure, pre-closure, semipre-closure, regular closure and b-closure) of A and is denoted by scl(A) (resp. αcl(A), pcl(A), spcl(A), rcl(A) and bcl(A)). A subset A of (X,τ) is called clopen if it is both open and closed in (X, τ).

Definition 2.2

A subset A of a space (X, τ) is called **π-closed** [12] if A is a finite intersection of regular closed sets.

Definition 2.3

A subset A of a space (X, τ) is called

(1) **a g-closed set**[4] if $cl(A) \subset U$ whenever $A \subset U$ and U is open in (X, τ).
(2) **a gp-closed set [**5] if $pcl(A) \subset U$ whenever $A \subset U$ and U is open in (X, τ).
(3) **a gs-closed set** [10] if $scl(A) \subset U$ whenever $A \subset U$ and U is open in (X, τ).
(4) **a gb-closed set** [1] if $bcl(A) \subset U$ whenever $A \subset U$ and U is open in (X, τ).
(5) **a πg-closed set** [9] if $cl(A) \subset U$ whenever $A \subset U$ and U is π-open in (X, τ).
(6) **a πgp-closed set** [11] if $pcl(A) \subset U$ whenever $A \subset U$ and U is π-open in (X, τ).
(7) **a πgs-closed set** [3]) if $scl(A) \subset U$ whenever $A \subset U$ and U is π-open in (X, τ).
(8) **a πgb-closed set** [7] if $bcl(A) \subset U$ whenever $A \subset U$ and U is π-open in (X, τ).
(9) **a πgb*-closed set** [22] if $int(bcl(A)) \subset U$ whenever $A \subset U$ and U is π-open in (X, τ).

Complement of π-closed set is called **π-open set.**

Complement of g-closed, gp-closed, gs-closed, gb-closed, gα-closed, πgα-closed, πgp-closed, πgs-closed, πgb-closed and πgb*-closed sets are called g-open, gp-open, gs-open, gb-open, gα-open, πgα-open**,** πgp-open**,** πgs-open**,** πgsp-open, πgb-open and πgb*-open sets respectively.

Definition 2.5

A function $f : (X, \tau) \to (Y, \sigma)$ is called continuous (resp.α-continuous, pre-continuous, semi-continuous, b-continuous, g-continuous, gα-continuous, gp-continuous, gs-continuous, gb-continuous) if $f^{-1}(V)$ is closed (resp. α-closed, pre-closed, semi-closed, b-closed, gα-closed, gp-closed, gs-closed, gb-closed) in (X, τ) for every closed set V in (Y, σ).

Definition 2.6

A function $f : (X, \tau) \to (Y, \sigma)$ is called π-continuous(resp. $\pi\alpha$-continuous, πg-continuous, πgp-continuous, πgs-continuous, πgb-continuous) if $f^{-1}(V)$ is π-closed(resp. $\pi\alpha$-closed, πg-closed, πgp-closed, πgs-closed, πgb-closed) in (X, τ) for every closed set V of (Y, σ).

Theorem 2.7 [22]

Every closed, α-closed, pre-closed, semi-closed, b-closed, g-closed, gp-closed, gs-closed, gb-closed, gα-closed, b*-closed, π-closed, πg-closed, πgα-closed, πgp-closed, πgs-closed and πgb-closed set is πgb*-closed. And the converse need not be true.

IV. πgb*-CONTINUITY

Definition 3.1

A function $f : (X, \tau) \to (Y, \sigma)$ is called **πgb*-continuous** if $f^{-1}(V)$ is πgb*-closed in (X, τ) for every closed set V of (Y, σ).

Definition 3.2

A function $f : (X, \tau) \to (Y, \sigma)$ is called **πgb*-irresolute** if $f^{-1}(V)$ is πgb*-closed in (X, τ) for every πgb*-closed set V in (Y, σ).

Definition 3.3A function $f : (X, \tau) \to (Y, \sigma)$ is called **πgb*-closed** if f(V) is πgb*-closed in (Y, σ) for every πgb*-closed set V in (X, τ).

Theorem 3.4

Every continuous function is πgb*-continuous.

Proof

Let $f : (X, \tau) \to (Y, \sigma)$ be a continuous function. Let V be a closed set in Y. Since f is continuous $f^{-1}(V)$ is closed in X. As every closed set is πgb*-closed, $f^{-1}(V)$ is πgb*-closed. Hence f is πgb*- continuous.

