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An Efficient Algorithm for Power Load Flow Solutions by Schur Complement and Threshold Technique

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ABSTRACT: This paper deals with a algorithm developed for load flow problems in electrical power systems. Jacobian matrix in Newton Raphson method require large computer memory and take more computation time in calculations. Schur complement method can be reduce memory required by divided Jacobian matrix into two separated matrices with reasonable computation time. The proposed algorithm used this method with threshold technique in order to test different power systems. Results has been compared with Newton Raphson method and Fast Decoupled method in term of influence of convergence properties and algorithm efficiency.

KEYWORDS: Power system network, Load flow, Element incidence matrix, Schur complement method, Software Development.

I.INTRODUCTION

Power flow analysis is necessary for planning, operation, economic scheduling and exchange of power between utilities. Power flow analysis is required for many other analyses such as transient stability, optimal power flow and contingency studies. The principal information of power flow analysis is to find the magnitude and phase angle of voltage at each bus and the real and reactive power flowing in each transmission lines. Power flow problem could be viewed as a multi-input multi-output system. Each input could have effect on each of the outputs, more or less. Thus it makes it more difficult to appreciate the correlation between these variables.

In view of the topological specialty of transmission and distribution networks, many researchers has proposed several special load flow techniques [1-4]. However, an acceptable load flow method should meet the requirements such as high speed and low storage requirements, highly reliable, and accepted versatility and simplicity. The starting of studies for the load flow calculations was begin with the Ward & Hale [1] , while the Newton-Raphson (N-R) method [2], [3] are being widely used. The problem of the computation iteration time has been solved by the developments of the Fast Decoupled Load Flow [4], then Fast Load Flow Method Retaining Nonlinearity [5], and so on. A large variety of proposed solution for power flow problem addressing with different targets like reduce computation time [6-9], ill-conditioned cases and robustness [10–12], and optimal power flow (OPF) problems [10-15].

Based on matrix theory, this paper used Schur complement which shows that the indices obtained from reduced Jacobian matrix and 2 * 2 matrix are also independent of bus number order which has better properties to find solution and mostly convergence smoothly. Reduce effect of reactive power values on correlation of voltage angle variables has been considered by filtering low effect values with corresponding to other high value. Consequently, less memory and reasonable computation time required to optimize the results. It should be pointed out that modification proposed in this paper is compared to the Newton Raphson method and Fast Decoupled method to got reasonable accuracy, competition times and less computer memory respectively.

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II. LOAD FLOW FOR POWER SYSTEM

This method requires the construction of admittance (n x n) matrix, where n is number of bus system. The diagonal elements of the admittance matrix represent the self-admittance of the bus and the off diagonal represent the mutual admittance between buses.

$$Y=\begin{bmatrix} Y_{11} & \cdots & Y_{jk} \\ \vdots & \ddots & \vdots \\ Y_{kj} & \cdots & Y_{kk} \end{bmatrix} \quad \text{……………………… (1)}$$

The real and reactive power calculated at specific bus using initial guess and specified voltage magnitude and angle. The iteration methods were used to solve this non linear equations like (Gauss Sidle and Newton Raphson). Newton Raphson was more efficient and robust and basically it calculated powers as follows[16]:

$$P_k^{calc\,(x)} = \sum_{j=1}^{n} |V_k||V_j||Y_{kj}|\cos(\Theta_{kj} - \delta_k + \delta_j) \quad \text{……………………… (2)}$$

$$Q_k^{calc\,(x)} = -\sum_{j=1}^{n} |V_k||V_j||Y_{kj}|\sin(\Theta_{kj} - \delta_k + \delta_j) \quad \text{……………………… (3)}$$

Finalized the iteration will depend on tolerance of the power mismatch which calculated by their formulae :

$$\Delta P_k^{(x)} = P_k^{sch} - P_k^{calc\,(x)} \quad \text{……………………… (4)}$$

$$\Delta Q_k^{(x)} = Q_k^{sch} - Q_k^{calc\,(x)} \quad \text{………………………(5)}$$

The Jacobian matrix of this method can be represented by:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix} \quad \text{………………………(6)}$$

The elements of the Jacobian matrix are partial derivative values of either P or Q with respect to either |V| or δ.

$$J = \begin{bmatrix} \frac{\partial P}{\partial \delta} & \frac{\partial P}{\partial |V|} \\ \frac{\partial Q}{\partial \delta} & \frac{\partial Q}{\partial |V|} \end{bmatrix} \quad \text{………………………(7)}$$

