

# A New Ranking Scheme for Multi and Single Objective Problems

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**ABSTRACT:** Evolutionary algorithms (EAs) have received a lot of interest in last two decade due to the ease of handling the multiple objectives. But one of criticism of EAs is lack of efficient and robust generic method to handle the constraints. One way to handle the constraint is use the penalty method .in this paper we have proposed a method to find the objective when the decision maker (DM) has to achieve the certain goal. The method is variant of multi objective real coded Genetic algorithm inspired by penalty approach. It is evaluated on eleven different single and multi objective problems found in literature. The result show that the proposed method perform well in terms of efficiency, and it is robust for a majority of test problem

**KEYWORDS:** Multi objective problem, Goal attainment method, Decision maker (DM), ID , dM, rank

## I. INTRODUCTION

During the last few decades Evolutionary Algorithms (EAs) have proved to become an important tool to solve difficult search and optimization problems. Most real-world problems have constrained and EAs do not have a efficient and generic constraint handling techniques. The constrained optimization problem may be handled as a multi objective optimization problem as indicated by Coello Coello [2], Michalewicz [9] and Fonseca and Fleming [5]. Furthermore, EAs based on non-dominated sorting for multiobjective problems have received large interest during the past decades. Therefore it seems natural to look upon the constrained optimization problem as a multi objective problem. One of the interesting constraints handling method based on non-domination is presented by Deb et al. [7], Multi objective approaches of constrained problems based on Shaffers VEGA [11] is found in [12] [10]. To directly apply a multi objective EA based on non-domination on a constrained optimization problem leads to a search of the best compromises of the objective value and constraint satisfaction, This whole set of solutions is usually not interesting since it is the optimal and feasible solution that is searched. Therefore it will not be efficient to directly apply a multi objective EA on a constrained problem; still the idea to handle the constrained problem with some variant of a multi objective EA is interesting. One of the most crucial steps in a multi objective EA is how to rank individuals. In this paper an alternative ranking scheme for the constrained single and multi objective problem is introduced. This ranking scheme is generic and no new parameters are introduced. The ideas of the ranking scheme are borrowed from the non-domination ranking for multiple objectives by Goldberg [6], Ander's ranking method [1] and Fonseca and Fleming ranking approach [4]. The paper first defines the constrained optimization problem, and hereafter the proposed method is presented in more detail.

## II. PRELIMINARIES

### a) Constraint optimization problem.

In this section the constraint optimization problem and its terminology is define. The constraint optimization problem or non -linear programming with 'k' inequality constraint and 'm' equality constraint is formulated as

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Minimize  
 $F(X)$  (1)

Such that

$g_i(X) \leq 0 \quad i = 1 \dots k$  (2)

$h_i(X) = 0 \quad i = 1 \dots m$  (3)

$X = [x_1, x_2, x_3, x_4 \dots \dots x_n]$  is a vector of 'n' design variables such that  $X \in S \subseteq R^n$ . The search space S is define as an n-dimensional rectangle by upper and lower for design variables,  $x_i^l \leq x_i \leq x_i^u \quad i = 1 \dots n$ . the feasible region  $F \subseteq S$  is the region of S for which the inequality and equality constraint are satisfied. The optimal solution is denoted by  $X^*$ . a constraint is said to be active at point  $X^*$  if  $g_j(X^*) = 0$ . By default all equality constraint are active at all point of feasible space.

### b) Incorporate the decision making in problem

Most of the times the decision maker (DM) himself in such position that he can decide what should be goal for the given problem.

So the equation 1 can be written as

$F(X) \leq G(x)$  (4)

it is depend upon the DM to which extent he want fulfillment of the goal .this can be achieved by using the following equation

$f_i(x) - wz \leq G_i(x)$  (5)

' $f_i$ ' is the  $i^{th}$  objective function . and ' $G_i$ ' is the goal associate with  $i^{th}$  objective function , 'w' is the weight assign to the 'i'th objective, and  $z \in S \subseteq R^n$ .

### III. THE PROPOSED APPROACH FOR CONSTRAINT OPTIMIZATION

In this section the proposed scheme is introduced .the new ranking scheme is used to formulate a scalar valued function that is used to rank individual in current population. Then selection, crossover, mutation and reinsertion are used in a standard manner for a real coded GA in this paper. Our main focus for this section is to define a new ranking scheme.

The approach is based on the following criteria.

