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A STUDY OF CERTAIN INTEGRAL K-TRANSFORMS

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Abstract: The main object of this paper is to establish some generalization results of K-transform by using chain of this transform. Some examples of the results are also given.
Keywords: K-transform General class of polynomial, Bessel function. (2000 Mathematics subject classification: 33C99)

I. INTRODUCTION

If $g(y)$ and $f(x)$ are related by the integral equation

$$g(y)=\int_0^\infty f(x)k_v(xy)\sqrt{(xy)}\,dx \tag{1.1}$$

Then $g(y)$ is said to be the K-transform of order v of $f(x)$ and regard y as a complex variable.
We shall denote (1.1.) symbolically as

$$g(y)=M^v[f(x)] \tag{1.2}$$

This transform was introduced by Meijer [3], Maheshwari [2] have studied the properties of the aforesaid transform by considering certain chains of this transform.
Srivastava [4] introduced the general class of polynomials (see also Srivastava and Singh [5])

$$S_n^m[x]=\sum_{k=0}^{[n/m]}\frac{(-n)_{mk}}{k!}A_{n,k}x^k,\ n=0,1,2,... \tag{1.3}$$

Where m and n are arbitrary integers the coefficients $A_{n,k}(n,k\geq 0)$ are arbitrary constants real or complex.

II. MAIN RESULTS

Theorem 1. If

$$M^v[f_1(x)]=g(y) \tag{2.1}$$

$$M^v[f_2(x)S_n^m(\sqrt{x})]=\pi f_1\left(\frac{1}{y}\right) \tag{2.2}$$

Then

$$f(k)M^{2v}\left\{x^{k+\frac{3}{2}}f_2\left(\frac{x^2}{4}\right)\right\}=4y^{\frac{3}{2}}g(y^2) \tag{2.3}$$

Provided $x^{\left(\pm v\pm k+\frac{1}{2}\right)}f_2(x)$ are bounded and absolutely integrable $(0,\infty)$ and $f(k)=\sum_{k=0}^{[n/m]}\frac{(-n)_{mk}}{k!}A_{n,k}$.

Further, let

$$M^{2v}\left[S_n^m(\sqrt{x})f_3(x)\right]=\frac{\pi}{4}y^{-\frac{3}{2}}f_2\left(\frac{1}{4y^2}\right) \tag{2.4}$$

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$$M^{2v}\left[S_n^m(\sqrt{x})f_4(x)\right]=\frac{\pi}{4}y^{-\frac{3}{4}}f_3\left(\frac{1}{4y^2}\right) \quad (2.5)$$

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$$M^{2^{n-2}v}\left[S_n^m(\sqrt{x})f_n(x)\right]=\frac{\pi}{4}y^{-\frac{3}{4}}f_{n-1}\left(\frac{1}{4y^2}\right) \quad (2.6)$$

Then

$$f(k)M^{2^{n-1}v}\left[x^{k+\frac{3}{2}}f_n\left(\frac{x^2}{4}\right)\right]=4y^{\frac{3}{2}(2^{n-1}-1)}g\left(y^{2(n-1)}\right) \quad (2.7)$$

Provided $x^{\left(\pm 2^{n-1}v\pm k+\frac{1}{2}\right)}f_2(x)$ are bounded and absolutely integrable $(0,\infty)$ and $f(k)=\sum_{k=0}^{[n/m]}\frac{(-n)_{mk}}{k!}A_{n,k}$.

Proof: Taking $x^{\left(\pm 2^{n-2}v+k+\frac{1}{2}\right)}f_n(x), n=2,3,\dots,n$

Then by definition of K-transform, we obtain

$$M^v\left[f_1(z)\right]=\int_0^\infty f_1(z)k_v(zp)\sqrt{(zp)}dz$$

Write $f_1(z)$ from (2.2), we get

$$=\frac{1}{\pi}\int_0^\infty\left\{\int_0^\infty f_2(x)S_n^m(\sqrt{x})k_v\left(x/z\right)\sqrt{(x/z)}dx\right\}k_v(zp)\sqrt{(zp)}dz$$