Typically, the Jacobian matrix is inversed and added to the left side of the equation, the final form looks like the following:

$$\begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix}^{-1} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \quad \text{………………………(8)}$$

All unknown voltage magnitudes and angles would initially require a guess would be 1 and 0 respectively. As far as concern, Buses and parameters classified as :

Bus	Known Parameter	Unknown Parameter
Slack or Swing Bus	|V| and δ	P and Q
Generator or PV or Regulated bus	P and |V|	Q and δ
Load or PQ Bus	P and Q	|V| and δ

Table 1: Classification of Buses in Electrical Network

If iteration continue then the new voltage magnitude and angle can be updated with the following equations:

Voltage angle mismatch $\delta_k^{(x+1)} = \delta_k^{(x)} + \Delta\, \delta_k^{(x)}$ ……………………(9)

Voltage magnitude mismatch $|V_k^{(x+1)}| = |V_k^{(x)}| + \Delta\, |V_k^{(x)}$ …………………..(10)

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III. THE SCHUR COMPLEMENT

In linear algebra and the theory of matrices, The sub-domain problems usually involve interior, local interface and external interface variables. The Schur complement technique is a procedure to eliminate the interior variables in each sub-domain and derive a global, reduced in size, linear system involving only the interface variables [17]. Suppose that the square matrix (M) dimensioned (r+s)×(r+s), is partitioned into four sub matrix blocks as A, B, C and D respectively r×r, r×s, s×r and s×s matrices, and D is invertible. Let

$$M = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

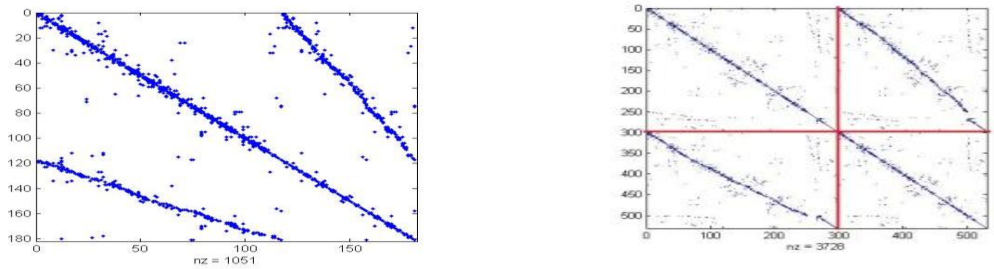
A and D are square matrices, but B and C are not square unless r = s. so that inverse of M is a (r+s)×(r+s) matrix:

$$\begin{pmatrix} A_{r*r} & B_{r*s} \\ C_{s*r} & D_{s*s} \end{pmatrix}^{-1} = \begin{pmatrix} (A - BD^{-1}C)^{-1} & -(A - BD^{-1}C)^{-1}BD^{-1} \\ -(D - CA^{-1}B)^{-1}CA^{-1} & (D - CA^{-1}B)^{-1} \end{pmatrix}$$

Then the Schur complement of the block D or A of the matrix M is the r×s matrix:

$$A - BD^{-1}C \quad \text{or } D - CA^{-1}B$$

Because the data dependencies are not regular as in Fig.(1), Schur method can be used to separate the unknown variables in interface unknown variables and sub-domain internal unknown variables [18]. That mean, no need for large memory when inverse matrix which will be more powerful in iteration methods by separate their variables. In practice, one needs D to be well-conditioned in order for this algorithm to be numerically accurate.

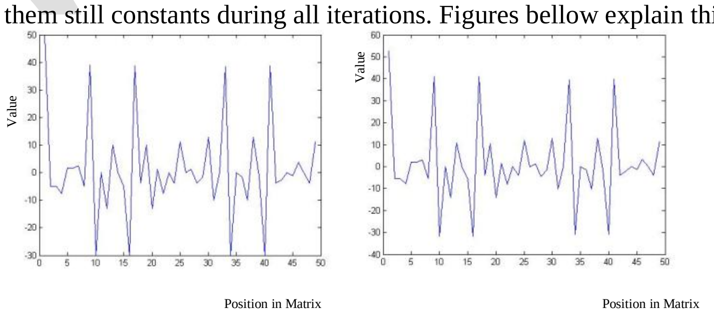


Jacobian for 118 Bus Jacobian for 300 Bus

Fig. (1) Jacobian Matrix for different systems

IV. RESULT AND ALGORITHM

After study and analysis Newton Raphson method to understanding the behaviour of Jacobian Matrix during iterations for different systems (5, 14 and 30 bus), it was noted that values of few elements will slightly changed at beginning of iterations while mostly of them still constants during all iterations. Figures bellow explain this fact:



