



International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Vol. 2, Special Issue 1, December 2013

Beamforming and Transmission Power Optimization

Reeta Chhatani ¹, Alice Cheeran ²

PhD Scholar, Victoria Jubilee Technical Institute, Mumbai, India¹

Professor, Victoria Jubilee Technical Institute, Mumbai, India²

Abstract: Beamforming-array signal processing is a powerful means of increasing capacity and coverage of the mobile communication networks. It supports the next generation broadband cellular system to efficiently carry populated traffic over a limited spectrum due to its co-channel interference mitigation capability.

This paper presents performance comparison of three beamforming methods namely null steering, minimum variance distortionless response (MVDR) and minimum mean square error (MMSE) beamforming. It is observed that MMSE beamforming is most suitable for practical non-line-of-sight multipath environment.

The work is further extended to downlink beamforming problem for multicell network which is formulated as minimization of transmitted power by all base stations (BSs) subject to SINR constraints at each mobile. The conventional per cell basis beamforming and coordinated multicell beamforming are implemented to achieve downlink transmit power optimization using downlink beamforming. Both algorithms use uplink (UL)-downlink (DL) duality concept to solve complicated DL beamforming by using dual uplink problem.

Per cell basis beamforming treats intercell interferences as a part of background noise while coordinated multicell beamforming considers intercell interferences for weight vector calculation. Thus, per cell basis optimization does not lead to a joint optimal solution which is possible by coordinated multicell beamforming. The important feature of coordinated beamforming algorithm is that, it leads to distributed implementation in time division duplex (TDD) system. The distributed implementation is less complex and requires only beamforming level coordination unlike centralized implementation which requires system level coordination and thus it is more complex.

Thus, in this paper per cell basis beamforming and coordinated multicell beamforming has been implemented and compared for mobile wimax multicell network. The simulation results proved that coordinated beamforming outperforms per cell basis beamforming for different values of SINR and user locations.

Keywords: Array signal processing, beamforming, multicell network, uplink-downlink duality.

I. INTRODUCTION

IN recent years there is an enormous rise in the traffic for mobile and personal communication systems. This is due to dynamic business and social applications which has driven the popularity of mobile data services and created a need for all-time connectivity at any time and anywhere. Far beyond simple voice communication, today's wireless networks must support applications such as internet access, video streaming etc. Thus major demands for next generation mobile communication system are,

- 1) broadband data rates.
- 2) high capacity to support increasing traffic load over limited spectrum.
- 3) high quality of service in affordable prize.

To achieve high data rates and high capacity it is needed to reduce interferences and noise which greatly affects the performance of cellular system, and increase the desired signal power. With limited spectrum availability, achieving this goal is difficult due to co-channel interferences. So more advanced technologies are desperately needed to fulfill the need of next generation wireless mobile communication.

Beamforming- antenna array signal processing is one of the most powerful and promising technology which reduces co-channel interferences and noise and thereby ensures high signal to interference noise ratio (SINR). Beamforming antenna has the ability to focus energy in direction of desired users and nulls in direction of interferences. Such directional transmission reduces co-channel interferences and allows co-channel users to be separated by spatial multiplexing, thereby increasing capacity of cellular network. Beamforming systems require less transmission power as compared to omni-directional antennas. Thus it helps to reduce transmission power without degrading quality of service. Also the directional transmission increases coverage by each base station (BS) thereby reducing capital expenditure and operating expenditure which gives cost efficient network deployment. Thus beamforming has proved



International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Vol. 2, Special Issue 1, December 2013

its benefits for next generation mobile cellular networks and helps mobile wireless operators to provide high speed data services as per the demand in affordable price to subscribers by supporting large number of users over limited available spectrum.

In this paper, performance evaluation has been carried out for beamforming design methods namely null steering, minimum variance distortionless response (MVDR) and minimum mean square error (MMSE) beamforming. The work is further extended to downlink (DL) / transmit beamforming problem for multicell network, which is of great interest nowadays as in conventional wireless system antenna arrays are typically deployed at base station, whereas the mobile terminals are equipped with single antennas. In this work downlink beamforming problem is formulated as minimization of transmitted power by all base stations (BSs) subject to SINR constraints at each mobile station (MS). Downlink (DL) / transmit beamforming is more complicated than uplink (UL) / receive beamforming as all beamformers must be optimized jointly. Thus, complicated downlink problem is solved by dual uplink problem which is easier to solve. The UL-DL duality concept is used to solve DL beamforming problem efficiently.

The conventional per cell basis beamforming algorithm and coordinated multicell beamforming algorithm are compared to study the benefits of BS coordination in multicell network for power optimization using downlink beamforming. The coordinated multicell beamforming algorithm considered in this paper is implemented in distributive fashion in time-division duplex (TDD) system as it only requires beamforming level coordination and thus it is easy to implement.

A. Related Work

The overview of beamforming technology and different beamforming methods for narrowband and broadband signals have been investigated by L. Godara, 1997 [1]. The implementation of different beamforming algorithms is also presented in literature [2], [3], [4]. Out of different beamforming algorithm null steering, MVDR and MMSE beamforming is found more attractive for wireless communication systems.

Recently transmit/downlink beamforming problem has been proposed in many works [5]-[17]. The downlink beamforming problem can be formulated in two ways as:

- 1) Reduce the transmitted power by BS subject to SINR constraints at each MS. Thus in this lower limit on SINR is fixed. For this limit transmitted power is optimized so as to reduce interference to the other users in cell and out of cell.
- 2) Optimized SINR subject to transmit power constraint of BS. In this target transmission power of BS is fixed and SINR at MS is optimized to achieve high throughput rate (bps/Hz) [5]-[8].

For DL beamforming weight vector calculation channel state information (CSI) is required. In frequency division duplex (FDD) system, UL and DL channels are different. Thus for DL beamforming, DL channel is estimated at MS and is feedback to BS for weight vector calculation. Performance of DL beamforming is greatly affected due to limited bandwidth of feedback channel. This limitation is overcome by efficient weight vector codebook design. In Grassmannian weight vector codebook design method the correlated vectors are removed to reduce the size of codebook [9] and codebook is made available to both BS and MS. When signal arrives at MS, it estimates the best weight vector from predetermined codebook which matches the estimated channel. The index of this weight vector is feedback to BS. By using this feedback, BS selects the weight vector from codebook to achieve optimum transmission. For temporally correlated channels feedback compression algorithm is proposed to compress feedback CSI in alternate feedback intervals [10].

In time division duplex (TDD) system UL and DL channels are reciprocal and thus complicated DL transmit problem can be solved by UL receive beamforming by using UL-DL duality concept [11]-[14], [17]. In single cell scenario downlink power optimization is done on per cell basis which considers intercell interference as a part of background noise [13], [14]. This single cell DL beamforming problem can also be formulated as a second order cone programming (SOCP) problem [15]. This helps to interpret UL-DL duality via Lagrangian theory of convex optimization [14], [16], [17].

For multicell DL beamforming, coordination between different BSs in network is needed, which is achieved in centralized or distributive manner. In centralized co-ordination scheme, the BS's in different cells act as single BS with distributed antennas and schedules the user by estimating weight vector corresponding to best channel out of all available channels. At particular instant BS which has the best channel serves the user and weights are calculated for that channel. Thus centralized coordination requires, to exchange large data among all BSs, all BSs need to have access to data destined for MSs of other BSs and also scheduling decision will be jointly made at the BSs. This increases feedback overhead and complexity [18]-[22].

Unlike centralized beamforming which requires signal level coordination, distributive coordination requires beamforming level coordination [17]. In distributive coordination, each BS performs beamforming for its own users by



International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Vol. 2, Special Issue 1, December 2013

considering intercell interference for weight vector calculation. Thus there is no centralized beamforming and user scheduling which makes system implementation much easier than centralized coordinated network.