- If no feasible individual exists in the current population , the search should be directed towards the feasible region.
- If majority of individuals in population are feasible , the search should be directed towards the individuals which satisfy the goal.
- A feasible individual closer to the goal value always better than a feasible individual further from the goal.

This ranking scheme is consist of finding the two ranks of each individual base on their objective values and the base of their goal value. First we will find the  $rank_1$  of the individual .  $rank_i$  is higher than  $rank_j$  if  $i < j$ . In order to find out the  $rank_1$  and  $rank_2$ , we find out the value of two parameter ID and dM. where  $ID_k$  is the list of individual which are dominated by the  $k^{th}$  individual and  $dM_k$  is the number of individual are dominate  $k^{th}$  individual. At time of finding  $rank_1$  value of  $k^{th}$  individual the  $ID_k$  and  $dM_k$  value depend on it's objective values and at time of  $rank_2$  the  $ID_k$  and  $dM_k$  value depends on its goal value. Suppose the problem is in form eq(1) and constraint are in the form of eq(2) or eq(3),and problem with goal value in form of eq(5).

#### a) Procedure to find ID and dM value in rank<sub>1</sub>

Suppose there are 'q' number of objectives and at certain instant assume the two q-dimensional objective vectors are  $y_a(a_1, a_2 \dots a_q)$  and  $y_b(b_1, b_2 \dots b_q)$ .  $y_a$  is said to be better than  $y_b$

- if ( $y_a[a_i] < y_b[b_i]$ )  $\forall i = 1 \dots q$ .
- The  $y_a$  dominate more number of objective values of  $y_b$ .

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If both  $Y_a$  and  $Y_b$  dominate equal number of objective values or all the objective values of both  $Y_a$  and  $Y_b$  are equal than they are said to be non-dominating individuals. On the basis of above criteria found the value of ID and dM value of all individual in the population.

### b) Procedure to find ID and dM value in rank<sub>2</sub>

Again we are considering the  $Y_a(Y_{a,1}, Y_{a,2}, \dots, Y_{a,q})$  and  $Y_b(Y_{b,1}, Y_{b,2}, \dots, Y_{b,q})$  and goal vector is  $(g_1, g_2, \dots, g_q)$ . also consider that  $Y_a$  is such that it meets a number from 1 to 'q-k' number of specified goal. There are three cases .

• **Case A**

$$\exists k = 1 \dots q - 1, \forall i = 1 \dots k, \forall j = k + 1 \dots q \quad \text{If } (Y_{a,i} \leq g_i) \wedge (Y_{a,j} > g_j)$$

$Y_a$  Meets goals from 1...k and it will preferable to  $Y_b$ . If it dominates  $Y_b$  with respect to its k+1..... q . For the case where all of the k+1....q components are equal to  $Y_b$ ,  $Y_a$  is still be preferable to  $Y_b$ , if it dominates  $Y_b$  with respect to remaining components or if the remaining components of  $Y_b$  do not meets the goal. i.e

$$(Y_a(k+1 \dots q) < Y_b(k+1 \dots q)) \vee \{(Y_a(k+1 \dots q) = Y_b(k+1 \dots q)) \wedge [Y_a(k \dots 1) < Y_b(k \dots 1) \vee \sim(Y_b(k+1 \dots q) \leq g(k+1 \dots q))]\}$$

• **Case B**

$\forall i = 1 \dots q$  if  $(Y_{a,i} > g_i)$ .  $Y_a$ , Does not Satisfies none of the goal. Then,  $Y_a$  is preferable to  $Y_b$  if and only if dominates  $Y_b$ ,  $Y_a(1 \dots q) < Y_b(1 \dots q)$

• **Case C**

$\forall j = 1 \dots q$  if  $(Y_{a,j} \leq g_j)$ .  $Y_a$  meets all of the goals, which means that it is a satisfactory though not necessarily optimal, solution .  $Y_a$  is preferable to  $Y_b$  if and only if it dominates  $Y_b$  or  $Y_b$  is not satisfactory .

$$(Y_a(1 \dots q) < Y_b(1 \dots q)) \vee \sim(Y_b(1 \dots q) \leq g(1 \dots q))$$

On the basis of above criteria found out the ID and dM values for all individual in population.

Once the Id and dM for both rank<sub>1</sub> and rank<sub>2</sub> values are found, assign the rank<sub>1</sub> and rank<sub>2</sub> on basis of dM value .

