

Analysis of Population Growth of India and Estimation for Future

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ABSTRACT: Since times, uncontrolled human population growth has been posing a threat to the earth's resources and to the habitats themselves. Population of India has been increasing dramatically since late 1900's. India as a developing nation should deal quite tactfully with this issue. We work here to estimate the population growth of India since 2009 to 2012 using the logistic model approach and give a comparison with actual population of India for the same time period. An error equation is also deduced based on the trend line for the specified time period and the population of India for the year 2013 and onwards till 2025 is estimated. The time required for India to reach its carrying capacity is also discussed. A relevant conclusion is also made based on the obtained results. This work will provide an insight to the changing trends with respect to India's population growth.

KEYWORDS: Population growth, Estimation of Population, Future prediction, Logistic model, Population trend.

NOTATIONS

t	Time t in years
P_t	Population at time t
P_0	Population at reference time $t=0$ i.e. year 2007
K	Population carrying capacity
k	Maximum growth rate
e_t	Error in population estimation at time t
dp/dt	Rate of change of population with respect to time t .

I. INTRODUCTION

A population model is a type of mathematical model that is applied to the study of population dynamics. Models allow a better understanding of how complex interactions and processes work. Modeling of dynamic interactions in nature can provide a manageable way of understanding how numbers change over time or in relation to each other. Many, if not all, of Earth's processes affect human life. The Earth's processes are greatly stochastic and seem chaotic to the naked eye. However, a plethora of patterns can be noticed and are brought forth by using population modeling as a tool. There are various methods to develop a population model.

A simple exponential model gives sufficient approximation in estimation of population but does not define a saturation point. Hence the population estimated increases exponentially without any upper limit giving an unrealistic figure for longer time period. This is because it does not consider the environmental factors and hence suitable for very short period of time. Logistic model tells that the population growth rate decreases as the population reaches the carrying capacity or saturation point of the environment. The logistic model is more accurate than the exponential model [1]. Much work has been done to further develop these models so as to predict population growth accurately. In this paper we propose to estimate the country's population for the years 2013 and onwards till 2025.

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II. LITERATURE SURVEY

J. N.Kapur and Q. J. A. Khan [2] talk about the simple model of population growth in their work “Some Mathematical Models for Population Growth”. This simple model only considers the excess of births over deaths per unit time and does not account for the limitation of resources. Hence a suitable parameter is considered to modify the simple model using logistic law. This form is known as simple logistic model. The authors also indicate that this modified model explained very well growth of bacterial colony in a nutrient medium and also been successfully used to fit the data for human population. An S-shape curve is obtained when the population is plotted against time. Apart from this the authors have proposed their own models to predict the population of bacteria.

In the book “Spreadsheet Exercises in Ecology and Evolution”, the authors Donovan and Welden [3] tell that the Logistic model with explicit carrying capacity is most convenient way to study population growth as the related equation contains few parameters. They also hint that the solution for basic equation of continuous-logistic model can be obtained by integrating the equation. More details of this solution are discussed later in this paper.

A report “Population Growth: Trends, Projections, Challenges and Opportunities” published by Planning Commission, Government of India [4] shows us the trend of decreasing birth from 1901. The report also tells us that the death rate is much lesser than the birth rate hinting net increasing population of India.

Carl Haub and O. P. Sharma [5] have shown the increasing population trend in their work “India’s Population Reality: Reconciling Change and Tradition”. It can be seen that the population of India has drastically increased in late 1900’s. C M Lakshmana [6] in his paper “Population, Development and Environment in India” show that the population growth of India has decreased from 2001 to 2011 as compared to the previous decade. He also points out that the increasing population of India is main reason behind the environment degradation. He suggests that immediate remedial actions have to be taken to reverse the degrading trend.

From [2] and [3], it can be concluded that with predate population data Logistic model works accurately in estimation of population of India. From [4], [5] and [6], we conclude that it is necessary to estimate the population of India yearly rather than projecting it as per census. This is important to understand the changing population trend for planning and implementation of policies related to population, environment and economy of India.

Hence we intend to estimate the population growth of India using logistic model from year 2009 to 2012 by using available predate population data for the year 2007 and 2008. To analyze the error pattern by comparing the estimated population with actual population and to deduce an error equation based on error pattern. We also intend to incorporate the error equation with the logistic model and project the population growth of India for the year 2013 and onwards till 2025.