Jacobin at 1st iteration Jacobin at 10th iteration

Fig. (2) Jacobin Matrix at beginning and end of iterations for 5 Bus systems

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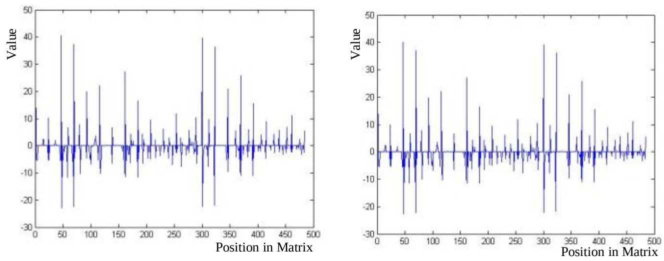
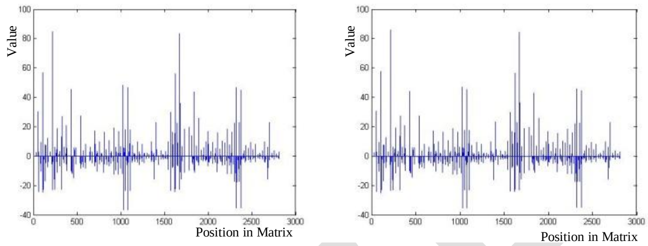


Jacobin at 1st iteration   Jacobin at 10th iteration

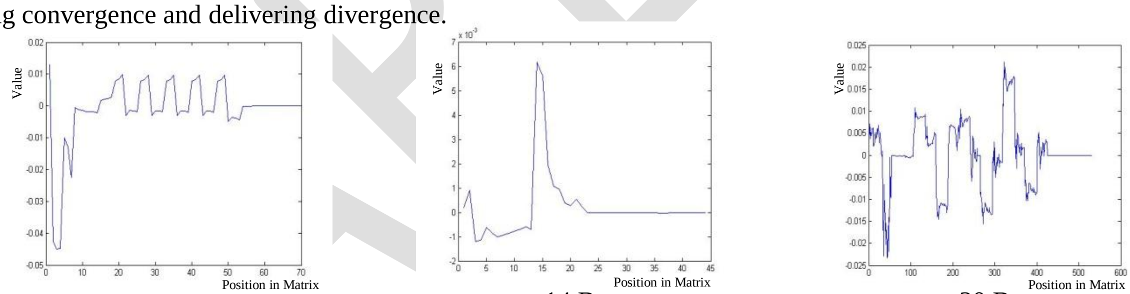
Fig. (3) Jacobian Matrix at beginning and end of iterations for 14 Bus systems



Jacobin at 1st iteration   Jacobin at 10th iteration

Fig. (4) Jacobian Matrix at beginning and end of iterations for 30 Bus systems

That mean convergence and divergence will followed the main element values of Jacobian (derivative values) and it is not changed at each iterations. Problems concerning the convergence or divergence tend to be difficult because there are many of roots for each case. Most of ill conditional cases appear to surf ace when system has low R and X line values which effect on solution and it will be fluctuated (Up – Down) near point of root as in fig.(5). These factors will constraining convergence and delivering divergence.



5 Bus   14 Bus   30 Bus

Fig.(5). Newton Raphson convergence for 5, 14 and 30 Bus

The Schur complement (SC) method can be eliminate or reduce the off-diagonals effects, when it has high values, of Jacobian matrix which make it close to be singular. This matrix will divided to 4 sub-matrices. That mean, it will have separated equations for voltage and voltage angel as follows:

$$\Delta\delta = X_1(\Delta P - Y_1\Delta Q) \qquad \text{.......................................(11)}$$

$$\Delta V = X_2(\Delta Q - Y_2\Delta P) \qquad \text{.......................................(12)}$$

Where :