Lower the dM value higher the rank. At this stage each individual has rank<sub>1</sub> and rank<sub>2</sub>. P is the size of population . N

is number of feasible solutions. Now new objective function  $\Phi(x_j)$  is formulated as

$$\Phi(x_j) = \frac{P-N}{P} \text{rank}_1(x_j) + \frac{N}{P} \text{rank}_2(x_j) \tag{6}$$

### c) Procedure to find ID<sub>final</sub> and dM<sub>final</sub> value

Found out the  $\Phi$  value for each individual of the population by using eq(6).

•  $x_j$  dominates the  $x_i$ , if the value  $\Phi$  of  $x_j$  is less than  $x_i$ .

The ID<sub>final</sub> and dM<sub>final</sub> is found on the basis of above criteria and this values are used to find out final rank of individuals of the population.

The computational complexity of ranking scheme is  $P^2$ . Note that if no feasible solution is present in the population (N=0), the ranking according to the constraint and objective (rank<sub>1</sub>), and when P=N the ranking is according to goal (rank<sub>2</sub>). and search is directed toward the more compact feasible solution space . The most fit individual is one which has lower value of rank<sub>1</sub> and rank<sub>2</sub> , because it has lower objective value and fulfil all the constraint (rank<sub>1</sub>) and it has goal value which is more nearer to the assign goal value by DM (rank<sub>2</sub>). all these observation are consistent with the previously listed criteria .

An interesting feature for the proposed ranking scheme is that the search direction depends on the number of feasible solution. eq(6) has a similar structure as a penalty based approach but it should be pointed out that no parameter out that requires problem dependent fine is introduced. The "weight" for the two objective in eq(6) only depend on the size and number of feasible individuals in current population .

The proposed ranking procedure for NLP problems is summarized below

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- Assign the goal and formulate the problem according to eq(1) and eq(5)
- Find the value of ID and dM for  $rank_1$  and assign  $rank_1$  to population .
- Find the value of ID and dM for  $rank_2$  and assign  $rank_2$  to population.
- Calculate the objective  $\varphi(x_j)$  according to eq(6).
- Find the  $ID_{final}$  and  $dM_{final}$  value and assign rank to population.

### IV. AN ILLUSTRATIVE EXAMPLE

In the section the ranking based on eq(6) is discussed for a simple multiobjective constraint problem. the purpose of this section to show the import effect of the method not to solve the problem. first the result of an actual search is presented. Then the imposed search direction is discussed with the help of hypothetical population .

The problem is as follow. A quadratic function is to be minimized and the feasible solution are constrained by three circles .problem is stated as

$$\begin{aligned}
 &\text{minimize } f(x) = x_1^2 + x_2^2 \\
 &\text{subject to} \\
 &g_1(x) = x_1^2 + (x_2 - 3)^2 \leq 1.5^2; \\
 &g_2(x) = (x_1 - 1)^2 + (x_2 - 1)^2 \leq 1.5^2; \\
 &g_3(x) = (x_1 + 1)^2 + (x_2 - 1)^2 \leq 1.5^2 \\
 &-5 \leq x_1 \leq 5 \\
 &-3 \leq x_2 \leq 7
 \end{aligned}$$

For the unconstraint problem the optimal solution is  $x^* = [-0.1679, -0.01329]$  the population size set to 10 and , the maximum evaluation is set to 50. the crossover and mutation probability are set to be 0.6 and 1/n respectively .the initial and final generation according to eq(6) is given in figure 1.

At start it is found that  $x_2$  is generated between the range of -4 to 7 and  $x_1$  is generated between the range of -2 to 5 . The individuals are scattered in between above range , and no solution (individual) is feasible solution . in first evaluation the search is moving towards the to find the feasible solution but now the range of both variables i.e  $x_2, x_1$  are -2 to 2.5 and -1 to 4.5, respectively. At  $second^{nd}$  and  $third^{rd}$  still there was no feasible solution but range of solution space is moving toward to get feasible solution . it was 0 to 4 , -1.5 to 2 and 0 to 4.5 , -0.5 to 2 , respectively . up to to this stage the eq(6) assign the final rank of all the individuals is equal to  $rank_1$  , because there was no feasible solution .but at  $fourth^{th}$  evaluation the feasible solution is found now the final ranking will be take both  $rank_1$  and  $rank_2$  in an account as the number of feasible individuals increases search is move towards to individuals which will satisfy the goal i.e the goal value of individuals is more close to goal value (assign by DM).