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Remark 3.5
The converse of the above theorem need not be true as seen from the following example.

Example 3.6
Consider $X = \{a, b, c\}$, $\tau = \{\varphi, \{a\}, \{a, b\}, \{a, c\}, X\}$ and $Y = \{a, b, c\}$ with the topology $\sigma = \{\varphi, Y, \{b, c\}\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be defined by f(a) = a, f(b) = c, f(c) = b, then f is πgb*-continuous but it is not continuous.

Theorem 3.7
Every pre-continuous function is πgb*-continuous.

Proof
Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a pre-continuous function. Let V be a closed subset of Y. Since f is pre-continuous $f^{-1}(V)$ is pre-closed in X. As every pre-closed set is πgb*-closed, $f^{-1}(V)$ is πgb*-closed. Hence f is πgb*-continuous.

Remark 3.8
The converse of above theorem need not be true which can be shown by the following example.

Example 3.9
Let $X = \{a, b, c, d\}$ with topology $\tau = \{\varphi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}, X\}$ and $Y = \{a, b, c, d\}$ with the topology $\sigma = \{\varphi, Y, \{a\}, \{c\}, \{a, c\}\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity function. Then f is πgb*-continuous but it is not pre-continuous.

Theorem 3.10
Every semi-continuous function is πgb*-continuous.

Proof
Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a semi-continuous function. Let V be a closed subset of Y, since f is semi-continuous $f^{-1}(V)$ is semi-closed in X. As every semi-closed set is πgb*-closed, $f^{-1}(V)$ is πgb*-closed. Hence f is πgb*-continuous.

Remark 3.11
The converse of the above theorem need not be true as seen from the following example.

Example 3.12
Let $X = \{a, b, c\}$ with topology $\tau = \{\varphi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\}$ and $Y = \{a, b, c\}$ with topology $\sigma = \{\varphi, Y, \{c\}\}$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ as f(a) = b, f(b) = a and f(c) = c. Then f is πgb*-continuous but not it is semi-continuous.

Theorem 3.13
Every b-continuous function is πgb*-continuous.

Proof
Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a b-continuous function. Let V be a closed subset of Y. Since f is b-continuous $f^{-1}(V)$ is b-closed in X. As every b-closed set is πgb*-closed, $f^{-1}(V)$ is πgb*-closed. Hence f is πgb*-continuous.

Remark 3.14
The converse of the above theorem need not be true which can be seen from the following example.

Example 3.15
Let $X = \{a, b, c, d\}$ with topology $\tau = \{\varphi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}, X\}$ and $Y = \{a, b, c, d\}$ with topology $\sigma = \{\varphi, Y, \{d\}\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be defined as f(a) = b, f(b) = a, f(c) = b, f(d) = d. Then f is πgb*-continuous but it is not b-continuous.

Theorem 3.16
Every *g*-continuous function is πgb*-continuous.

Proof
Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a g-continuous function. Let V be a closed subset of Y. Since f is g-continuous $f^{-1}(V)$ is g-closed in X. As every g-closed set is πgb*-closed, $f^{-1}(V)$ is πgb*-closed. Hence f is πgb*-continuous.

Remark 3.17
The converse of the above theorem need not be true which can be seen from the following example.

Example 3.18
Let $X = \{a, b, c, d\}$ with topology $\tau = \{\varphi, \{a\}, \{d\}, \{a, d\}, \{c, d\}, \{a, c, d\}, X\}$ and $Y = \{a, b, c, d\}$ with topology $\sigma = \{\varphi, Y, \{a\}, \{a, b, d\}\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be defined as f(a) = a, f(b) = d, f(c) = c, f(d) = b. Then f is πgb*-continuous but not g-continuous.