$$X_1 = A - BD^{-1}C$$
$$X_2 = D - CA^{-1}B$$
$$Y_1 = BD^{-1}$$
$$Y_2 = CA^{-1}$$

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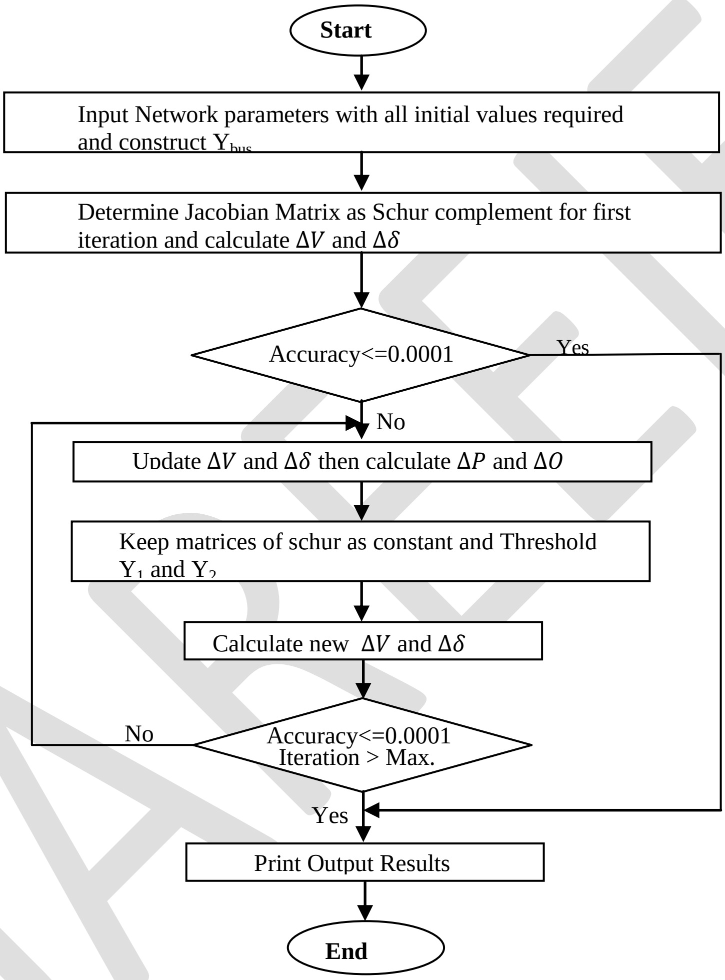
The off-diagonal matrices like B and C will melt inside inverse of matrix B as term ($BD^{-1}C$), While second term ($CA^{-1}B$) will has same values of term above if we eliminated rows and columns for (PV Buses) in matrix A which are already disappeared in Matrix D. It can be considered that X1 equal X2 for same dimensions.

All matrices X1, X2, Y1 and Y2 will slightly changed after each iteration which can considered as constant matrix. In same time, X1 and X2 will tend to be as symmetric matrices and more linearity in their element values. Threshold technique can be used with Y1 and Y2 which has more differences between element values. According to above procedures, the load flow algorithm can be presented as follows:



This algorithm has been applied successfully to reduce computation time of load flow iterations, memory required and convergence strategy. Different number of buses has been tested (5, 14,30 and 118) and results has been compared with Newton Raphson (NR) method and Fast Decoupled (FD) method, which shows the efficiency of Schur complement to got good accuracy with less memory and time.

Threshold process has applied successfully to Y1 and Y2 through their values which has high differences to reduce fluctuation in calculations, memory required and computation time. With reference to threshold value which chosen, the value of element should be zero if it below threshold value and no change if it over threshold value. In most of cases, 70% of elements will has zero value if it applied threshold factors for each row. Threshold values for each row can be considered up to 30% of maximum value of elements in the row which will not effect at the accuracy of results. Following table shows results of program for different cases:

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No. Of Bus		5	14	30	118
SC	Accuracy	3.751e-5	0.000106	0.000448	0.000862
	Time(s)	0.2496	0.2808	0.3276	0.3588
NR	Accuracy	1.01*e-10	0.00072	3.535e-8	4.2797e-5
	Time(s)	0.312	0.2808	0.3564	0.4524
FD	Accuracy	5.185e-5	0.000749	0.000841	Diverge
	Time(s)	0.1560	0.2028	0.2340	2.246

Table (1) Comparison results

The ability of dividing Jacobian matrix by two sub-matrices will allowed to reduce effect of fluctuation up and down near the solution. At same time, it can be observed that performance of algorithm more flexible to find solution with reasonable accuracy and less time. The two separated matrices can be controlled easily, especially when study the stability of system is required. All these features will recommended to use this algorithm to find load flow solution in real applications and in real time calculation for large systems.