Interchanging the order of integration which is justified under the conditions mentioned in the theorem and use the series representation of general class of polynomial, we get

$$=\frac{1}{\pi}f(k)\int_0^\infty\left\{\int_0^\infty\sqrt{p}\,k_v(zp)k_v\left(z/p\right)dz\right\}x^{\frac{k+1}{2}}f_2(x)dx$$

Now evaluating the inner integral by ([1], p.146), we get

$$=\frac{1}{\pi}f(k)\int_0^\infty \pi p^{-\frac{1}{2}}k_{2v}(2\sqrt{(xp)}\,x^{\frac{k+1}{2}}f_2(x)dx$$

Or

$$=g(y)=f(k)\int_0^\infty y^{-\frac{1}{2}}k_{2v}(2\sqrt{(ty)}\,t^{\frac{k+1}{2}}f_2(t)dt \quad (2.8)$$

Writing $y=y^2$ and $t=\frac{t^2}{4}$, we obtain from (2.8)

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$$4y^{\frac{3}{2}}g(y^2)=f(k)M^{2v}\left\{x^{k+\frac{3}{2}}f_2\left(\frac{x^2}{4}\right)\right\}$$

Proceeding successively we assume the result (2.7).
Also let

$$\pi y^{-\frac{3}{2}}f_n\left(\frac{1}{4y^2}\right)=\int_0^\infty f_{n+1}(x)S_n^m(\sqrt{x})k_{2^{n-1}v}(xy)\sqrt{(xy)}dx \tag{2.9}$$

Substituting the expression for $f_n\left(\frac{x^2}{4}\right)$ from (2.9) in (2.7), interchanging the order of integration, using the series representation of general class of polynomial and evaluating the later integral by ([1], p.146), we obtain

$$y^{\frac{3}{2}(2^{n-1}-1)}g(y^{2^{n-1}})=\frac{1}{\sqrt{y}}f(k)\int_0^\infty t^{k+\frac{1}{2}}f_{n+1}(t)k_{2^n v}(ty)\sqrt{(ty)}dt \tag{2.10}$$

Writing $y=y^2$ and $t=\frac{t^2}{4}$, we obtain from (2.10)

$$y^{\frac{3}{2}(2^n-1)}g(y^{2^n})=f(k)\int_0^\infty t^{k+\frac{3}{2}}f_{n+1}\left(\frac{t^2}{4}\right)k_{2^n v}(ty)\sqrt{(ty)}\,dt$$

i.e. $f(k)M^{2v}\left\{x^{k+\frac{3}{2}}f_{n+1}\left(\frac{x^2}{4}\right)\right\}=y^{\frac{3}{2}(2^n-1)}g(y^{2^n})$.

We thus find that if (2.7) is true for $n=2$, it is also true for $(n+1)$ i.e. for the next higher order. But we have seen that it is true for $n=2$ and so it is true for $n=3$ and so on. Hence (2.7) is true for all positive integral values of n except 1.

Theorem 2. If

$$M^v\left[f_1(x)\right]=g(y) \tag{2.11}$$

$$M^v\left[S_n^m(\sqrt{x})f_2(x)\right]=\pi y^{-2}f_1\left(\frac{1}{y}\right) \tag{2.12}$$

Then

$$f(k)M^{2v}\left\{x^{-\frac{1}{2}-k}f_2\left(\frac{x^2}{4}\right)\right\}=y^{-\frac{1}{2}}g(y^2) \tag{2.13}$$

Provided $x^{\left(\pm v\pm k\pm\frac{1}{2}\right)}f_2(x)$ are bounded and absolutely integrable in $(0,\infty)$ and $A_{n,k}\,(n,k\geq 0)$ are constant real or complex.