B. Organization

The remainder of the paper is organized as follows. Section II contains beamforming design methods. Downlink beamforming and transmission power optimization is presented in section III. Section IV gives simulation results followed by concluding remarks in section V.

II. BEAMFORMING DESIGN METHODS

This paper presents three methods of beamformer design namely Null steering beamforming, Minimum variance distortionless response (MVDR) and Minimum mean square error (MMSE). Depending on system requirement one of the methods can be selected for beamformer design.

A. System Model

A single cell with M mobile users and base station with L-element antenna array is considered. Let there be d desired signal sources and l interference sources transmitting on same frequency channel simultaneously. Then the signal received by antenna array becomes,

$$x(t) = \sum_{j=1}^d a_j(\varphi) s_j(t) + \sum_{k=1}^l a_{\text{int}-k}(\varphi) s_k(t) + n(t) \quad (1)$$

where $a_i(\varphi)$ is a array steering vector denoting the amplitude gain and phase shift of the signal of i^{th} user. j denote the desired users and k denote interferences. $s_i(t)$ is signal vector and $n(t)$ is random noise component with zero mean and unity variance.

The beamformer system output can be written as,

$$y(t) = w^H x(t) \quad (2)$$

where w is weight vector and superscript H denote conjugate transpose.

B. Null Steering Beamforming Algorithm

Null-Steering Beamformer forms a beam in the desired direction and nulls in interference directions. Two modes of null-steering beamforming are considered, null steering beamforming for single desired user and multi-beamforming for multiple desired users. Thus, the null-steering beamforming problem can be formulated as,

1) Estimation of Direction of Arrivals (DOAs) using DOA Algorithm: Minimum variance distortionless response (MVDR) DOA algorithm is used for DOA estimation for which the power spectrum is given as [1],

$$P_{MV}(\varphi) = \frac{1}{a_\varphi^H R^{-1} a_\varphi} \quad (3)$$

where R is array correlation matrix defined by $R = E [x(t) x^H(t)]$.

2) Validation Process: Once DOAs are estimated for all incoming signal, next is validation process which separates out desired signal from interferences. If after validation process, it is found that there is only single desired user and others are interferences then null steering mode is invoked. If it is found that there is more than one desired user then multi-beamforming mode is invoked.

3) Null Steering Beamforming: The open loop null steering algorithm has developed to satisfy following constraints [23], [24]:

- a) to make $w^H a_i = 0$;
- b) to maximize $\| w^H a_d \|^2$

where a_d array steering vector for a desired user and a_i array steering vector for a interferences.

An optimal weight for a null steering array is given as [23],



International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Vol. 2, Special Issue 1, December 2013

$$w = [I_L - a_1(a_1^H a_1)^{-1} a_1^H] a_d \quad (4)$$

where I_L is $L \times L$ identity matrix.

4) Multi-Beamforming: During signal validation process, if it is observed that there are more than one desired user then multibeam mode is triggered. The weight vectors for all desired users are calculated by using (4). In this mode, due to the presence of multiple desired users the time window is defined for each user. Certain percentage of duty cycle is allocated to each user such that for the assigned time period beam is formed towards that user [3]. Thus for certain percentage of time duration one user is served and for next interval the other user is served.

C. Minimum Variance Distortionless Response (MVDR) Beamforming Algorithm

The MVDR beamformer does not require the knowledge of the directions of the interferences for weight vector calculation. It requires only the direction of the desired signal.

MVDR weight vector is given as [25], [26]

$$w = \frac{R^{-1} a_d(\theta)}{a_d(\theta)^H R^{-1} a_d(\theta)} \quad (5)$$

These weights are the solution of the following optimization problem:

$$\text{minimize } w^H R w \quad \text{subject to } w^H a_d(\varphi) = 1. \quad (6)$$

Thus, the beamformer weights are selected by minimizing the mean output power while maintaining unity response in the look direction. The constraint ensures that the signal passes through the beamformer undistorted. Therefore, the output signal power is the same as the look-direction source power. The minimization process then minimizes the total noise, including interferences and uncorrelated noise. Minimization of total output noise, while keeping the output signal constant is same as maximizing the output SINR. As MVDR beamformer maximize SINR only in one direction, this beamformer is not suitable for non-line-of-sight (NLOS) multipath environment where desired signal get scatter in many directions. Thus it can be implemented for rural environment where multipath signals do not exist.

D. Minimum Mean Square Error (MMSE) Beamforming Algorithm

In multipath fading environment, MMSE beamformer gives the optimum result. This beamformer utilizes reference signal which is correlated to desired signal and uncorrelated with interferences. In MMSE algorithm the array output is compared with reference signal, thus weight vectors are calculated such that beams are produced in the direction of multipath signal those matches with reference signal unlike MVDR. Thus, the MMSE beamforming is optimum candidate for NLOS urban environment, as it not only reduces the interference but also multipath fading is mitigated.

In MMSE beamforming array output is subtracted from an available reference signal $r(t)$ to generate an error signal $\varepsilon(t) = r(t) - w^H x(t)$, which is used to control the weights. Weights are adjusted such that the mean square error between the array output and the reference signal is minimized. The optimal weight vector is given by Wiener–Hoff equation [25], [26]

$$w = R^{-1} z \quad (7)$$

$$z = E[x(t) r(t)^H] \quad (8)$$

where z is the cross-correlation between the reference signal $r(t)$ and the signal vector $x(t)$.

III. DOWNLINK BEAMFORMING AND TRANSMISSION POWER OPTIMIZATION

In conventional wireless system, antenna arrays are typically deployed at base station, whereas the mobile terminals are equipped with single antennas. Thus, beamforming has to be performed in downlink to achieve desired performance.

International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Vol. 2, Special Issue 1, December 2013

Downlink beamforming is more complicated than uplink as in DL all beamformers must be optimized jointly. Due to nonorthogonal transmission in cell, the specific choice of one of the user's beamformer may affect the crosstalk experienced by other users. Hence, the SINR values of all users are coupled, which makes downlink beamforming a complicated task. The uplink beamforming is achieved independently at each BS while in downlink beamforming BS has to consider SINR at the all MSs to optimize the beamformer output. As a result, downlink beamforming has to be done jointly in the entire network.

In this paper downlink beamforming problem is formulated for minimization of transmitted power across all BS subject to SINR constraints at each mobile receiver. This paper presents per cell basis optimization and coordinated multicell optimization algorithm for downlink beamforming of multicell wireless network.

A. Problem Formulation

1) System Model: This paper considers a multicell multi-user spatial multiplex system with N cells and K users per cell, with N_t antennas at each base-station and a single antenna at each remote user. Multiuser downlink transmit beamforming is employed at each base-station. Let $x_{i,j}$ be a complex scalar denoting the information signal for the j^{th} user in the i^{th} cell, and $w_{i,j} \in C^{N_t \times 1}$ be its associated beamforming vector. The received signal at the j^{th} remote user in the i^{th} cell, denoted as $y_{i,j}$, is a summation of the intended signal, intracell interference, and intercell interference:

$$y_{i,j} = \sum_l h_{i,i,j}^H w_{i,l} x_{i,l} + \sum_{m \neq i,n} h_{m,i,j}^H w_{m,n} x_{m,n} + n_{i,j} \quad (9)$$

where $h_{i,i,j} \in C^{N_t \times 1}$ is the vector channel from the base station of the i^{th} cell to the j^{th} user in the i^{th} cell, and $n_{i,j}$ is the additive white circularly symmetric gaussian complex noise.