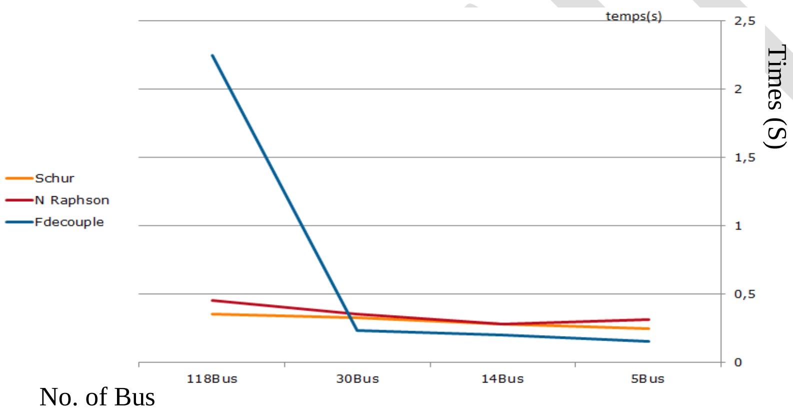

Fig.(6) Compression of time calculations

V.DISCUSSION AND CONCLUSION

In this a new algorithm, It has been used schur complement to develop the load flow program, which solve the non linear power flow questions, by separate the Jacobian matrix into two matrices. The first matrix represents the relationship between voltage angles and apparent powers while second matrix represents relationship between voltages and apparent powers. These two matrices are combined to form a direct approach with better convergence, especially when Jacobian matrix is singular. Threshold technique is used to reduce calculations for elements with low values in matrices which has less effective on accuracy of results. This technique is extremely efficient, so that it can be change threshold factor up to 50% which equalized 70% of matrix elements to zero values. This algorithm has been tested different IEEE standard systems and compare it results with Newton Raphson and Fast decoupled methods. Reasonable computation time with less memory and smooth convergence resulted by utilizing this algorithm.

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APPENDIX A

Parts of the Jacobian matrix can be calculated with following equations:

$$\frac{\partial P_k}{\partial \delta_k}^{(x)} = \sum_{j\neq k} |V_k||V_j||Y_{kj}|\sin(\theta_{kj} - \delta_k + \delta_j)$$

J1 diagonal element

$$\frac{\partial P_k}{\partial \delta_j}^{(x)} = -|V_k||V_j||Y_{kj}|\sin(\theta_{kj} - \delta_k + \delta_j)$$

J1 off diagonal element

$$\frac{\partial P_k}{\partial |V_k|}^{(x)} = 2|V_k||Y_{kk}|cos\theta_{kk} + \sum_{j\neq k} |V_j||Y_{kj}|\cos(\theta_{kj} - \delta_k + \delta_j)$$

J2 diagonal element

$$\frac{\partial P_k}{\partial |V_j|}^{(x)} = |V_k||Y_{kj}|\cos(\theta_{kj} - \delta_k + \delta_j)$$

J2 off diagonal element

$$\frac{\partial Q_k}{\partial \delta_k}^{(x)} = \sum_{j\neq k} |V_k||V_j||Y_{kj}|\cos(\theta_{kj} - \delta_k + \delta_j)$$

J3 diagonal element

$$\frac{\partial Q_k}{\partial \delta_j}^{(x)} = -|V_k||V_j||Y_{kj}|\cos(\theta_{kj} - \delta_k + \delta_j)$$

J3 off diagonal element

$$\frac{\partial Q_k}{\partial |V_k|}^{(x)} = -2|V_k||Y_{kk}|sin\theta_{kk} - \sum_{j\neq k} |V_j||Y_{kj}|\sin(\theta_{kj} - \delta_k + \delta_j)$$

J4 diagonal element

$$\frac{\partial Q_k}{\partial |V_j|}^{(x)} = -|V_k||Y_{kj}|\sin(\theta_{kj} - \delta_k + \delta_j)$$

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J4 off diagonal element

The general form of the method will be:

$$\begin{bmatrix} \triangle P_2 \\ \vdots \\ \triangle P_{n-1} \\ \triangle Q_2 \\ \vdots \\ \triangle Q_{n-ng-1} \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & & \frac{\partial P_2}{\partial |V_2|} & \\ & \ddots \; \frac{\partial P_{n-1}}{\partial \delta_{n-1}} & & \ddots \; \frac{\partial P_{n-1}}{\partial |V_{n-ng-1}|} \\ \frac{\partial Q_2}{\partial \delta_2} & & \frac{\partial P_k}{\partial |V_k|} & \\ & \ddots \; \frac{\partial Q_{n-ng-1}}{\partial \delta_{n-1}} & & \ddots \; \frac{\partial P_k}{\partial |V_k|} \end{bmatrix} \begin{bmatrix} \triangle \delta_2 \\ \vdots \\ \triangle \delta_{n-1} \\ \triangle |V_2| \\ \vdots \\ \triangle |V_{n-ng-1}| \end{bmatrix}$$

N bus general form of N-R method with

The matrix size is calculated by $2 * nbus - ng - 2 * ns$, where nbus is the total number of bus of network, ng is the total number of generator bus in network and ns is the number of slack bus.