III. LOGISTIC DIFFERENTIAL EQUATION AND ITS ANALYTICAL SOLUTION

The basic logistic differential equation [3] is given as

$$\frac{dP}{dt} = kP \left(1 - \frac{P}{K}\right) \dots \dots (1)$$

The logistic equation (1) can be solved for population P by integrating both sides. Rearranging the terms in equation (1), we have

$$\frac{dP}{P(1 - P/K)} = k dt$$

Integrating both sides we get,

$$\int \frac{dP}{P(1 - K/P)} = \int k dt$$

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The term in the integration sign on the left hand side of the above equation can be rewritten as:

$$\frac{dP}{P(1 - P/K)} = \frac{dPK}{P(K - P)} = \frac{dP}{P} + \frac{dP}{K - P}$$

Hence

$$\int \frac{dP}{P} + \int \frac{dP}{K - P} = \int k dt$$

$$\ln|P| - \ln|K - P| = kt + C$$

$$\ln \left| \frac{K - P}{P} \right| = -kt - C$$

$$\left| \frac{K - P}{P} \right| = e^{-kt - C}$$

$$\frac{K - P}{P} = Ae^{-kt}, \text{ where } A = \pm e^{-C}, \text{ a constant}$$

When reference time is considered $t=0$ and $P=P_0$, substituting this in above equation we get

$$A = \frac{K - P_0}{P_0} \dots \dots (2)$$

Hence the logistic equation can be re written as

$$P = \frac{K}{1 + Ae^{-kt}} \dots \dots (3)$$

IV. POPULATION ESTIMATION BY LOGISTIC MODEL

The actual population of India from year 2007 to 2012 is obtained from World Bank homepage [7] and the same is shown below in Table 1 along with the notations used.

Table 1: Actual Population of India from 2007 to 2012 [7]

Year	Notation of time t	Actual Population for the time t	Notation of population for the time t
2007	t_0	1159095250	P_0
2008	t_1	1174662334	P_1
2009	t_2	1190138069	P_2
2010	t_3	1205624648	P_3
2011	t_4	1221156319	P_4
2012	t_5	1236686732	P_5

The notations of time and population for the year 2013 and onwards till year 2025 are continued after the notations for year 2012 in similar manner.

Carrying capacity ‘K’: Gretchen C. Daily and Paul R. Ehrlich [8] in their paper mention about India’s carrying capacity as 2 billion.

The constant ‘A’: The term ‘A’ in equation (2) is obtained by substituting the carrying capacity and reference population that is population of India for the year 2007 (from table 1). The obtained value of term ‘A’ is 0.725483734.

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Maximum growth rate 'k': The term 'k' in logistic equation (3) is a constant and can be determined by using the population of India for the year 2007 and 2008. The estimation of "k" is as shown:
The logistic equation (3) is rewritten for year 2008, that is when $t=t_1$ and $P=P_1$ as

$$P_1 = \frac{K}{1 + Ae^{-kt_1}}$$

Substituting the values of terms in above equation from table1, we get,

$$1174662334 = \frac{2000000000}{1 + 0.725483734 \times e^{-k \times 1}}$$

$$1 + 0.725483734 \times e^{-k} = \frac{2000000000}{1174662334} = 1.702616950$$

$$0.725483734 \times e^{-k} = 0.702616950$$

$$e^{-k} = \frac{0.702616950}{0.725483734} = 0.968480638$$

Taking natural log on both sides of above equation we get,

$$-k = \ln(0.968480638) = -0.032026788$$

Simplified logistic equation: For the purpose of estimation of population of India from year 2009 onwards, the logistic equation (3) can be simplified by substituting values of terms like K, A and e^{-k} . Hence the simplified logistic equation obtained is:

$$P_t = \frac{K}{1 + A(e^{-k})^t} = \frac{2000000000}{1 + 0.725483734 \times 0.968480638^t} \dots \dots (4)$$

Population estimation from year 2009 and onwards: Substituting the values of time and population from tabe 1 in equation (4), the estimated population is obtained. The table 2 below shows the estimated population and actual population for the year 2009 and onwards till year 2012 along with the error in population estimation.

Table 2: Error in population estimation form 2009 to 2012

Year	Actual Population of India	Population estimated by logistic model	Error in estimation
2009	1190138069	1190142588	-4520
2010	1205624648	1205528867	95781
2011	1221156319	1220814208	342111
2012	1236686732	1235991841	694891

Figure 1 below shows the plot of actual population and estimated population versus time.