Transmit Beamforming Problem: Let $w_{i,j}$ be the beamforming vectors, then the SINR $\Gamma_{i,j}$ for the j^{th} user in the i^{th} cell can be expressed as [17]

$$\Gamma_{i,j} = \frac{|w_{i,j}^H h_{i,i,j}|^2}{\sum_{l \neq j} |w_{i,l}^H h_{i,i,j}|^2 + \sum_{m \neq i,n} |w_{m,n}^H h_{m,i,j}|^2 + \sigma^2} \quad (10)$$

Let $\gamma_{i,j}$ be the SINR target for the j^{th} user in i^{th} cell. A total transmit power minimization problem subject to SINR constraints can then formulate as follows:

$$\begin{aligned} & \text{minimize} \quad \sum_{i=1}^K \alpha_i w_{i,j}^H w_{i,j} \\ & \text{subject to} \quad \Gamma_{i,j} \geq \gamma_{i,j} \quad \forall i = 1 \dots N, j = 1 \dots K \quad (11) \end{aligned}$$

where α_i is the power constraint of each BS and the minimization is over the $w_{i,j}$'s.

In conventional per cell basis optimization, multicell beamforming problem is solved by considering out-of-cell interference as a part of background noise and beamformers of all out of cell mobile stations (MSs) are fixed. For calculation of beamforming weights conventional beamformer only considers the interference of users in same cell and optimizes transmitted power. It does not consider intercell interference for calculation of weight vector. Thus, per-cell basis optimization does not necessarily lead to a joint optimal solution. Significant performance improvement may be obtained if base-stations coordinate in jointly optimizing all of their beamformers at the same time.

Joint optimal solution i.e. coordinated beamforming for multicell in which each BS calculates the weight vector for each user by considering channel and power of the other users in its own cell as well as the neighboring cell. Such coordination helps to mitigate interference in the scenario when two users belonging to two different cells are present along the same direction and are at cell edge. In such situation the two base station schedules the users such that beam for both users are not formed simultaneously [27]. For such coordination channel state information is required to exchange between two base stations. If users are not at cell edge but both are at same spatial location coordinated beamforming is done reduced the transmitted power subject to SINR at each user.

International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Vol. 2, Special Issue 1, December 2013

B. Conventional Per Cell Basis Beamforming

To solve the downlink beamforming problem with per cell basis optimization, power minimization algorithm for single cell which has proposed by Martin Schubert and Holger Boche [13] has been extended for multicell network. It can be proved that the SINR at each user is inversely proportional to cross-talk between the users. The optimal beamforming weights are calculated to reduce cross-talk between the users and to achieve feasible SINR region (i.e. $SINR \geq \gamma$) and once it is achieved transmission power is minimized. An illustration of the algorithm is given in fig. (1).

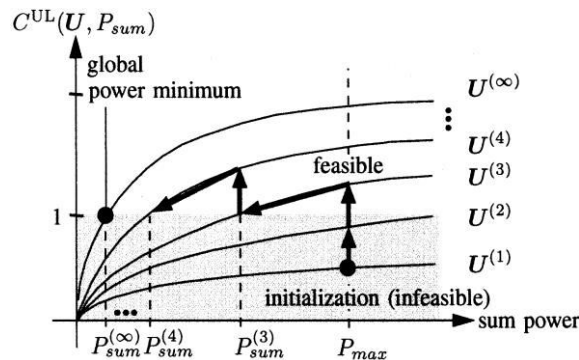


Fig. 1: Schematic Illustration of Per-Cell Optimization Algorithm [13]

Let there be K users per cell in multicell network. The beamforming is performed on per cell basis. The beamforming weight vectors u_i are collected in a matrix $U = [u_1, \dots, u_K]$. The downlink transmission power vectors p_i are stack in a vector $p = [p_1, \dots, p_K]^T$ and the total transmission power per cell is given by $\|p\|_1$. The uplink transmit powers of all mobiles are stacked in a vector $q = [q_1, \dots, q_K]^T$. The total transmission power per cell of both uplink and downlink is limited by P_{max} i.e. maximum allotted transmission power per cell for considered cellular network.

The downlink spatial covariance matrices are given by,

$$R_i = E(h_i h_i^H) \quad 1 \leq i \leq K \quad (12)$$

The TDD system has reciprocity between uplink and downlink channels, i.e. both channels are described by the same covariance matrices as in (12).

Due to spatial non-orthogonal transmission, users are generally coupled by cross-talk. The coupling is characterized by a nonnegative matrix Ψ which is a function of beamforming weight vector U .

$$\begin{aligned} [\Psi(U)]_{ik} &= u_k^H R_i u_k \quad k \neq i \\ &= 0 \quad k = i \end{aligned} \quad (13)$$

The uplink and downlink SINRs, are given by the following two expressions, respectively

$$SINR_i^{UL}(u_i, q) = \frac{q_i u_i^H R_i u_i}{u_i^H (\sum_{k=1, k \neq i}^K q_k R_k + \sigma_i^2 I) u_i}, \quad \forall i \quad (14)$$

$$SINR_i^{DL}(U, p) = \frac{p_i u_i^H R_i u_i}{\sum_{k=1, k \neq i}^K p_k u_k^H R_i u_k + \sigma_i^2}, \quad \forall i \quad (15)$$

$$\text{Let } C^{DL}(U, P_{max}) = \frac{SINR_i^{DL}(U, p)}{\gamma_i} \quad 1 \leq i \leq K \quad (16)$$



International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Vol. 2, Special Issue 1, December 2013

$$P_{\max} = \|p\|_1$$

$$\sigma = [\sigma_1^2, \dots, \sigma_K^2]^T$$

$$\text{and } D = \text{diag} \left\{ \left(\frac{\gamma_1}{u_1^H R_1 u_1} \right), \dots, \dots, \left(\frac{\gamma_K}{u_K^H R_K u_K} \right) \right\} \quad (17)$$

An extended power vector is $p_{\text{ext}} = [p \ 1]^T$ and an extended coupling matrix is given as [13],

$$\chi(U, P_{\max}) = \begin{bmatrix} D\Psi(U) & D\sigma \\ \frac{1}{P_{\max}} \mathbf{1}^T D\Psi(U) & \frac{1}{P_{\max}} \mathbf{1}^T D\sigma \end{bmatrix} \quad (18)$$

It can be proved that the $C^{\text{DL}}(U, P_{\max})$ is inversely proportional to maximum eigenvalue of the extended coupling matrix χ [13].

$$C^{\text{DL}}(U, P_{\max}) = \frac{1}{\lambda_{\max}(\chi(U, P_{\max}))} \quad (19)$$

The optimal power vector p is the first K components of the dominant eigenvector of $\chi(U, P_{\max})$, which can be scaled so that its last component equals one.

For uplink with the same total power limit P_{\max} , the same targets, and the same fixed beamforming matrix, the equations are obtained as,

$$C^{\text{UL}}(U, P_{\max}) = \frac{1}{\lambda_{\max}(\Lambda(U, P_{\max}))} \quad (20)$$

where Λ is the extended uplink coupling matrix define as

$$\Lambda(U, P_{\max}) = \begin{bmatrix} D\Psi^T(U) & D\sigma \\ \frac{1}{P_{\max}} \mathbf{1}^T D\Psi^T(U) & \frac{1}{P_{\max}} \mathbf{1}^T D\sigma \end{bmatrix} \quad (21)$$

The optimal power vector q is the first K components of the dominant eigenvector of $\Lambda(U, P_{\max})$, which can be scaled so that its last component equals one.

Thus it can be observed that, $C(U, P_{\max})$ increases with decrease in dominant eigenvalue of coupling matrix.

1) Uplink–Downlink Duality: It can be proved that, both uplink and downlink has the same SINR achievable regions. Also, minimum transmit power needed to achieve a set of SINR constraints in a downlink channel is the same as the minimum total transmit power needed to achieve the same set of SINR targets in an uplink channel [11], [13]. This is known as uplink-downlink duality. A beneficial consequence of this duality is that, all SINR-based beamforming concepts designed for uplink reception immediately carry over to downlink transmission. Thus, the solution of the downlink beamforming problem is equivalently obtained by solving an easier-to-handle uplink problem.