From the evaluation 5 onwards the number of feasible solution are increases hence the search will move to solution region which satisfy all the constraint and the more close to goal in order to study the behaviour during this situation we are taking the cases when the number feasible solution are 15% of total population and when the feasible solutions are 45% of population .

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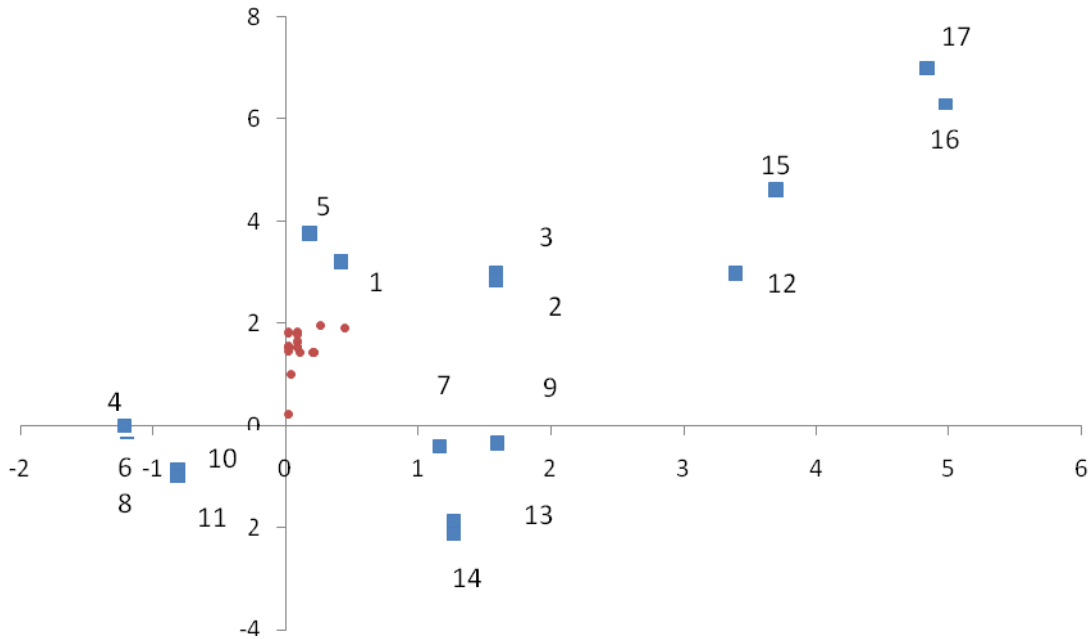


Figure 1. the numbering shows the ranking at starting of evolution. And dots shows the population at the of final evolution

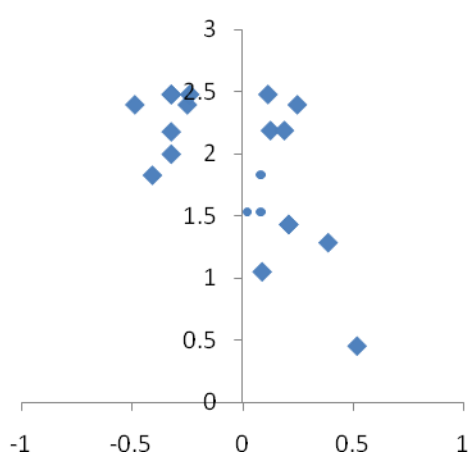


Figure 3

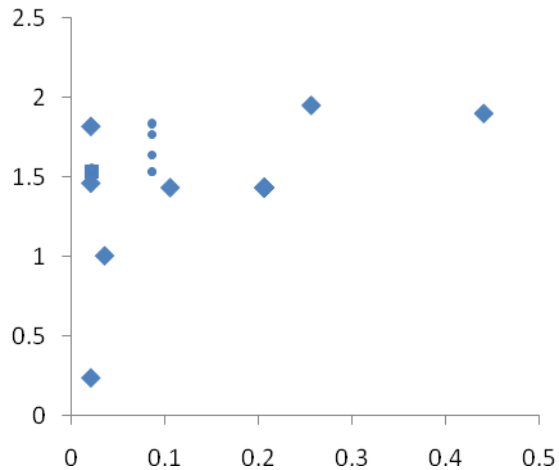


Figure 2.

circle shows the feasible solution and diamond shows the rest of individuals in population .square represent the most better individual (final\_rank =0)

From the figure 2 the number of feasible solutions is less hence the weight of rank<sub>2</sub> is less but still it moving towards the region where more number of feasible individuals can found which has values closer to the goal (by DM), and in figure 3 there are almost double in number of feasible individuals compare to figure 2. As the number of evaluation

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increases the number of feasible individuals increases and more better solution will found. the number of feasible individuals are found during evaluation is represented in the Figure 3.