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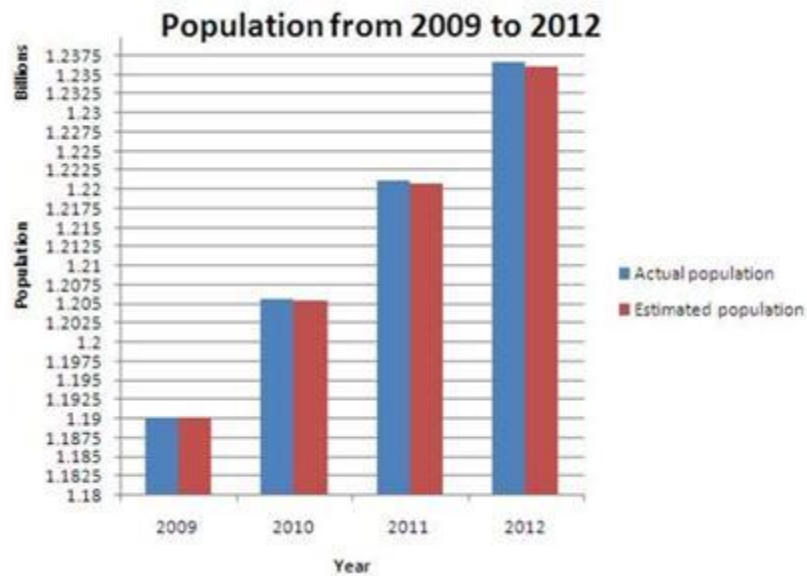


Figure 1: Comparison of actual and estimated population of India

V. ERROR EQUATION, POPULATION CORRECTION, RESULTS AND DISCUSSION

Case 1: The Figure 2 below shows the plot of error in estimation of population of India and also the best fit trend line curve. It should be noted that the trend line is a polynomial line with equation of order 3. Also the R^2 value is unity indicating that it is the best fit trend line. A 3rd order polynomial equation is deduced using the available error from year 2009 to 2012. Indeducing the equation the variable time t value used is $t=1$ for 2009, $t=2$ for year 2010 and so on. The deduced equation is

$$-6596.5t^3 + 112593.5t^2 - 191304t + 80787 = e_t \dots \dots (5)$$

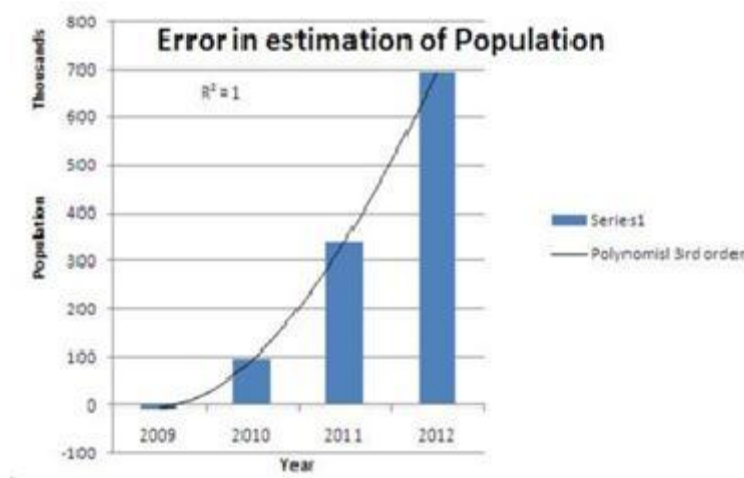


Figure 2: Error in estimated population with trend line 1

To validate the equation (5), it is cross checked by substituting the value of time t for years at random pick. The obtained errors are

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Year 2013, $t= 5$, $e_5= 1114542$, Year 2014, $t=6$, $e_6= 1561485$, Year 2015, $t=7$, $e_7= 1996141$, Year 2020 $t= 12$, $e_{12}= 2599851$, Year 2018, $t=10$, $e_{10}= 2830597$, Year 2019, $t=11$, $e_{11}= 2810215$, Year 2107, $t=100$, $e_{100}= -5489614613$. It can be observed from error e_{10} , e_{11} and e_{12} for the year 2018, 2019 and 2020 that the error value is decreasing and for the year 2107 the error value reaches to around 5.5 billion with a negative sign. As per logistic model the population for year 2107 is estimated to 1.94 billion. This implies that the corrected population for the year 2107 would be approximately $1.94 \text{ billion} + (-5.5 \text{ billion}) = -3.56 \text{ billion}$.

This value seems to unrealistic and treacherous. Hence the polynomial trend line of order 3 is discarded.