Thus uplink–downlink duality gives,

$$C^{\text{DL}}(U, P_{\max}) = C^{\text{UL}}(U, P_{\max}), \quad (22)$$

In uplink, maximization of SINR is achieved by MMSE beamforming with weight vector given as,

$$u_i = \arg \max \frac{u_i^H R_i u_i}{u_i^H Q_i(q_{\text{ext}}) u_i} \quad (23)$$



International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Vol. 2, Special Issue 1, December 2013

where

$$Q_i(q_{\text{ext}}) = \sum_{\substack{k=1 \\ k \neq i}}^K [q_{\text{ext}}]_k R_k + I \quad (24)$$

The matrices Q_i are nonsingular and symmetric, thus (23) is solved by the dominant generalized eigenvectors of the matrix pairs $(R_i, Q_i(q_{\text{ext}}))$, $1 \leq i \leq K$.

2) Algorithm for Per Cell Basis Optimization:

- a) Initialize: $n = 0$, $q = 0$, $C = 0$, $p_sum = \text{max power allotted/cell for considered network}$.
- b) $P_max = p_sum$
- c) $R_i = R_i/\sigma_i^2$, $1 \leq i \leq K$.
- d) $\sigma_i^2 = 1$, $1 \leq i \leq K$.
- e) $u_i = \text{Dominant generalized eigenvectors } (R_i, Q_i(q))$,
 $1 \leq i \leq K$.
- f) $u_i = u_i/\|u_i\|_2$, $1 \leq i \leq K$.
- g) if $C < 1$ then (1st stage)
Calculate dominant eigenvalue and associated eigenvector of uplink extended coupling matrix $\Lambda(U, P_{\text{max}})$.
 $C = 1/\lambda_{\text{max}}$
else (2nd stage)
 $q = (I - D\Psi^T(U))^{-1}D1$
 $P_{\text{sum}} = \|q\|_1$
end
- h) Calculate intercell interferences and are added to noise vector.
- i) Repeat (c)-(h) until all relative SINR are balanced.
- j) Repeat steps (b)-(i) until $p_sum(n-1) - p_sum(n) < \epsilon$
- k) Downlink power allocation
 $p^{\text{opt}} = (I - D\Psi(U))^{-1}D1$

The feasibility of SINR requirements is always checked by 1st stage of algorithm. If the problem is feasible then $C > 1$ and if it is infeasible, then $C < 1$ after convergence, so algorithm is terminated in controlled manner.

C. Coordinated Multicell Beamforming

For coordinated multicell beamforming UL-DL duality is proved by using Lagrange dual theory of convex optimization.

The SINR target constraints in (11) are nonconvex i.e. the problem has local optimal point which is not necessarily global optima. Such nonconvex problem is converted into convex by using Lagrangian technique [14], [17].

1) Uplink-Downlink Duality by Lagrangian Dual for Multicell Coordinated Beamforming: By using Lagrangian duality concept it can be proved that optimal solution for dual of DL problem is same as that of UL problem when constraints are satisfied with equality [17]. Due to such strong duality [16], the optimal transmit beamforming problem in (11) for the downlink multiuser multi-cellular network can be solved via a dual uplink channel in which the SINR constraints remain the same and the noise power is scaled by α_i . Mathematically, a Lagrangian dual of the downlink optimization problem given by (11) is the following uplink problem [17]:

$$\begin{aligned} & \text{minimize } \sum_{i=1}^K \lambda_{i,j} \sigma^2 \\ & \text{subject to } \zeta_{i,j} \geq \gamma_{i,j} \forall i = 1 \dots N, j = 1 \dots K \end{aligned} \quad (25)$$

where the minimization is over $\lambda_{i,j}$, and

International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Vol. 2, Special Issue 1, December 2013

$$\zeta_{i,j} = \max_{\hat{w}_{i,j}} \frac{\lambda_{i,j} |\hat{w}_{i,j}^H h_{i,i,j}|^2}{\sum_{(m,l) \neq (i,j)} \lambda_{m,l} |\hat{w}_{i,j}^H h_{i,m,l}|^2 + \alpha_i \|\hat{w}_{i,j}\|^2} \quad (26)$$

The optimal $\hat{w}_{i,j}$ has the interpretation of being the receiver beamformer of the dual uplink channel, and is a scaled version of the optimal DL weight vector $w_{i,j}$. The optimal $\lambda_{i,j}$ has the interpretation of being the dual uplink power, and it corresponds to the dual variable associated with the SINR constraint of (11). $\zeta_{i,j}$ denote the uplink SINR for j^{th} user in i^{th} cell.

2) Weight Vector Calculation: The optimal receive beamformers $\hat{w}_{i,j}$ in (25) that maximize the SINR are the minimum-mean-squared-error (MMSE) receivers, which can be expressed as

$$\hat{w}_{i,j} = (\sum_{m,l} \lambda_{m,l} \sigma^2 h_{i,m,l} h_{i,m,l}^H + \sigma^2 \alpha_i I)^{-1} h_{i,i,j} \quad (27)$$

The weight vector $w_{i,j}$ and $\hat{w}_{i,j}$ are scaled versions of each other given by,

$$w_{i,j} = \sqrt{\delta_{i,j}} \hat{w}_{i,j} \quad (28)$$

where

$$\delta = F^{-1} \mathbf{1} \sigma^2 \quad (29)$$

$\mathbf{1}$ is the $NK \times 1$ all ones-vector and F is the following $NK \times NK$ matrix:

$$F = \begin{bmatrix} F^{11} & F^{12} & \dots & F^{1N} \\ F^{21} & F^{22} & \dots & F^{2N} \\ \vdots & \vdots & \dots & \vdots \\ F^{N1} & F^{N2} & \dots & F^{NN} \end{bmatrix} \quad (30)$$

where the (j,n) th entry of each $K \times K$ sub-matrix F^{im} is defined as follows,

$$F_{j,n}^{im} = \begin{cases} \frac{1}{\gamma_{i,j}} |\hat{w}_{i,j}^H h_{i,i,j}|^2 & \text{if } m = i \text{ and } n = j, \\ - \sum_{n \neq j} |\hat{w}_{i,n}^H h_{i,i,j}|^2 & \text{if } m = i \text{ and } n \neq j, \\ - \sum_{m \neq i, n} |\hat{w}_{m,n}^H h_{m,i,j}|^2 & \text{if } m \neq i \end{cases} \quad (31)$$

3) Algorithm for Coordinated Multicell Beamforming [17]: Coordinated downlink beamforming problem is solved by distributive implemented iterative function evaluation algorithm, which is given as follows,

a) Optimal uplink power $\lambda_{i,j}$ is calculated using iterative function evaluation which is given by,

$$\lambda_{i,j} = \frac{1}{\left(1 + \frac{1}{\gamma_{i,j}}\right) h_{i,i,j}^H \Sigma_i^{-1} h_{i,i,j}} \quad (32)$$

where Σ_i is the covariance matrix of the received signal in the uplink direction at the base-station i which includes the intended signal, the interference, and a scaled version of the background noise which is given by,

$$\Sigma_i \triangleq \alpha_i I + \sum_{m,n} \lambda_{m,n} h_{i,m,n} h_{i,m,n}^H \quad (33)$$

International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Vol. 2, Special Issue 1, December 2013

- b) Repeat (a) until it converges to optimal uplink power. This uplink per-cell iteration process always converges to unique fixed point.
- c) Calculate the optimal uplink receive beamformers $\widehat{w}_{i,j}$ using (27) for optimal uplink power $\lambda_{i,j}$
- d) Calculate optimal downlink transmit beamformers using (28).