Further if

$$M^{2v}\left[S_n^m(\sqrt{x})f_3(x)\right]=\pi y^{-\frac{3}{2}}f_2\left(\frac{1}{4y^2}\right) \tag{2.14}$$

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$$M^{2v}\left[S_n^m(\sqrt{x})f_4(x)\right]=\pi y^{-\frac{3}{2}}f_3\left(\frac{1}{4y^2}\right) \quad (2.15)$$

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$$f(k)M^{2^{n-1}v}\left[f_n(x)\right]=\pi y^{-\frac{3}{2}}f_{n-1}\left(\frac{1}{4y^2}\right) \quad (2.16)$$

Then

$$f(k)M^{2^{n-1}v}\left\{x^{-\frac{1}{2}}f_n\left(\frac{x^2}{4}\right)\right\}=y^{-\left(2^{n-1}-\frac{1}{2}\right)}g(y^{2^{n-1}}) \quad (2.17)$$

Provided $x^{\left(\pm 2^{n-2}v\pm k\pm\frac{1}{2}\right)}f_n(x), n=2,3,\ldots,n$, are bounded and absolutely integrable in $(0,\infty)$ and $A_{n,k}(n,k\geq 0)$ are real or complex.

Proof: In proving this theorem, we make use of the well known result ([1], p.146)

$$\int_0^\infty x^{-\frac{5}{2}}k_v\left(\frac{a}{x}\right)k_v(xy)\sqrt{xy}\,dx=\frac{\pi}{a}k_{2v}(2\sqrt{ay})$$

$\operatorname{Re}(a)>0, \operatorname{Re}(y)>0$.

Proof of the theorem is omitted, as being similar to that of theorem 1.

III. SPECIAL CASES

Let

$$f_1(x)=\sqrt{\pi}\,2^{-v}a^{(2v-1)}x^{2v}J_{v-\frac{1}{2}}\left(\frac{a^2x}{2}\right)S_n^m(\sqrt{x})$$

Then making use of result ([1], p. 137), we obtain from (2.1)

$$g(y)=f(k)\frac{\sqrt{\pi}a^{(4v-2)}}{y^{\left(3v+k+\frac{1}{2}\right)}}\Gamma\left(2v+k+\frac{1}{2}\right)\left(1+\frac{a^2}{4y^2}\right)^{-2v-k-\frac{1}{2}}$$

$$\operatorname{Re}(v)>-\frac{1}{4}, \operatorname{Re}(y)>\left|\operatorname{Im}\frac{a^2}{4}\right|.$$

From (2.2) and ([1], p. 148), we obtain

$$f_2(k)=\frac{x^{\left(v-k-\frac{1}{2}\right)}}{\pi}f(k)I_{2v}(a\sqrt{x})J_{2v-1}(a\sqrt{x})$$

$\operatorname{Re}(v)>0, \operatorname{Re}(y)>0.$

Taking $n=2$, we obtain from (2.7)

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$$f(k)M\left\{\frac{x^{(2v+k+\frac{1}{2})}}{2^{(2v-1)}\pi}I_{2v-1}J_{2v-1}(ax/2)\right\}$$

$$=f(k)\frac{4\sqrt{\pi}a^{(4v-2)}}{y^{6v-k-\frac{1}{2}}}\Gamma\left(2v+k+\frac{1}{2}\right)\left(1+\frac{a^4}{4y^4}\right)^{-2v-\frac{1}{2}}$$

$\mathrm{Re}(v)>0, \mathrm{Re}(y)>\mathrm{Re}(a/2)$.

4. APPLICATION

Let

$$f_1(x)=\sqrt{\pi}\,S_n^m(\sqrt{x})2^{-(v-k-\frac{1}{2})}a^{(2v+1)}x^{(2v+2)}J_{v-\frac{1}{2}}(\frac{a^2x}{2}),$$

Then making use of the result ([1], p. 137), we obtain

$$g(y)=\frac{2\sqrt{\pi}\,a^{4v}}{\Gamma\left(\frac{1}{2}+v+\frac{k}{2}\right)}f(k)y^{-\left(3v+\frac{3}{2}\right)}\Gamma\left(2v+\frac{3}{2}+k\right)\Gamma\left(v+\frac{3}{2}+\frac{k}{2}\right)$$

$${}_2F_1\left(2v+\frac{3}{2}+k,v+\frac{3}{2}+\frac{k}{2};v+\frac{1}{2}+\frac{k}{2};-\frac{a^2}{4}\right)\ \mathrm{Re}(v)>-\frac{3}{4},\mathrm{Re}(y)>\left|\mathrm{Im}\frac{a^2}{2}\right|.$$