Case 2: The figure 3 shows the plot of error in estimation of population of India and also the next best fit trend line curve. It should be noted that the trend line is a polynomial equation of order 2. Also the R^2 value is 0.999 indicating that it is the best fit trend line. A 2nd order polynomial equation is deduced using the available error from year 2009 to 2012. In deducing the equation the variable time t value used is similar to case 1. The deduced equation is

$$63120t^2 - 81142t + 11524 = e_t \dots \dots (6)$$

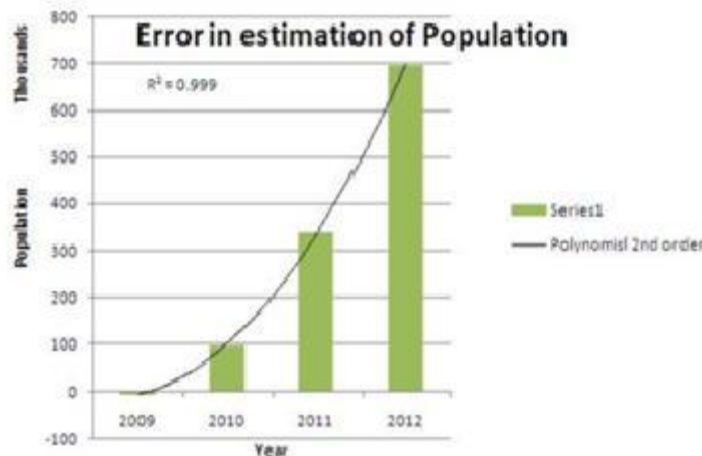


Figure 3: Error in estimated population with trend line 2

To validate the equation (6), it is cross checked by substituting the value of time t for years at random pick. The obtained errors are

Year 2013, $t=5$, $e_5= 1183814$, Year 2014, $t=6$, $e_6= 1796992$, Year 2015, $t=7$, $e_7= 2536410$, Year 2020, $t=12$, $e_{12}= 8127100$, Year 2018, $t=10$, $e_{10}= 5512104$, Year 2019, $t=11$, $e_{11}= 6745482$, Year 2107, $t=100$, $e_{100}= 623097324$. It can be observed from error e_{10} , e_{11} and e_{12} for the year 2018, 2019 and 2020 that the error values are in good agreement. For the year 2107 the error value reaches to around 0.62 billion. As per logistic model the population for year 2107 is estimated to 1.94 billion. This implies that the corrected population for the year 2107 would be approximately $1.94 \text{ billion} + 0.62 \text{ billion} = 2.56 \text{ billion}$.

This value seems to unrealistic and treacherous as the population is crossing the carrying capacity. Hence the polynomial trend line of order 2 is discarded.

Case 3: The figure 4 below shows the plot of error in estimation of population of India and also another trend line curve. It should be noted that the trend line is a linear line. Also the R^2 value is 0.944 indicating that it is a good trend line. A linear equation is deduced using the available error from year 2009 to 2012. In deducing the equation the variable time t value used are similar to case 1 and case 2. The deduced equation is

$$234450t - 304070 = e_t \dots \dots (7)$$

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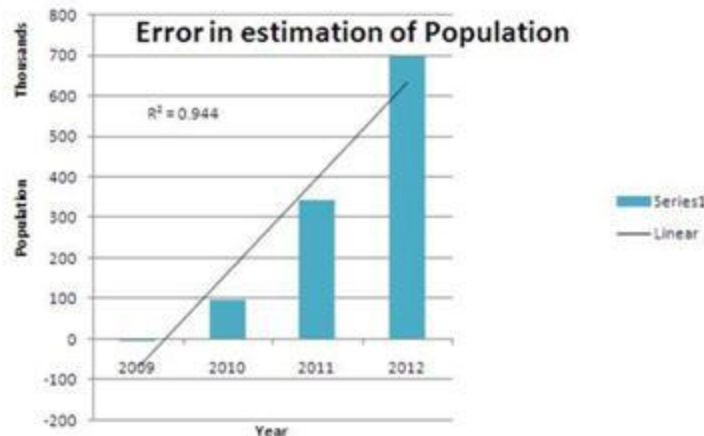


Figure 4: Error in population estimation with trend line 3

To validate the equation (7), it is cross checked by substituting the value of time t for years at random pick. The obtained errors are Year 2013, t=5, $e_5 = 868180$, Year 2014, t=6, $e_6 = 1102630$, Year 2015, t=7, $e_7 = 1337080$, Year 2020, t=12, $e_{12} = 2509330$, Year 2018, t=10, $e_{10} = 2040430$, Year 2019, t=11, $e_{11} = 2274880$, Year 2107, t=100, $e_{100} = 23140930$.

It can be observed from error e_{10} , e_{11} and e_{12} for the year 2018, 2019 and 2020 that the error values are in good agreement. For the year 2107 the error value reaches to around 0.024 billion. As per logistic model the population for year 2107 is estimated to 1.94 billion. This implies that the corrected population for the year 2107 would be approximately $1.94 \text{ billion} + 0.024 \text{ billion} = 1.964 \text{ billion}$.