In a TDD system, the covariance matrix is estimated locally at each base station in the uplink direction. Channel reciprocity implies that the signal and interference covariance matrices at the uplink receiver are same as that of downlink. Thus, the iterative updates of $\lambda_{i,j}$ and the estimation of Σ_i performed on a per-cell basis, i.e. the iterative function evaluation process can be done locally without the need of explicit inter-base-station coordination. The base-station coordination is achieved implicitly via uplink power control i.e. the update of $\lambda_{i,j}$'s, which affect all other Σ_i 's. Thus, this method needs only the beamforming level coordination and can be implemented in distributive manner. It requires much less overhead and is more practical to implement.

IV. SIMULATIONS

This section presents, performance analysis of null steering beamforming, MVDR beamforming and MMSE beamforming. Also compares performance of per cell basis optimization algorithm and coordinated multicell optimization algorithm for mobile wimax network.

A. Performance Analysis of Beamforming Algorithms

This paper considers uniform circular array (UCA) with 8-element and radius of array is $0.5 \times$ wavelength. The 5 incoming signal sources are considered for simulation with 10 dB SNR for all incoming sources.

Azimuth angle of incoming signals = $\{0^\circ, 50^\circ, 100^\circ, 150^\circ, 200^\circ\}$.

1) Null Steering Beamforming

- a) MVDR DOA Estimation: MVDR DOA estimates all incoming signals. Fig. 2 (a) and (b) shows the polar plot and power spectrum of MVDR DOA algorithm, respectively.

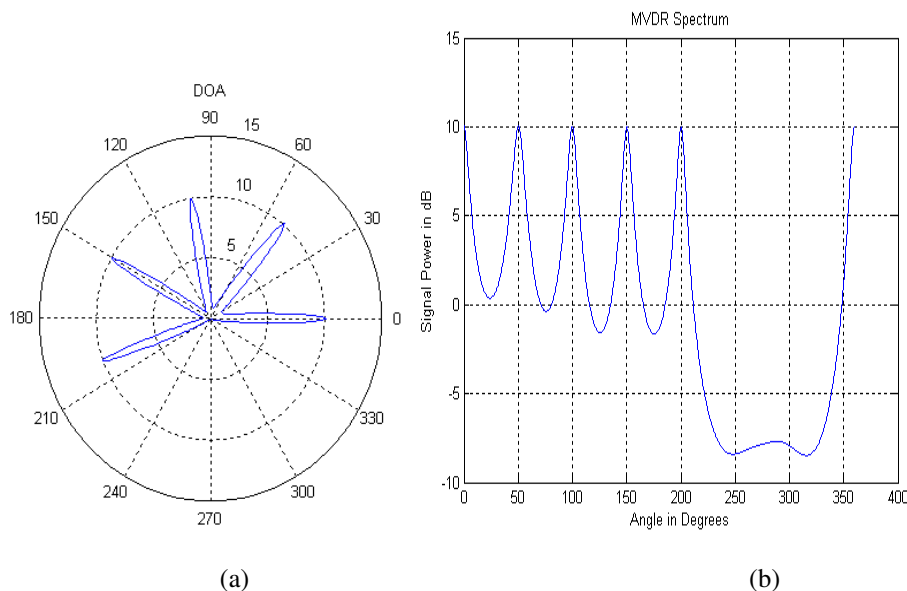


Fig. 2: (a) Polar Plot (b) Power Spectrum of MVDR DOA Estimation

- b) Null Steering Beamforming for Single Desired User: Let direction of desired source is $\varphi = 200^\circ$ and direction of interferences are $(0^\circ, 50^\circ, 100^\circ, 150^\circ)$.

Weights are calculated using (4) to produce a beam in the direction of desired user (200°) and null in the direction of interferences ($0^\circ, 50^\circ, 100^\circ, 150^\circ$). Fig. 3 (a) and (b) shows the polar plot and power spectrum of null steering

International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Vol. 2, Special Issue 1, December 2013

beamforming respectively. It can be observed that output SNR is 26.46 dB, thus approximately gain of 16 dBi is achieved in direction of desired user.

- c) Multi-Beamforming: Two desired sources in directions $\phi_1 = 150^\circ$ and $\phi_2 = 200^\circ$ are considered. Validation process trigger multi-beamforming algorithm. Weights are calculated to produce a beam in the direction of desired users ($\phi_1 = 150^\circ$ and $\phi_2 = 200^\circ$) and null in the direction of interferences ($0^\circ, 50^\circ, 100^\circ$).

Fig. 4 (a) and (b) shows the polar plot and power spectrum of multi-beamforming respectively. It can be observed that output SNR of 26.5 dB is achieved for both desired users with null depth of approximately -30 dB for interferences.

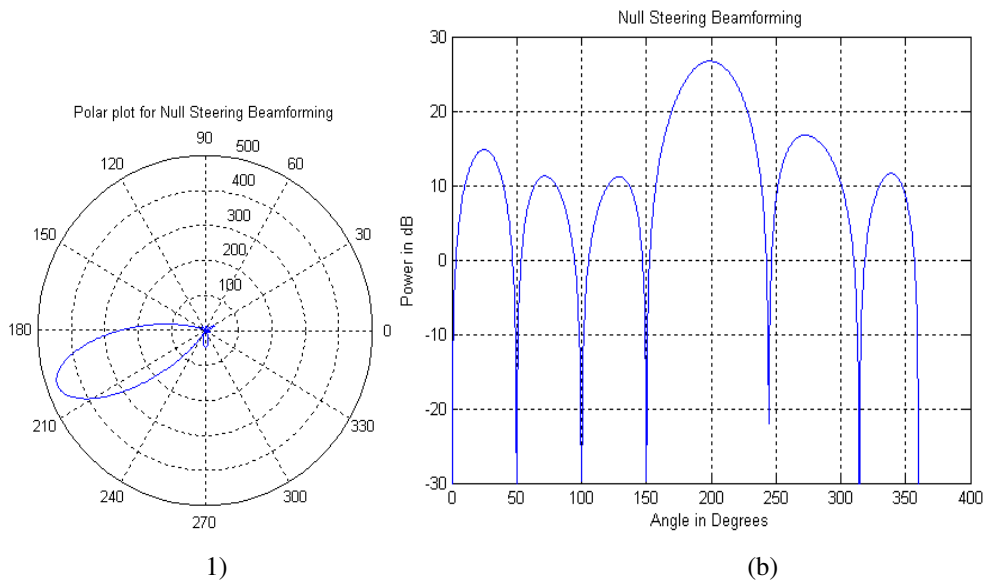


Fig. 3: (a) Polar Plot (b) Power Spectrum of Null Steering Beamforming

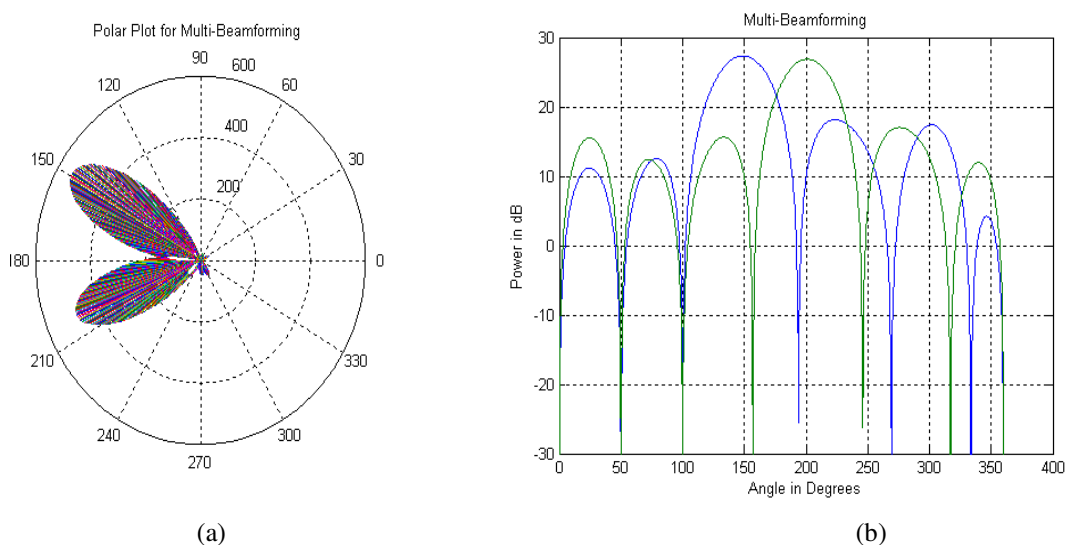


Fig. 4: (a) Polar plot (b) Power Spectrum of Multi-Beamforming

International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Vol. 2, Special Issue 1, December 2013