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Theorem 3.19
Every gp-continuous function is πgb*-continuous.
Proof
Let f : (X, τ) → (Y, σ) be a gp-continuous function. Let V be a closed subset of Y. Since f is gp-continuous $f^{-1}(V)$ is gp-closed in X. As every gp-closed set is πgb*-closed, $f^{-1}(V)$ is πgb*-closed. Hence f is πgb*-continuous.
Remark 3.20
The converse of the above theorem need not be true as seen from the following example.
Example 3.21
Let X = { a, b, c } with topology τ = { φ, {a, b}, X } and let Y = { a, b, c } with topology σ = { φ, Y, {b}, {c}, { b, c}}. Let f : (X, τ) → (Y, σ) be defined as f(a) = a, f(b) = c, f(c) = b. Then f is πgb*-continuous but it is not gp-continuous.
Theorem 3.22
Every gs-continuous function is πgb*-continuous.
Proof
Let f : (X, τ) → (Y, σ) be a gs-continuous function. Let V be a closed subset of Y. Since f is gs-continuous $f^{-1}(V)$ is gs-closed in X. As every gs-closed set is πgb*-closed, $f^{-1}(V)$ is πgb*-closed. Hence f is πgb*-continuous.
Remark 3.23
The converse of the above theorem need not be true it can be seen from the following example.
Example 3.24
Let X = { a, b, c } with topology τ = { φ, {a}, {a, b}, { a, c}, X } and Y = { a, b, c } with topology σ = { φ, Y, { b}}.
Let f : (X, τ) → (Y, σ) be defined as f(a) = c, f(b) = b, f(c) = a. Then f is πgb*-continuous but is not gs-continuous
Theorem 3.25
Every gb-continuous function is πgb*-continuous.
Proof
Let f : (X, τ) → (Y, σ) be a gb-continuous function. Let V be a closed subset of Y. Since f is gb-continuous $f^{-1}(V)$ is gb-closed in X. As every gb-closed set is πgb*-closed, $f^{-1}(V)$ is πgb*-closed. Hence f is πgb*-continuous.
Remark 3.26
The converse of above theorem need not be true as seen from the following example.
Example 3.27
Let X = { a, b, c, d } with topology τ = { φ, {a}, {b}, {a, b}, { a, b, c}, {a, b, d} X } and Y = { a, b, c, d } with topology σ = { φ, Y, { d}}. Let f : (X, τ) → (Y, σ) be defined as f(a) = b, f(b) = c, f(c) = a, f(d) = d. Then f is πgb*-continuous but is not gb-continuous.
Theorem 3.28
Every πg-continuous function is πgb*-continuous.
Proof
Let f : (X, τ) → (Y, σ) be a πg-continuous function. Let V be a closed subset of Y. Since f is πg-continuous $f^{-1}(V)$ is πg-closed in X. As every πg-closed set is πgb*-closed, $f^{-1}(V)$ is πgb*-closed. Hence f is πgb*-continuous.
Remark 3.29
The converse of the above theorem need not be true it can be seen from the following example.
Example 3.30
Let X = { a, b, c, d } with topology τ = { φ, {a}, {b}, {a, b}, { b, c}, {a, b, c}, X } and Y = { x, y, z } with topology σ = { φ, Y, { x, y}, {x, z}, {x}}. Define f : (X, τ) → (Y, σ) as follows f(a) = y, f(b) = f(d) = x, f(c) = z then f is πgb*-continuous but it is not πg-continuous.

Theorem 3.31
Every πgp-continuous function is πgb*-continuous.
Proof
Let f : (X, τ) → (Y, σ) be a πgp-continuous function. Let V be a closed subset of Y, since f is πgp-continuous $f^{-1}(V)$ is πgp-closed in X. As every πgp-closed set is πgb*-closed, $f^{-1}(V)$ is πgb*-closed. Hence f is πgb*-continuous.
Remark 3.32

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The converse of the above theorem need not be true it can be seen from the following example.

Example 3.33

Let $X = \{a, b, c, d\}$ with topology $\tau = \{\varphi, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, X\}$ and $Y = \{x, y, z\}$ with topology $\sigma = \{\varphi, Y, \{x, y\}, \{x, z\}, \{x\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be defined as $f(a) = y$, $f(b) = f(d) = x$, $f(c) = z$ then f is πgb*-continuous but it is not πgp-continuous.

Theorem 3.34

Every πgs-continuous function is πgb*-continuous.

Proof

Let $f : (X, \tau) \to (Y, \sigma)$ be a πgs-continuous function. Let V be a closed subset of Y, since f is πgs-continuous $f^{-1}(V)$ is πgs-closed in X. As every πgs-closed set is πgb*-closed, $f^{-1}(V)$ is πgb*-closed. Hence f is πgb*-continuous.