Figure 1 satisfy the criteria 1 i.e start wth random population and finding the feasible solution . figure 2 satisfy the criteria 2. i.e here have more than one feasible solution finding solution feasible solution closer to goal value(By DM).figure 3 satisfy the criteria 3 i.e the individual closer to goal is better than one which is away from goal.

## V. CONSTRAINED MULTI AND SINGLE OBJECTIVE TEST PROBLEMS

It is always difficult to make fair comparisons between different EAs. Two different strategies may well have different optimal settings for the optimization algorithm parameters on the same problem. Another difficulty is to determine how to compare different algorithms. A naive but obvious way to compare algorithms is to compare the best solution found in the same number of function evaluations. A measure of the robustness of the algorithms is indicated by the spread of the best solutions found if the optimization is run several times independently. Here it is chosen to compare the results for the proposed ranking scheme with previously reported results for other EAs by other authors on a set of problems. In this section we discuss the various test problems which are implemented by using the proposed method . graph shows the feasible solution at different iterations.

Test problem	No of variables	No of objective functions	No of constraint value	Goal values
1	8	1	6	7000
2	7	1	4	680
3	5	1	6	30665
4	13	1	9	15
5	2	2	2	0.5,1
6	7	2	11	2920,715
7	6	2	6	499,20
8	1	2	0	4.09,3.06
9	2	2	2	10,1
10	2	2	2	4.09,3.06
11	.3	5	7	66000,75,2170000 270000,1100

