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A Study on Disctrete Model of Three Species Syn-Eco-System with Unlimited Resources for the First and Second Species

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ABSTRACT: The system comprises of a commensal (S_1) common to two hosts S_2 and S_3. S_2 is a commensal of S_3 and S_3 is a host of both S_1, S_2. Further the first and second species have unlimited resources while the third has limited resources. The basic equations for this model constitute as three first order non-linear coupled ordinary difference equations. All possible equilibrium points are identified based on the model equations at two stages and criteria for their stability are discussed. Further the numerical solutions are computed for specific values of the various parameters and the initial conditions.

AMS Classification : 92D25, 92D40

Keywords: Commensal, equilibrium state, host, stable, oscillatory.

I. INTRODUCTION

Ecology is the study of the interactions between organisms and their environment. The organisms include animals and plants, the environment includes the surroundings of animals. So ecology relates to the study of living beings (animals and plants) in relation to their habits and habitats. This discipline of knowledge is a branch of evolutionary biology purported to explain how or to what extent the living beings are regulated in nature. Allied to the problem of population regulation is the problem of species distribution- prey-predator, competition, commensalism and so on.

Mathematical modeling has been playing an important role for the last half a century in explaining several phenomena concerned with individuals and groups of populations in nature. Significant research in the field of theoretical ecology has been formulated by Lotka[13] and by Volterra [17]. Since then, several mathematicians and ecologists contributed to the growth of this area of knowledge.

The general concept of modeling has been presented in the treatises of Freedman[6], Braun [5], Paul Colinvaux[14] and Kapur[11]. Srinivas[16] studied competitive ecosystem of two species and three species with limited and unlimited resources. Later, Lakshminarayan [12] studied prey-predator ecological model with partial cover for the prey and alternate food for the predator. Stability analysis of competitive species was carried out by Archana Reddy, Pattabhi Ramacharyulu and Krishna Gandhi [3] and by Bhaskara Rama Sarma and Pattabhi Ramacharyulu [4]. Acharyulu KVLN and Pattabhi Ramacharyulu[1], [2] obtained fruitful results on some Mathematical models of ecological Ammensalism. Phani Kumar [15] studied some mathematical models of ecological commensalism. The present author Hari Prasad [7] - [10] et al, discussed three and four species ecological models.

The present investigation is a study on discrete model of three species (S_1, S_2, S_3) syn-eco system. The system comprises of a commensal (S_1), two hosts S_2 and S_3 . **Commensalism** is a symbiotic interaction between two or more populations which live together, and in which only one of the populations (commensalism) is benefited while the

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other(host) is not effected, for example the clownfish shelters among the tentacles of the sea anemone, while the sea anemone is not affected..

II. BASIC EQUATIONS OF THE MODEL

A. NOTATION ADOPTED

S_1 : Commensal of S_2 and S_3

S_2 : Host of S_1 and commensal of S_3

S_3 : Host of S_1 and S_2

$N_i(t)$: The population strength of S_i at time t, i = 1,2,3.

t : Time instant.

a_i : Natural growth rate of S_i , i = 1,2, 3.

a_{33} : Self inhibition coefficient of S_3.

a_{12}, a_{13} : Interaction coefficients of S_1 due to S_2 and S_1 due to S_3.

a_{23} : Interaction coefficient of S_2 due to S_3

Further the variables N_1, N_2, N_3 are non-negative and the model parameters a_1, a_2, a_3, a_{33}, a_{12} , a_{13}, a_{23} are assumed to be non-negative constants.

B. BASIC EQUATIONS

Consider the growth of the species during the time interval (t, t + 1).

Equation for the first species (N_1):

$$N_1(t+1) = N_1(t) + a_1 N_1(t) + a_{12} N_1(t)\, N_2(t) + a_{13} N_1(t)\, N_3(t) \tag{1}$$

Equation for the second species (N_2):

$$N_2(t+1) = N_2(t) + a_2 N_2(t) + a_{23} N_2(t) N_3(t) \tag{2}$$

Equation for the third species (N_3):

$$N_3(t+1) = N_3(t) + a_3 N_3(t) - a_{33} N_3^2(t) \tag{3}$$

Species-Growth equations in the discrete form:

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The discrete form of the three species syn ecological model is

$$N_1(t+1) = \alpha_1 N_1(t) + a_{12} N_1(t)\, N_2(t) + a_{13} N_1(t)\, N_3(t) \tag{4}$$

$$N_2(t+1) = \alpha_2 N_2(t) + a_{23} N_2(t) N_3(t) \tag{5}$$

$$N_3(t+1) = \alpha_3 N_3(t) - a_{33} N_3^2(t) \tag{6}$$

where $\alpha_i = a_i + 1$, $i = 1,2,3$ (7)

III. EQUILIBRIUM STATES

For a continuous model the equilibrium states are defined by $\frac{dN_i}{dt} = 0$, $i = 1,2,3$, the equilibrium states for a discrete model are defined in terms of the period of no growth.

i.e, $N_i(t + r) = N_i(t)$, $r = 1,2,3,\ldots\ldots\ldots$, where r is the period of the equilibrium state.