Remark 3.35

The converse of the above theorem need not be true as seen from the following example.

Example 3.36

Let $X = \{a, b, c\}$ with topology $\tau = \{\varphi, \{b\}, \{c\}, \{a, b\}, \{b, c\}, X\}$. Define $f : (X, \tau) \to (Y, \sigma)$ as identity function then f is πgb*-continuous but it is not πgs-continuous.

V. πGB*-CONTINUITY AND ITS CHARACTERISTICS

Theorem 4.1

Let $f : X \to Y$ be a function. Then the following statements are equivalent:

(1) f is πgb*-continuous;
(2) The inverse image of every open set in Y is πgb*-open in X.

Proof

(1) $\Rightarrow$ (2)

Let U be open subset of X. Then (Y – U) is closed in Y. Since f is πgb*-continuous, $f^{-1}(Y-U) = X - f^{-1}(U)$ is πgb*-closed in X. Hence $f^{-1}(U)$ is πgb*-open in X.

(2) $\Rightarrow$ (1)

Let V be a closed subset of Y. Then (Y – V) is open in Y hence by hypothesis (2) $f^{-1}(Y-V) = X - f^{-1}(V)$ is πgb*-open in X. Hence $f^{-1}(V)$ is πgb*-closed in X. Therefore, f is πgb*-continuous.

Theorem 4.2

Every πgb*- irresolute function is πgb*-continuous.

Proof

Let $f : X \to Y$ be πgb*-irresolute function. Let V be closed set in Y, then V is πgb*-closed in Y. Since f is πgb*-irresolute $f^{-1}(V)$ is πgb*-closed in X. Hence f is πgb*-continuous.

Remark 4.3

The converse of the above theorem need not be true it can be seen from the following example.

Example 4.4

Consider $X = Y = \{a, b, c\}$, $\tau = \{\varphi, X, \{a\}, \{b\}, \{a, b\}\}$, $\sigma = \{\varphi, X, \{a\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity map. Then f is πgb*-continuous but it is not πgb*-irresolute.

Theorem 4.5

Let $f : X \to Y$ be a function. Then the following statements are equivalent:

(1) For each $x \in X$ and each open set V containing f(x) there exists a πgb*-open set U containing x such that $f(U) \subset V$.
(2) $f(\pi gb^*\text{-}cl(A)) \subset cl(f(A))$ for every subset A of X.

Proof

(1) $\Rightarrow$ (2)

Let $y \in f(\pi gb^*\text{-}cl(A))$ then, there exists an $x \in \pi gb^*\text{-}cl(A)$ such that $y = f(x)$. We claim that $y \in cl(f(A))$ and let V be any open neighborhood of y. Since $x \in \pi gb^*\text{-}cl(A)$ there exists an πgb*-open set U such that $x \in U$ and $U \cap A \neq \varphi$, $f(U) \subset V$. Since $U \cap A \neq \varphi$, $f(A) \cap V \neq \varphi$. Therefore, $y = f(x) \in cl\, f(A)$. Hence $f(\pi gb^* cl\, A) \subset cl\, f(A)$.

(2) $\Rightarrow$ (1)

Let $x \in X$ and V be any open set containing f(x). Let A = f^{-1}(Y-V), since f(πgb*-cl(A)) $\subset$ cl(f(A)) $\subset$ (Y – V) $\Rightarrow$ πgb*cl A $\subset$ f^{-1}(Y-V) = A. Hence πgb*-cl(A) = A. Since f(x) $\in$ V $\Rightarrow$ x $\in$ f^{-1}(V) $\Rightarrow$ x $\notin$ A $\Rightarrow$ x $\notin$ πgb*-cl(A). Thus there exists an open set U containing x such that U $\cap$ A = φ $\Rightarrow$ f(U) $\cap$ f(A) = φ. Therefore f(U) $\subset$ V. **Definition 4.6**

A topological space (X, τ) is a **πgb*-space** if every πgb*-closed set is closed.

Theorem 4.7

Every πgb*-space is πgb*-$T_{1/2}$ space.

Proof

Let (X ,τ) be a πgb*-space and let A$\subset$ X be πgb*-closed set in X.