- 2) MVDR Algorithm: A desired source is in direction $\phi = 200^\circ$. Weights are calculated using (5) to produce a unity response in the direction of desired user ($\phi = 200^\circ$) and null in the direction of interferences ($0^\circ, 50^\circ, 100^\circ, 150^\circ$). Fig. 5 (a) and (b) shows the polar plot and power spectrum of MVDR beamforming respectively. Thus, MVDR beamforming produces unity gain response in the direction of desired user and minimizes the total noise power.
- 3) MMSE Algorithm: For this simulation multipath environment is considered. Due to multipath fading a desired source is arriving from direct path $\phi = 0^\circ$ with SNR = 10 dB and from reflected path $\phi = 100^\circ$ with SNR = 9 dB. Weights are calculated using (7) to produce a beam in the direction of direct and reflected path ($\phi = 0^\circ$ and $\phi = 100^\circ$ respectively) and null in the direction of interferences ($50^\circ, 150^\circ, 200^\circ$). Fig. 6 (a) and (b) shows the polar plot and power spectrum of MMSE beamforming respectively. It can be observed that MMSE beamforming captures both the direct as well as reflected signal.

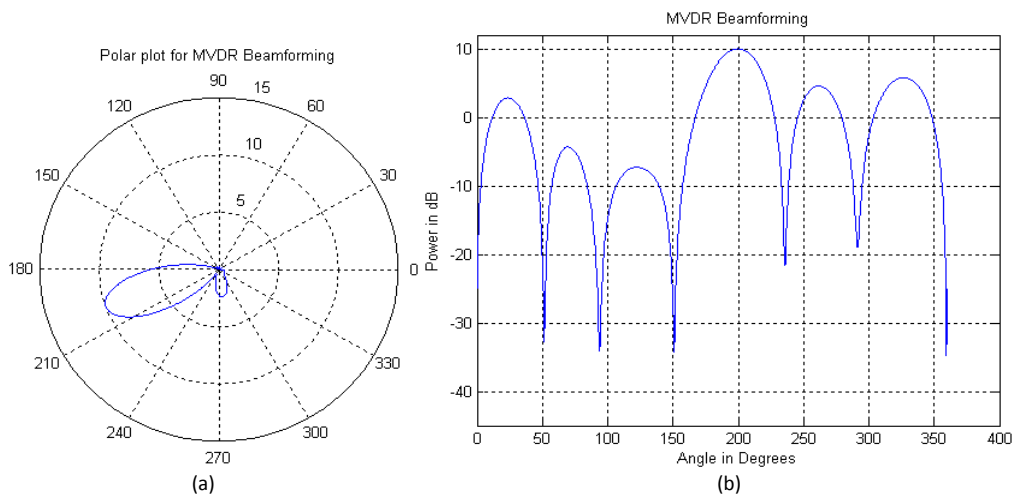


Fig. 5: (a) Polar plot (b) Power Spectrum of MVDR Beamforming

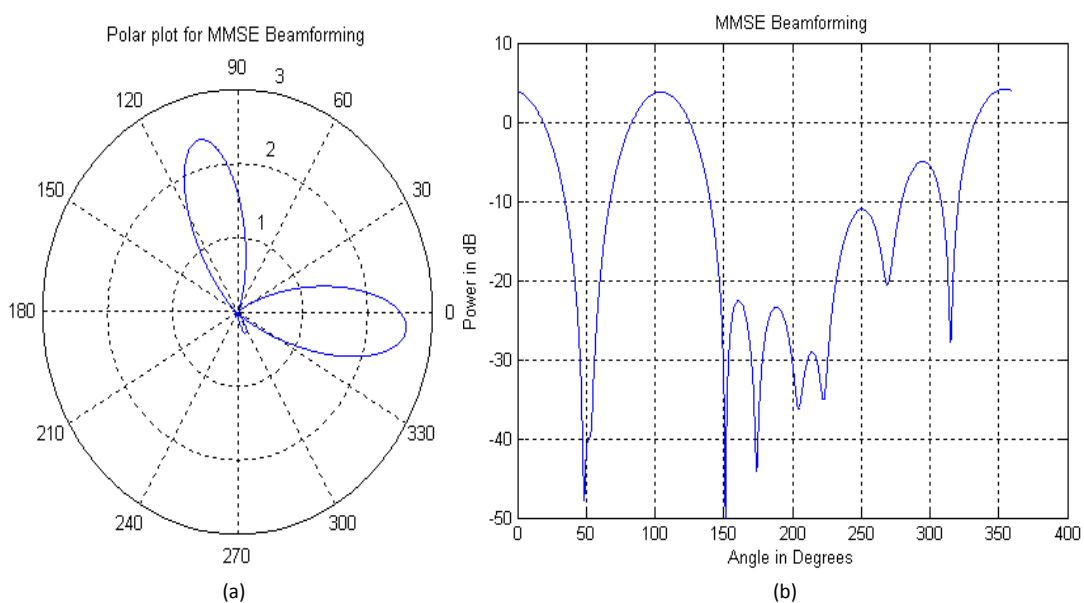


Fig. 6: (a) Polar plot (b) Power Spectrum of MMSE Beamforming

International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Vol. 2, Special Issue 1, December 2013

Simulation results of all three beamforming algorithms are tabulated in table I.

TABLE I
PERFORMANCE ANALYSIS OF BEAMFORMING ALGORITHMS

Beamforming Algorithm	I/p SNR (dB)	O/p SNR (dB)	Gain (dBi)	Null Depth (dB)	Comments
Null Steering (Both modes of operation)	10	26.46	16	-30	This open loop method is able to cancel a strong jammer in a mobile communications system. But it requires knowledge of directions of both desired and interfering signals.
MVDR	10	10	0	-35	This open loop method cancels out correlated arrivals. It is not suitable for multipath environment.
MMSE	10 ($\varphi = 0^\circ$ direct path) 9 ($\varphi = 100^\circ$ reflected path)	9.9565	Less than 0 dBi in desired direction	-50	This closed loop method is suitable for multipath correlated arrival. Its gain in the look direction is not unity, but the MMSE beamformer provides a stronger noise rejection than the MVDR and null steering beamformer.

B. Performance Comparison of Per Cell Basis Beamforming and Coordinated Multicell Beamforming

Fig. 7 shows two cell network for downlink beamforming power optimization problem.

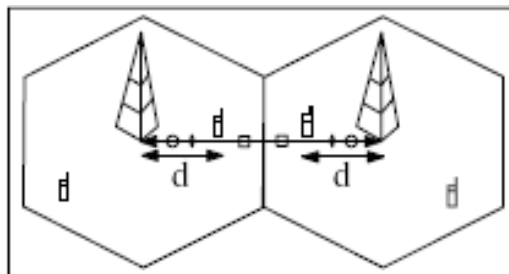


Fig. 7: Two Cell Network Model

The mobile wimax network parameters are considered for simulation. The distance between two BSs is 2.8 km. BS transmission power/cell is 43 dBm. Thus, total available power in 2 cell network is 46.02 dBm. Each BS has 12 antennas with gain of 15 dBi and single antenna is at MS. The channel model considered is ITU vehicular test environment model with log-normal shadowing and Rayleigh fading. The weighting factors corresponding to the base-station antenna power constraints are set to be $\alpha_i = 1$. d is the distance of the user from its own base station on the straight line connecting the two base-stations. Two users per cell are considered. One user is at distance d from its BS and the other user is located randomly elsewhere in the cell. The per cell basis algorithm and coordinated multicell algorithm both are implemented for considered network model.