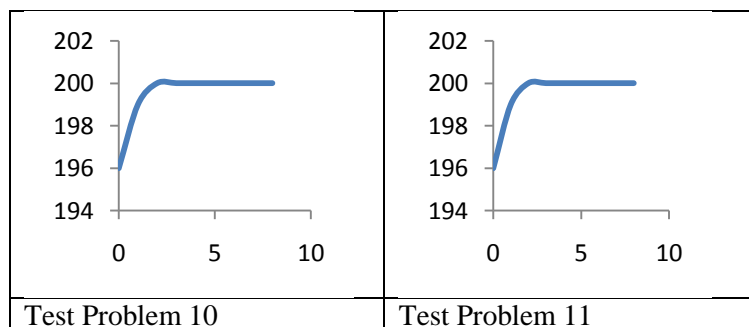
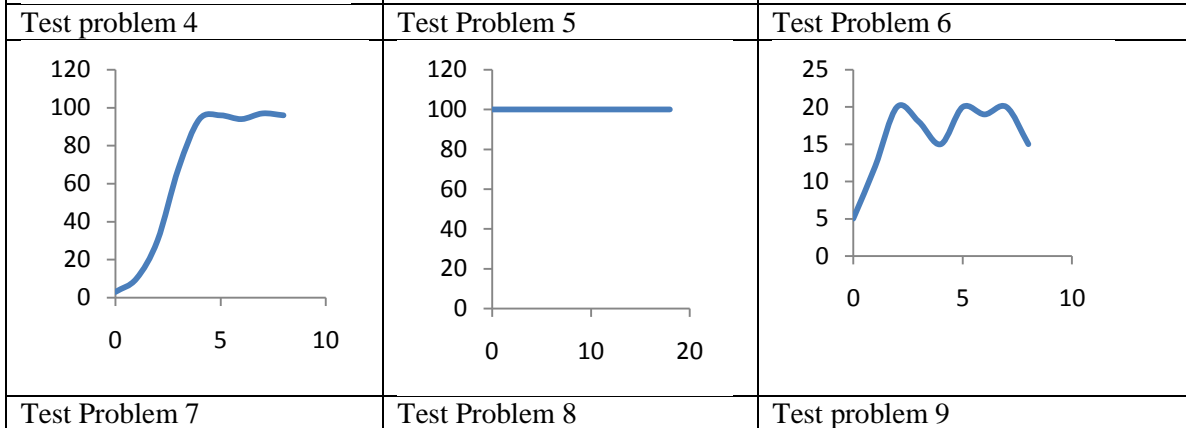
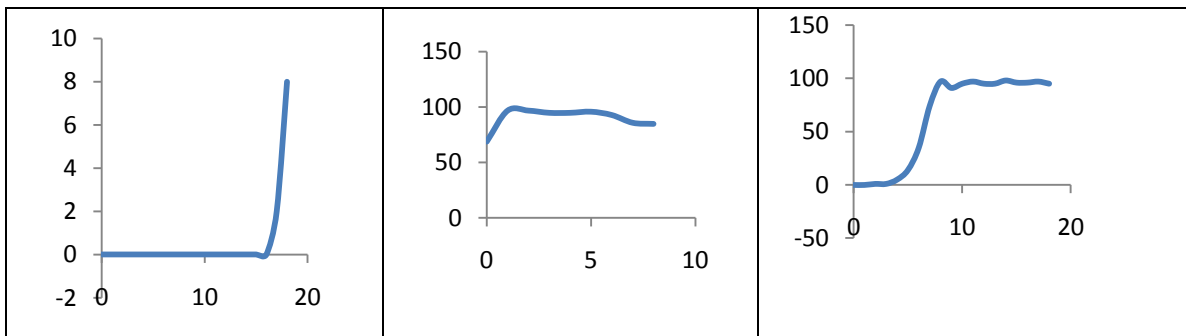
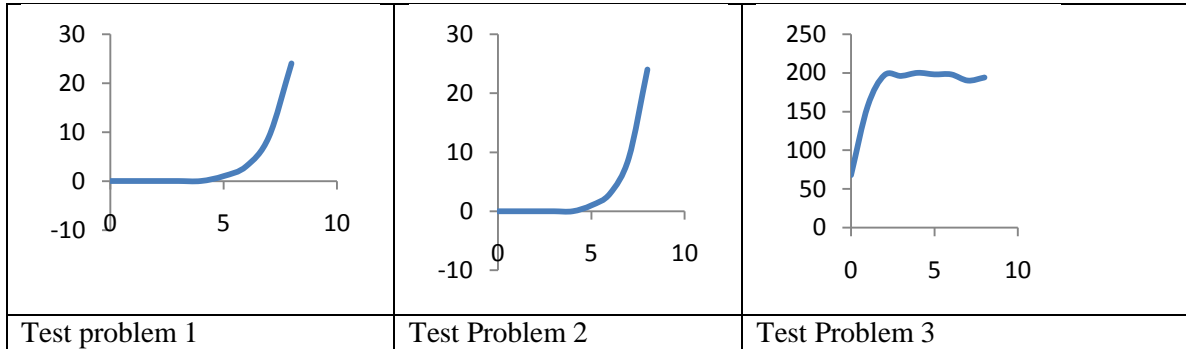
**Table 1:** problem description and input values taken during testing.

Our main purpose to solve the above problem by using propose ranking scheme therefore rest of parameters are taken as simple as possible here we have taken the NSGA II as our basic algorithm and the tournament selection method as selection method, PMXCrossover PolynomialMutation used as crossover and mutation respectively .This scheme will give more better answer if it used with more advance algorithm like adaptive genetic algorithm. The graphs shows the iteration Vs feasible solutions

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## VI. CONCLUSIONS

A general ranking scheme without problem specific extra parameters for constrained optimization problem has been presented. The performance for an algorithm with this ranking scheme has also been compared to the result of other



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ranking schemes with same algorithm on eleven problems, which are previously used by other authors. The results encourage further research since the method performs better than many other methods for the tested constrained single and multiple objective problems. It was only in the problem # 2 and #4 that this method did not perform well. It could not match up to the results for the other method for these problems (#2 and #4) either. The cause of this is an open question for further research. This method performed better in multi-objective (#5 to #11) problems as compare to other methods. It should also be mentioned that no effort has been made to study the optimal parameter settings such as population size, generation gap, mutation probability, etc.

## VII. FUTURE WORKS

Our main purpose here is to proposed the new ranking scheme not to test the problems the result of this ranking scheme will more better if this scheme is tested with more advance algorithm like adaptive genetic algorithm and differential evolutionary algorithm . Here the goal value of the objective is arranged at start of ranking and throughout the ranking the goal values are constant. If the goal values also able to change as the generation increment then more better result can be obtain because the in real world the conditions changes very rapidly.

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