A. ONE PERIOD EQUILIBRIUM STATES (STAGE-I)

$$N_i(t + 1) = N_i(t),\ i = 1,2,3 \tag{8}$$

$$\text{i.e,}\quad \left.\begin{array}{l} N_1(t+1) = \alpha_1 N_1(t) + a_{12} N_1(t)\, N_2(t) + a_{12} N_1(t)\, N_2(t) \\ N_2(t+1) = \alpha_2 N_2(t) + a_{23} N_2(t)\, N_3(t) \\ N_3(t+1) = \alpha_3 N_3(t) - a_{33} N_3^2(t) \\ \text{But } N_1(t+1) = N_1(t), N_2(t+1) = N_2(t), N_3(t+1) = N_3(t) \end{array}\right\} \tag{9}$$

The system under investigation has **two** equilibrium states given by

(i) Fully washed out state

$$E_1 : \overline{N}_1 = 0,\ \overline{N}_2 = 0,\ \overline{N}_3 = 0$$

(ii) The state in which only the third species survives

$$E_2 : \overline{N}_1 = 0,\ \overline{N}_2 = 0,\ \overline{N}_3 = \frac{\alpha_3 - 1}{a_{33}}, \text{ when } \alpha_3 > 1$$

B. STABILITY OF EQUILIBRIUM STATES

Stability of $E_1(0,0,0)$:

$N_1(t) = N_1(t+1) = N_1(t+2) = \ldots\ldots\ldots\ldots\ldots\ldots\ldots = 0$

$N_2(t) = N_2(t+1) = N_2(t+2) = \ldots\ldots\ldots\ldots\ldots\ldots\ldots = 0$

$N_3(t) = N_3(t+1) = N_3 t+2) = \ldots\ldots\ldots\ldots\ldots\ldots\ldots = 0$

i.e. $N_i(t + r)=0$, where r is an integer and $i = 1,2,3$

Hence, $E_1(0,0,0)$ is **stable**.

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Stability of E_2:

$N_1(t) = N_1(t+1) = N_1(t+2) = \ldots\ldots\ldots\ldots = 0$

$N_2(t) = N_2(t+1) = N_2(t+2) = \ldots\ldots\ldots\ldots = 0$

$N_3(t) = N_3(t+1) = N_3(t+2) = \ldots\ldots\ldots\ldots = \dfrac{\alpha_3 - 1}{a_{33}}$

i.e. $N_i(t+r) = 0$, $N_3(t+r) = \dfrac{\alpha_3 - 1}{a_{33}}$, where r is an integer and $i = 1,2$

Hence, E_2 is **stable**.

At this stage all the two equilibrium states E_1, E_2 are **stable**.

C. TWO PERIOD EQUILIBRIUM STATES (STAGE-II)

$$N_i(t+2) = N_i(t),\ i = 1,2,3 \qquad (10)$$

$$\text{i.e,}\quad \left.\begin{aligned} &N_1(t+2) = \alpha_1 N_1(t+1) + a_{12}N_1(t+1)N_2(t+1) + a_{13}N_1(t+1)N_3(t+1) \\ &N_2(t+2) = \alpha_2 N_2(t+1) + a_{23}N_2(t+1)N_3(t+1) \\ &N_3(t+2) = \alpha_3 N_3(t+1) - a_{33}N_3^2(t+1) \\ &\text{But } N_1(t+2) = N_1(t),\ N_2(t+2) = N_2(t) \text{ and } N_3(t+2) = N_3(t) \end{aligned}\right\} \qquad (11)$$

The system under investigation has **five** equilibrium states given by

(i) Fully washed out state

$E_1 : \overline{N_1} = 0,\ \overline{N_2} = 0,\ \overline{N_3} = 0$.