From (2.2) and ([1], p. 148), we obtain

$$f_2(x)=\frac{x^{\left(v+\frac{1}{2}+\frac{k}{2}\right)}}{\pi}f(k)I_{2v}(a\sqrt{x})J_{2v}(a\sqrt{x})$$

$$\mathrm{Re}(v)>-\frac{1}{2},\mathrm{Re}(y)>0.$$

Taking $n=2$, we obtain from (2.7)

$$f(k)M^{2v}\left[\frac{x^{\left(2v+\frac{5}{2}+\frac{k}{2}\right)}}{2^{(2v+1)}\pi}I_{2v}\left(\frac{ax}{2}\right)J_{2v}\left(\frac{ax}{2}\right)\right]=\frac{8\sqrt{\pi}a^{4v}}{\Gamma\left(\frac{1}{2}+v+\frac{k}{2}\right)}y^{-\left(6v+\frac{7}{2}\right)}$$

$$\Gamma\left(2v+k+\frac{3}{2}\right)\Gamma\left(v+\frac{k}{2}+\frac{3}{2}\right){}_2F_1\left(2v+\frac{3}{2}+k,v+\frac{3}{2}+\frac{k}{2};v+\frac{k}{2}+\frac{1}{2};-\frac{a^2}{4y^2}\right)$$

$$\mathrm{Re}(v)>-\frac{1}{2},\mathrm{Re}(y)>\mathrm{Re}\left(\frac{a}{2}\right).$$

Example 3. Let

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$$f_1(x)=\sqrt{\pi}\,S_n^m(\sqrt{x})2^{-v}a^{(2v-1)}x^{(2v-2)}J_{v-\frac{1}{2}}(\frac{a^2x}{2})$$

Then making use of the result ([1], p. 137), we obtain from (2.11)

$$g(y)=\frac{\sqrt{\pi}\,a^{(4v-2)}}{4\Gamma\left(\frac{1}{2}+v+\frac{k}{2}\right)}f(k)y^{-\left(3v-\frac{3}{2}\right)}\Gamma\left(2v-\frac{1}{2}+k\right)\Gamma\left(v-\frac{1}{2}+\frac{k}{2}\right)$$

$${}_2F_1\left(2v-\frac{1}{2}+k,v-\frac{1}{2}+\frac{k}{2};v+\frac{1}{2}+\frac{k}{2};-\frac{a^2}{4y^2}\right),$$

$$\mathrm{Re}(v)>-\frac{1}{4},\mathrm{Re}(y)>\left|\mathrm{Im}\frac{a^2}{2}\right|.$$

From (2.12) and ([1], p 148), we obtain

$$f_2(x)=\frac{x^{\left(v-\frac{1}{2}+\frac{k}{2}\right)}}{\pi}f(k)I_{2v-1}(a\sqrt{x})J_{2v-1}(a\sqrt{x})$$

$$\mathrm{Re}(v)>0,\mathrm{Re}(y)>0.$$

Taking $n=3$ we obtain from (2.17)

$$f(k)M^{2v}\left[\frac{x^{\left(2v-\frac{3}{2}+\frac{k}{2}\right)}}{2^{(2v-1)}\pi}I_{2v-1}\left(\frac{ax}{2}\right)J_{2v-1}\left(\frac{ax}{2}\right)\right]=\frac{8\sqrt{\pi}a^{(4v-2)}}{4\Gamma\left(\frac{1}{2}+v+\frac{k}{2}\right)}y^{-\left(6v-\frac{5}{2}\right)}$$

$$\Gamma\left(2v+k-\frac{1}{2}\right)\Gamma\left(v+\frac{k}{2}-\frac{1}{2}\right){}_2F_1\left(2v-\frac{1}{2}+k,v-\frac{1}{2}+\frac{k}{2};v+\frac{k}{2}+\frac{1}{2};-\frac{a^2}{4y^2}\right)$$

$$\mathrm{Re}(v)>\frac{1}{2},\mathrm{Re}(y)>\mathrm{Re}\left(\frac{a}{2}\right).$$

V. CONCLUSION

In this paper we study of certain integral K-transform by using chain of this transform. Some special cases and application of the results are also giving.

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