Fig. 8 shows, total transmitted power in network versus target SINR for both per cell basis optimization and coordinated beamforming. It is observed that coordinated beamforming achieves higher SINR target which are not feasible by per cell basis optimization. For considered 2-cell network scenario with power threshold of 46.02 dBm, per cell basis optimization achieves SINR upto 21 dB for distance d of 0.4 km, 9 dB for 0.8 km and 1 dB for 1.2 km. Thus it is proved that the coordinated beamforming system significantly outperforms the conventional per cell basis optimized system, especially at the high SINR target range.

Also coordination between BSs helps to serve cell edge users efficiently by scheduling them alternately. Thus, two BSs coordinate such that at a time only one of the edge users is served and other is turned off. This reduces co-channel interference, thus both users are served successfully with optimum transmission power.

International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Vol. 2, Special Issue 1, December 2013

For considered 2-cell network, as d increases beyond 1.2 km, high SINR becomes infeasible. For $d = 1.3$ km and target SINR = 25 dB, total transmission power required = 46.7720 dBm, which is infeasible as maximum available power in considered network is 46.02 dBm. Thus, users beyond 1.2 km are considered as cell edge users and scheduling of such users is done alternately by joint coordination between two BSs. Scheduling of edge users is done such that, during first time slot user 1 (cell edge user) of BS 1 is active while user 3 (cell edge user) of BS 2 is off and for next slot user 3 is active and user 1 is turned off. Thus, both users are served alternately to achieve high SINR values. Table II shows total transmitted power required for cell edge users to achieve SINR of 25 dB.

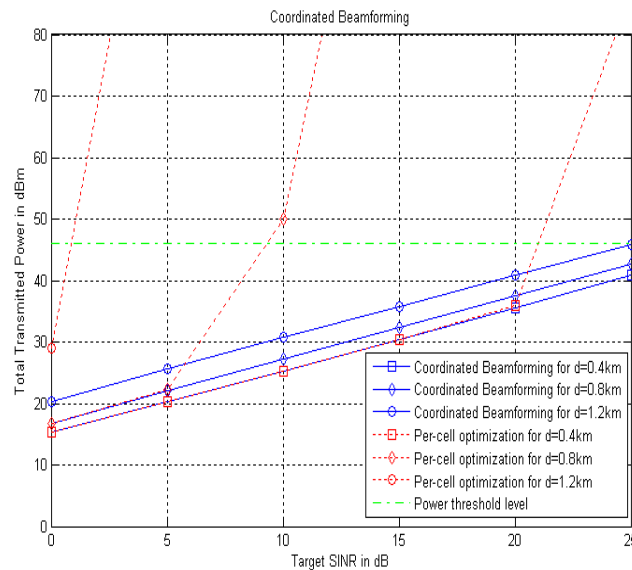


Fig. 8: Total Transmitted Power v/s Target SINR for Per-Cell Optimization and Coordinated Beamforming for 2 Cell Network with 2 Users per Cell.

Table II
Total transmission power for cell edge users to achieve SINR of 25 dB

Distance d (km)	Target SINR (dB)	Total Transmitted Power (dBm)	
		User 1 active User 3 turned off	User 3 active User 1 turned off
1.3	25	41.4661	45.0447
1.4	25	41.6976	45.7736

From table II it can be observed that, by serving cell edge users alternately total transmitted power required to achieve high SINR value (25 dB) is less than maximum available power in considered network (46.02 dBm). Thus, due to coordination between BSs, high SINR is feasible for cell edge users, which is not possible for per cell basis optimization as beamforming is done independently at each BS, considering out of cell users signals as a part of background noise.

2 cell network scenario is further extended to 7 cell network as shown in fig. 9.

International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Vol. 2, Special Issue 1, December 2013

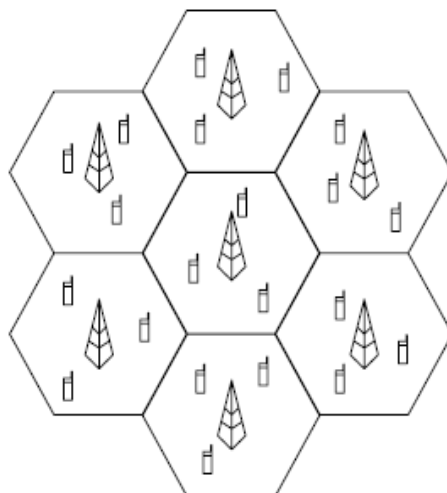


Fig. 9: 7 Cell Wireless Network

The per cell basis optimization and coordinated multicell beamforming has been implemented for 7 cell network with 2 randomly located users/cell. Thus, there are total 14 co-channel users in network. The total available power for 7 cell network is 51.46 dBm (43 dBm/cell) is the threshold level of maximum transmission power of network. Simulation has been carried for this network and results are compared in fig. 10.

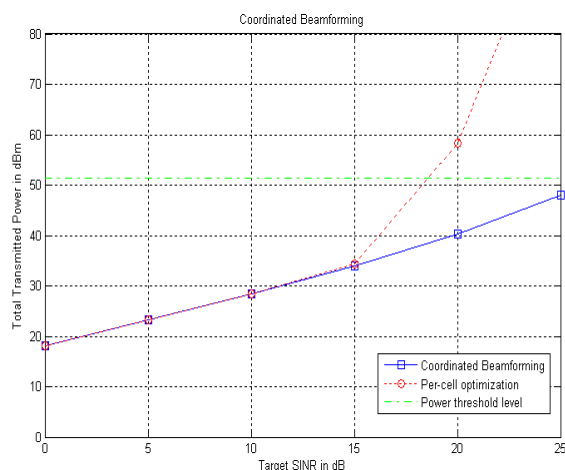


Fig. 10: Total Transmitted Power v/s Target SINR for Per-Cell Optimization and Coordinated Beamforming for 7 Cell Network with 2 Users per Cell.

From fig. 10, it can be observed that SINR upto 18 dB is feasible for per cell basis optimization while total transmission power required using coordinated beamforming to achieve SINR value upto 25 dB, is still below the threshold level of 51.46 dBm.

Fig. 11 shows simulation results of both algorithms for 7 cell network with 3 users per cell.

International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Vol. 2, Special Issue 1, December 2013

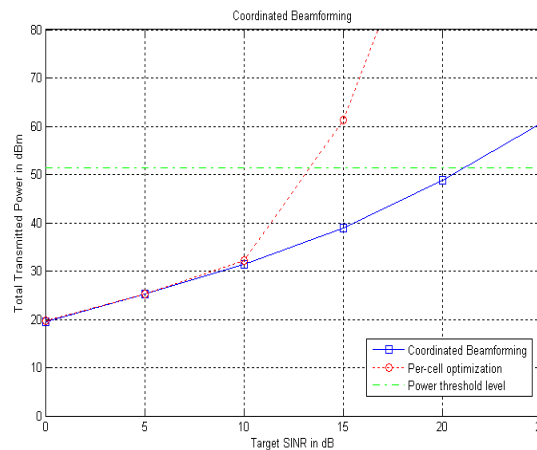


Fig. 11: Total Transmitted Power v/s Target SINR for Per-Cell Optimization and Coordinated Beamforming for 7 Cell Network with 3 Users per Cell.

From fig. 11, it can be observed that SINR upto 14 dB is feasible for per cell basis optimization and upto 21 dB is feasible for coordinated beamforming.