(ii) States in which only the third species survives

$E_2 : \overline{N_1} = 0,\ \overline{N_2} = 0,\ \overline{N_3} = \dfrac{\alpha_3 - 1}{a_{33}}$, when $\alpha_3 > 1$

$E_3 : \overline{N_1} = 0, \overline{N_2} = 0, \overline{N_3} = \dfrac{(\alpha_3 + 1) + \sqrt{(\alpha_3 + 1)(\alpha_3 - 3)}}{2a_{33}}$, when $\alpha_3 > 3$

$E_4 : \overline{N_1} = 0, \overline{N_2} = 0, \overline{N_3} = \dfrac{(\alpha_3 + 1) - \sqrt{(\alpha_3 + 1)(\alpha_3 - 3)}}{2a_{33}}$, when $\alpha_3 > 3$

$E_5 : \overline{N_1} = 0, \overline{N_2} = 0,\ \overline{N_3} = \dfrac{2}{a_{33}}$, when $\alpha_3 = 3$

The states E_3 and E_4 coincide when $\alpha_3 = 3$ and do not exist when $\alpha_3 < 3$.

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D. STABILITY OF EQUILIBRIUM STATES

The equilibrium states E_1 and E_2 are **stable** as established in **III-B.** Now we will discuss the stability of other equilibrium points except these two points in this stage.

Stability of E_3:

$$N_1(t) = N_1(t+1) = N_1(t+2) = \ldots\ldots\ldots = 0$$
$$N_2(t) = N_2(t+1) = N_2(t+2) = \ldots\ldots\ldots = 0$$

i.e. $N_i(t + r) = 0$, where r is an integer and i = 1,2

$$N_3(t) = N_3(t+2) = N_3(t+4) = \ldots\ldots\ldots = \frac{(\alpha_3+1)+\sqrt{(\alpha_3+1)(\alpha_3-3)}}{2a_{33}}$$

$$N_3(t+1) = N_3(t+3) = N_3(t+5) = \ldots\ldots\ldots = \frac{(\alpha_3+1)-\sqrt{(\alpha_3+1)(\alpha_3-3)}}{2a_{33}}$$

i.e, $N_3(t+2r) = \frac{(\alpha_3+1)+\sqrt{(\alpha_3+1)(\alpha_3-3)}}{2a_{33}}$ and $N_3(t+\overline{2r+1}) = \frac{(\alpha_3+1)-\sqrt{(\alpha_3+1)(\alpha_3-3)}}{2a_{33}}$

where r is an integer.

Hence, E_3 **oscillates** between $\frac{(\alpha_3+1)+\sqrt{(\alpha_3+1)(\alpha_3-3)}}{2a_{33}}$ and $\frac{(\alpha_3+1)-\sqrt{(\alpha_3+1)(\alpha_3-3)}}{2a_{33}}$, when $\alpha_3 > 3$

and is stable when $\alpha_3 = 3$.

Stability of E_4:

$$N_1(t) = N_1(t+1) = N_1(t+2) = \ldots\ldots\ldots = 0$$
$$N_2(t) = N_2(t+1) = N_2(t+2) = \ldots\ldots\ldots = 0$$

i.e. $N_i(t + r) = 0$, where r is an integer and i = 1,2

$$N_3(t) = N_3(t+2) = N_3(t+4) = \ldots\ldots\ldots = \frac{(\alpha_3+1)-\sqrt{(\alpha_3+1)(\alpha_3-3)}}{2a_{33}}$$

$$N_3(t+1) = N_3(t+3) = N_3(t+5) = \ldots\ldots\ldots = \frac{(\alpha_3+1)+\sqrt{(\alpha_3+1)(\alpha_3-3)}}{2a_{33}}$$

i.e, $N_3(t+2r) = \frac{(\alpha_3+1)-\sqrt{(\alpha_3+1)(\alpha_3-3)}}{2a_{33}}$ and $N_3(t+\overline{2r+1}) = \frac{(\alpha_3+1)+\sqrt{(\alpha_3+1)(\alpha_3-3)}}{2a_{33}}$

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where r is an integer.

Hence, E_4 **oscillates** between $\frac{(\alpha_3+1)-\sqrt{(\alpha_3+1)(\alpha_3-3)}}{2a_{33}}$ and $\frac{(\alpha_3+1)+\sqrt{(\alpha_3+1)(\alpha_3-3)}}{2a_{33}}$, when $\alpha_3 > 3$

and is stable when $\alpha_3 = 3$.

Stability of E_5:

$N_1(t) = N_1(t+1) = N_1(t+2) = \ldots\ldots = 0$

$N_2(t) = N_2(t+1) = N_2(t+2) = \ldots\ldots = 0$

$N_3(t) = N_3(t+1) = N_3(t+2) = \ldots\ldots = \frac{2}{a_{33}}$

i.e, $N_i(t+r) = 0$, $N_3(t+r) = \frac{2}{a_{33}}$, where r is an integer and $i = 1,2$

Hence, E_5 is **stable**.