Then A is closed $\Rightarrow$ A is b*-closed $\Rightarrow$ (X,τ) is a πgb*-$T_{1/2}$ space

Theorem 4.8

Let f : (X, τ) $\rightarrow$ (Y, σ) be a function then,

1. (1) If f is πgb*-irresolute and X is πgb*-$T_{1/2}$ space, then f is b*-irresolute.
2. (2) If is πgb*-continuous and X is πgb*-$T_{1/2}$ space, then f is b*-continuous.

Proof

1. (1) Let V be b*-closed in Y, then V is πgb*-closed in Y. Since f is πgb*-irresolute, f^{-1}(V) is πgb*-closed in X. Since X is πgb*-$T_{1/2}$ space, f^{-1}(V) is b*-closed in X. Hence f is b*-irresolute.
2. (2) Let V be closed in Y. Since f is πgb*-continuous, f^{-1}(V) is πgb*-closed in X. Since X is πgb*-$T_{1/2}$ space, f^{-1}(V) is b*-closed. Therefore f is b*-continuous.

Definition 4.9

A function f : X $\rightarrow$ Y is said to be **almost πgb*-continuous** if f^{-1}(V) is πgb*-closed in X for every regular closed set V of Y.

Theorem 4.10

For a function f : X $\rightarrow$ Y, the following statements are equivalent:

1. (1) f is almost πgb*-continuous.
2. (2) f^{-1}(V) is πgb*-open in X for every regular open set V of Y.
3. (3) f^{-1}(int(cl(V))) is πgb*-open in X for every open set V of Y.
4. (4) f^{-1}(cl(int(V))) is πgb*-closed in X for every closed set V of Y.

Proof

(1) $\Rightarrow$ (2)

Suppose f is almost πgb*-continuous. Let V be a regular open subset of Y. Since (Y – V) is regular closed and f is almost πgb*-continuous, f^{-1}(Y-V) = X – f^{-1}(V) is πgb*-closed in X. Hence f^{-1}(V) is πgb*-open in X.

(2) $\Rightarrow$ (1)

Let V be a regular closed subset of Y. Then (Y – V) is regular open. By the hypothesis, f^{-1}(Y-V) = X – f^{-1}(V) is πgb*-open in X. Hence f^{-1}(V) is πgb*-closed. Thus f is πgb*-continuous.

(2) $\Rightarrow$ (3)

Let V be an open subset of Y. Then int(cl(V)) is regular open in Y. By the hypothesis, f^{-1}(int(cl(V))) is πgb*-open in X.

(3) $\Rightarrow$ (2)

Let V be a regular open subset of Y. Since V = int(cl(V)) and every regular open set is open then f^{-1}(V) is πgb*-open in X.

(3) $\Rightarrow$ (4)

Let V be a closed subset of Y. Then (Y – V) is open in Y. By the hypothesis, f^{-1}(int(cl(Y-V))) = f^{-1}(Y-cl(int(V))) = X – f^{-1}(cl(int(V))) is πgb*-open in X. Therefore f^{-1}(cl(int(V))) is πgb*-closed in X.

(4) $\Rightarrow$ (3)

Let V be a open subset of Y. Then (Y – V) is closed. By the hypothesis f^{-1}(cl(int(Y-V))) = X – f^{-1}(int(cl(V))) is πgb*-closed in X. Therefore, f^{-1}(int(cl(V))) is πgb*-open in X.

Theorem 4.11

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Every πgb*-continuous function is almost πgb*-continuous.

Proof

Let f : X → Y be πgb*-continuous function. Let V be regular closed set in Y, then V is closed in Y. Since f is πgb*-continuous function f^{-1}(V) is πgb*-closed in X. Therefore f is almost πgb*-continuous.

Theorem 4.12

Every almost b*-continuous function is almost πgb*-continuous.

Proof

Let f : X → Y be almost b*-continuous function and let V be regular closed set in Y. Then, f^{-1}(V) b*-closed in X, hence f^{-1}(V) is πgb*-closed in X. Therefore f is almost πgb*-continuous.

Theorem 4.13

Let X be a πgb*-$T_{1/2}$ space. Then f : X → Y is almost πgb*-continuous if and only if f is almost b*-continuous.