This value seems to be realistic and satisfactory, though the value is approaching the carrying capacity of India it can still be used for short time period. Hence the linear trend line is acceptable. Though the R^2 value is less than case 1 and case 2 indicating decrease in accuracy of error estimation. The linear trend line gives realistic results.

Estimated and corrected population of India from 2013 to 2025: the table 3 below shows the estimated population, the projection of corrected population of India using the logistic model and the linear error equation from case 3 and percentage increase in population with respect to year 2009.

Table 3: Estimated and corrected population of India

Year	Notation of estimated population by logistic model	Estimated population by logistic model	Notation for error by error equation	Estimated error by error equation	Notation for corrected population estimation	Corrected population estimation	% increase in corrected population w.r.t. to year 2009
2013	P_6	1251055202	e_5	868180	$P_6 = P_6 + e_5$	1251923382	---
2014	P_7	1265997942	e_6	1102630	$P_7 = P_7 + e_6$	1267100572	0.01
2015	P_8	1280813934	e_7	1337080	$P_8 = P_8 + e_7$	1282151014	0.02
2016	P_9	1295497285	e_8	1571530	$P_9 = P_9 + e_8$	1297068815	0.04
2017	P_{10}	1310042342	e_9	1805980	$P_{10} = P_{10} + e_9$	1311848322	0.05
2018	P_{11}	1324443698	e_{10}	2040430	$P_{11} = P_{11} + e_{10}$	1326484128	0.06
2019	P_{12}	1338696196	e_{11}	2274880	$P_{12} = P_{12} + e_{11}$	1340971076	0.07
2020	P_{13}	1352794940	e_{12}	2509330	$P_{13} = P_{13} + e_{12}$	1355304270	0.08
2021	P_{14}	1366735291	e_{13}	2743780	$P_{14} = P_{14} + e_{13}$	1369479071	0.09
2022	P_{15}	1380512879	e_{14}	2978230	$P_{15} = P_{15} + e_{14}$	1383491109	0.11
2023	P_{16}	1394123595	e_{15}	3212680	$P_{16} = P_{16} + e_{15}$	1397336275	0.12
2024	P_{17}	1407563602	e_{16}	3447130	$P_{17} = P_{17} + e_{16}$	1411010732	0.13
2025	P_{18}	1420829329	e_{17}	3681580	$P_{18} = P_{18} + e_{17}$	1424510909	0.14

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The figure5 below shows a plot of corrected population of India from year 2013 to 2025.

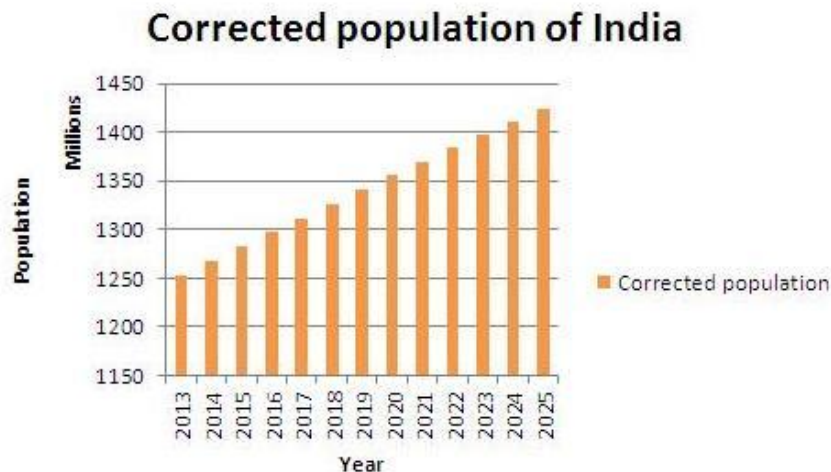


Figure 5: Corrected population of India

VI. CONCLUSION

Projection of population growth of India is important from sustainability point. The environment should be able to support the population, for this not only the environment but the nation should also contribute to control the population growth. The logistic approach is a good tool to predict population of India and any habitat in general. Though the estimated population consists of some errors the accuracy of prediction can be improved by incorporating an error equation. In this work an attempt of minimizing the error in prediction of population of India has been reduced to certain extent by incorporating a linear error equation. This error equation though holds good for short time period, certainly improves the result. The results can be improved by incorporating factors of correction like the constant 'A' and the constant 'k' in logistic equation which tend to change as the reference year or time zero year changes. It should also be noted that for India to reach its carrying capacity of 2 billion, it would take more than a century. To sum up all the discussion it is concluded that the logistic approach incorporated with error equation is a good tool in population estimation.

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