At this stage, in all five equilibrium states, only the three equilibrium states E_1, E_2, E_5 are **stable** and other two E_3, E_4 are **oscillatory**.

IV. NUMERICAL EXAMPLES

The numerical solutions of the discrete model equations (4), (5) and (6) computed for specific values of the various parameters and the initial conditions. For this MS EXCEL has been used and the results are illustrated in Figures 1 to 5.

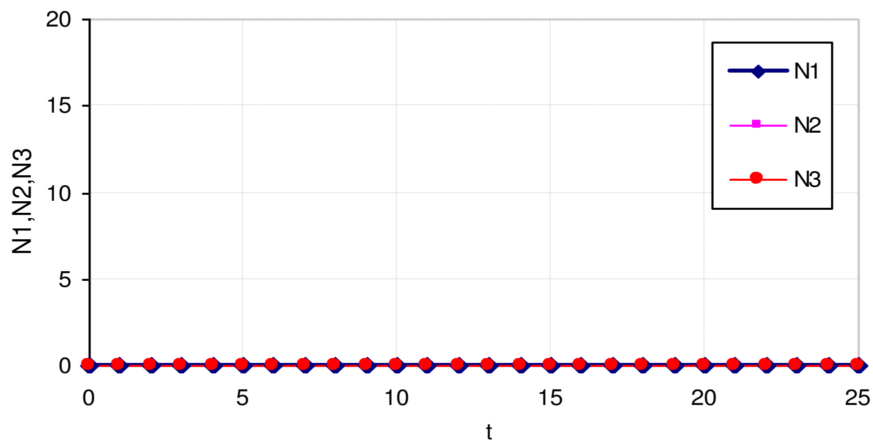

Figure 1: Variation of N_1, N_2 and N_3 against time(t) for
$\alpha_1 = 2.56$, $a_{12} = 0.646$, $a_{13} = 0.452$, $\alpha_2 = 2.92$, $a_{23} = 1.127$, $\alpha_3 = 3.4$, $a_{33} = 0.228$, $N_1(0) = 0$, $N_2(0) = 0$, $N_3(0) = 0$

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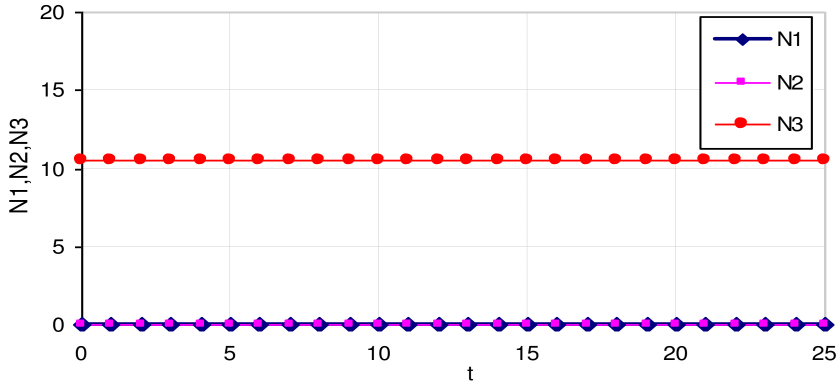

Figure 2: Variation of N_1, N_2 and N_3 against time(t) for
$\alpha_1 = 2.56$, $a_{12} = 0.646$, $a_{13} = 0.452$, $\alpha_2 = 2.92$, $a_{23} = 1.127$, $\alpha_3 = 3.4$, $a_{33} = 0.228$, $N_1(0) = 0$, $N_2(0) = 0$, $N_3(0) = 10.526$

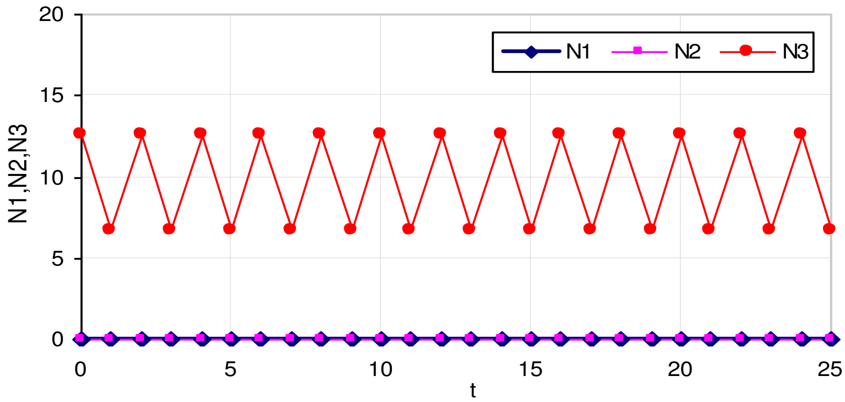

Figure 3: Variation of N_1, N_2 and N_3 against time(t) for
$\alpha_1 = 2.56$, $a_{12} = 0.646$, $a_{13} = 0.452$, $\alpha_2 = 2.92$, $a_{23} = 1.127$, $\alpha_3 = 3.4$, $a_{33} = 0.228$, $N_1(0) = 0$, $N_2(0) = 0$, $N_3(0) = 12.57$