Proof

Suppose f : X → Y is almost πgb*-continuous. Let A be a regular closed subset of Y. Then f^{-1}(A) is πgb*-closed in X. Since X is πgb*-$T_{1/2}$ space, f^{-1}(A) is b*-closed in X. Hence f is almost b*-continuous.

Conversely, suppose that f : X → Y is almost b*-continuous and A be a regular closed subset of Y. Then f^{-1}(A) is b*-closed in X. Since every b*-closed set is πgb*-closed, f^{-1}(A) is πgb*-closed. Therefore, f is almost πgb*-continuous.

VI. CONCLUSION

The study of πgb*-continuous function is derived from the definition of πgb*-closed set. This study can be extended to fuzzy topological spaces and bitopological spaces.

REFERENCES

[1] D. Andrijevic., "On b-open sets", Mat. Vesnik, 48, (1996), 59-64.
[2] S.P. Arya and T.M. Nour, "Characterizations of s-normal spaces", Indian J. Pure Appl. Math, 8, 21, (1990), 717-719.
[3] G. Aslim, A. Caksu Guler and T.Noiri, "On πgs-closed sets in topological spaces", Acta Math. Hungar., 112, 4, (2006), 275-283.
[4] N. Levine., "Generalized closed sets in topology", Rend. Circ. Mat. Palermo, 2, 19 (1970), 89-96.
[5] H. Maki, J. Umehara and T. Noiri., "Every topological space is pre-T1/2",Mem.Fac.Sci. Kochi Univ. Ser. A Math.,17, (1996), 33-42.
[6] Sinem Caglar Akgun and Gulhan Aslim., "On πgb- Closed Sets and Related Topics" International Journal of Mathematical Archive, 3, 5, (2012), 1873-1884.
[7] Sreeja D.and Janaki C., "On πgb-closed sets in topological spaces", International Journal of Mathematical Archieve, 2, 8, (2011), 1314-1320.
[8] M. S.Sarsak and N. Rajesh, "π-Generalized Semi-Preclosed Sets",International Mathematical Forum, 5 (2010), 573- 578.
[9] J.Dontchev and T.Noiri, "Quasi Normal Spaces and πg-closed sets", Acta Math. Hungar., 89, 3, (2000), 211-219.
[10] J.Dontchev.,"On generalizing semi-preopen sets", Mem. Fac. Sci.Kochi Univ. Ser. A Math., 16, (1995), 35-48.
[11] J. H Park, "On πgp-closed sets in Topological Spaces", Indian J.Pure Appl. Math.,(2004)
[12] Zaitsev V., "On Certain Class of Topological Spaces and their bicompactification", Dokl.Akad.Nauk SSSR 178(1968),pp.778-779.
[13] S.Muthuvel and Parimelazhagan., "b*-closed sets in topological spaces", Int.Journal of Math. Analysis, Vol. 6, 2012, no. 47, 2317-2323.
[14] Ahmad-Al-Omari and Mohd.Salmi Md Noorani , "On generalized b-closed sets", Bulletin of Mathematical Sciences Society, 32, (2009), 19-30.
[15] C.Janaki, "Studies on πg∝-closed sets in Topology", Ph.D Thesis, Bharathiar University, Coimbatore.(2009).
[16] Mashhour, A.S., Abd EI-Monsef, M.E. and EI-Deeb, S.N. "On pre-continuous and weak precontinuous mapping", Proc. Math. Phys. Soc. Egypt., 53, (1982),47 – 53.
[17]Maki, H., Devi, R. and Balachandran, K. "Associated topologies of generalized α-closed sets and α-generalized closed sets", *Mem. Fac. Sci. Kochi. Univ. Ser.A. Math.*, 5, (1994),51 – 63.
[18] Levine, N. "Semi open sets and semi continuity in topological spaces", Amer. Math. Monthly, 70,(1963). 36 – 41
[19] Njastad, o. On some classes of nearly open sets" , pacific j.Math.,15, (1965),961-970.
[20] Andrijevic, D. "Semipre-open sets",Mat.Vesnnik,38, (1986),24-32.
[21] Stone. M Application of the theory of Boolean rings to general topology,Trans.Amer.Math.soc.,41, (1937), 374-481.
[22] Dhanya R and A.Parvathi. "On πgb*-closed sets in topological spaces", IJIRSET, Vol. 3, (2014), Issue 5, 2319-8753.