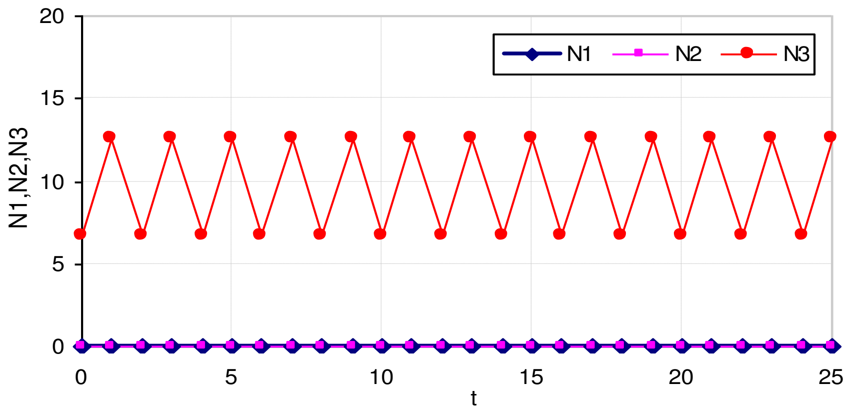

Figure 4: Variation of N_1, N_2 and N_3 against time(t) for
$\alpha_1 = 2.56$, $a_{12} = 0.646$, $a_{13} = 0.452$, $\alpha_2 = 2.92$, $a_{23} = 1.127$, $\alpha_3 = 3.4$, $a_{33} = 0.228$, $N_1(0) = 0$, $N_2(0) = 0$, $N_3(0) = 6.73$

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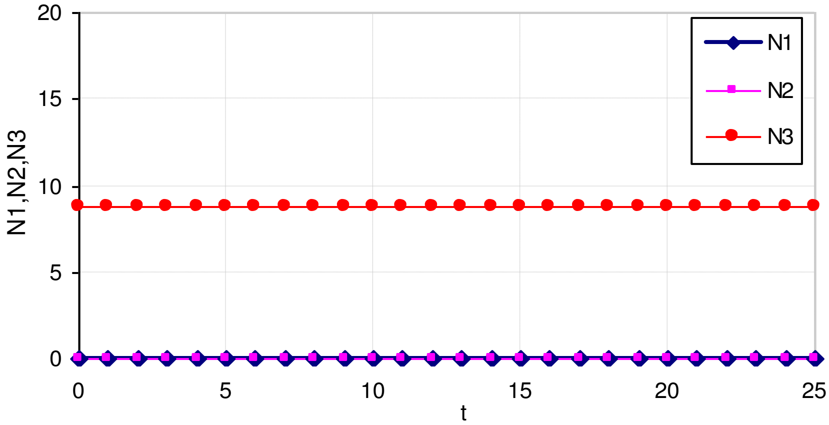

Figure 5: Variation of N_1, N_2 and N_3 against time(t) for
$\alpha_1 = 2.56$, $a_{12} = 0.646$, $a_{13} = 0.452$, $\alpha_2 = 2.92$, $a_{23} = 1.127$, $\alpha_3 = 3$, $a_{33} = 0.228$, $N_1(0) = 0$, $N_2(0) = 0$, $N_3(0) = 8.772$

V. CONCLUSION

The present paper deals with an investigation on a discrete model of three species syn eco-system. The system comprises of a commensal (S_1), two hosts S_2 and S_3 ie., S_2 and S_3 both benefit S_1, without getting themselves effected either positively or adversely. Further S_2 is a commensal of S_3, S_3 is a host of both S_1, S_2 and the first and second species have unlimited resources. All possible equilibrium points of the model are identified based on the model equations at two stages.

$$\text{Stage-I}: \; N_i(t+1) = N_i(t) \;\; ; i = 1, 2$$
$$\text{Stage-II}: \; N_i(t+2) = N_i(t) \;\; ; i = 1, 2$$

In Stage-I there are only two equilibrium points, while the Stage-II there would be five equilibrium points. All the two equilibrium points in Stage-I are found to be **stable** while in stage-II only three are **stable.** Further the numerical solutions for the discrete model equations are computed.

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