Thus, performance gain of coordinated beamforming is high for 2 users/cell (14 users in 7 cell network) than 3 users/cell (21 users in 7 cell network). This is because BSs are equipped with 12 antennas. Thus, for total 12 users in network coordinated beamforming has the capability of zero forcing out of cell users, thereby completely eliminating the out of cell interference while out of cell interference is always present for conventional per cell basis optimized network. For total 14 users in network, coordinated beamforming still shows high performance gain. With total 21 users in network, complete zero forcing is not possible, still coordination shows better performance than per cell optimization.

Thus, it is proved that coordinated multicell beamforming shows higher performance gain over per cell basis optimization and gives significant power saving and cochannel interference reduction which are most important aspects of mobile wireless cellular networks.

V. CONCLUSION

This paper presents Beamforming-array signal processing which has gained importance in wireless mobile communication network due to its ability to reduced co-channel interferences. Emerging broadband wireless systems, such as mobile wimax or 3GPP LTE-A, will re-use spectrum in every cell (reuse factor = 1) to maximize system spectrum efficiency. Such high reuse factor is possible due to mitigation of co-channel interferences which is possible due to beamforming technology. Also, increased SINR helps to achieve a broadband data rate which is the requirement of next generation wireless mobile communication systems. High directional beams increases range of coverage thereby reducing the number of base station required which provide cost efficient deployment.

This paper analyzes the performance of three beamforming algorithms namely null steering, minimum variance distortionless response (MVDR) and minimum mean square error (MMSE) beamforming algorithm. It is observed that out of three methods MMSE beamforming is most suitable for practical application like non-line of sight urban environment as it supports multipath arrivals.

The paper also implements the downlink/transmit beamforming problem which is of great interest currently. The downlink beamformers are optimized to minimize downlink transmitted power subject to fixed SINR constraint of each user. Complicated downlink/transmit optimization is solved by uplink/receive problem which is easier to handle by using uplink-downlink duality approach. The two power optimization algorithms namely per cell basis optimization and coordinated multicell beamforming are implemented and compared. In coordinated multicell beamforming Lagrangian dual of convex optimization theory is used to prove UL-DL duality.

It can be observed that the coordinated beamforming system significantly outperforms the conventional per cell basis optimized system, especially at the high SINR target range. As expected, maximum benefit is obtained when the users are close to the cell edge. With increasing users in network, per cell optimization problem becomes infeasible for relatively low SINR values as compared to coordinated beamforming. Also, with coordinated beamforming, due to



International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Vol. 2, Special Issue 1, December 2013

coordination between BSs, high SINR is achieved for the cell edge users which are served alternately by their respective BSs. Such coordination is not possible for per cell basis optimization.

Thus coordinated beamforming is more efficient than per cell optimization and also requires only beamforming level coordination between base stations. Thus, it can be implemented in a distributed fashion for TDD system. No system level coordination is required there by reducing feedback overhead and joint processing, still gives improved performance over per cell basis conventional beamforming. Thus, multicell downlink power minimization problem is solved efficiently by coordinated multicell beamforming.

This work can be extended to centralized coordinated beamforming in which all BSs acts as a single BS and centralized beamforming and scheduling is performed which is more complex to achieve. Performance of centralized coordinated beamforming can further be compared with distributed coordinated beamforming for future work.

REFERENCES

- [1] L. Godara, "Application of antenna arrays to mobile communications, part II: Beamforming and direction of-arrival considerations," *Proc. IEEE*, vol. 85, No. 8, Aug. 1997.
- [2] Y. Huang and M. Panique, "Performance analysis of a null steering algorithm," *Proc. IEEE*, No. 7, July 2007.
- [3] Y. Huang, "Design of a dynamic beamforming antenna for Wimax radio systems," *Proc. IEEE*, vol. 20, No. 6, Aug. 2008.
- [4] R. Weber, Y. Huang and W. Tidd, "Cognitive beamforming antenna," *Proc. IEEE*, vol. 20, No. 6, Oct. 2010.
- [5] R. Zakhour, Z. K. M. Ho, and D. Gesbert, "Distributed beamforming coordination in multicell MIMO channels," in *Proc. IEEE Veh. Tech. Conf.*, Barcelona, Spain, Apr. 2009.
- [6] E. Larsson and E. Jorswieck, "Competition versus cooperation on the MISO interference channel," *IEEE J. Sel. Areas Commun.*, vol. 26, no. 9, pp. 1059–1069, Sep. 2008.
- [7] L. Venturino, N. Prasad and X. Wang, "Coordinated linear beamforming in downlink multi-cell wireless networks," *IEEE Trans. on Wireless Communications*, vol. 9, No. 4, pp. 1451-1460, April, 2010.
- [8] J. Yang and D. Kim, "Multicell uplink downlink beamforming throughput duality based on Lagrangian duality with per base station power constraints," *IEEE communication letters*, vol 12, Apr. 2008.
- [9] D. Love, R. Heath, and T. Strohmer, "Grassmannian beamforming for multiple-input multiple-output wireless systems," *IEEE Trans. Inform. Theory*, vol. 49, no. 10, pp. 2735-2747, Oct. 2003.
- [10] K. Huang and R. Heath, "Limited feedback beamforming over temporally correlated channels," *IEEE transaction on signal processing*, vol. 57, May 2009.
- [11] F. Rashid-Farrokhi, K. J. R. Liu, and L. Tassiulas, "Transmit beamforming and power control for cellular wireless systems," *IEEE J. Sel. Areas Commun.*, vol. 16, no. 8, pp. 1437–1450, Oct. 1998.
- [12] E. Visotsky and U. Madhow, "Optimal beamforming using transmit antenna arrays," *Proc. IEEE Veh. Technol. Conf.*, vol. 1, July 1999, pp. 851-856.
- [13] M. Schubert and H. Boche, "Solution of the multiuser downlink beamforming problem with individual SINR constraints," *IEEE Trans. Veh. Technol.*, vol. 53, pp. 18–28, Jan. 2004.
- [14] W. Yu and T. Lan, "Transmitter optimization for the multi-antenna downlink with per-antenna power constraints," *IEEE Trans. Signal Processing*, vol. 55, no. 6, pp. 2646–2660, June 2007.
- [15] A. Wiesel, Y. C. Eldar, and S. Shamai, "Linear precoding via conic optimization for fixed MIMO receivers," *IEEE Trans. Signal Processing*, vol. 54, no. 1, pp. 161–176, Jan. 2006.
- [16] S. Boyd and L. Vandenberghe "Convex optimization," *Cambridge University Press*, pages 215-230, 2009.
- [17] H. Dahrouj and W. Yu, "Coordinated Beamforming for the Multicell Multi-Antenna Wireless System," *IEEE Trans. on Wireless Communications*, vol. 9, no. 5, may 2010.
- [18] W. Wong and S. Talwar, "Transmit beamforming and nulling approach with distributed scheduling to improve cell edge throughput," *Hindawi Publication*, Dec 2009.
- [19] H. Zhang and H. Dai, "Cochannel interference mitigation and cooperative processing in downlink multicell multiuser MIMO networks," *EURASIP J. Wireless Commun. and Networking*, vol. 2004, no. 2, pp. 222–235, Dec. 2004, article ID 202654.
- [20] S. Shamai and B. Zaidel, "Enhancing the cellular downlink capacity via co-processing at the transmitting end," in *Proc. IEEE Veh. Technol. Conf.*, vol. 3, 2001, pp. 1745–1749.
- [21] S. Jing, D. N. C. Tse, J. B. Soriaga, J. Hou, J. Smee, and R. Padovani, "Multicell downlink capacity with coordinated processing," *EURASIP J. Wireless Commun. and Networking*, vol. 2008, no. 5, Jan. 2008, article ID 586878.
- [22] M. Karakayali, G. Foschini, and R. Valenzuela, "Network coordination for spectrally efficient communications in cellular systems," *IEEE Wireless Commun.*, vol. 13, no. 4, pp. 56–61, Aug. 2006.
- [23] B. Friedlander, B. Porat, "Performance analysis of a null-steering algorithm based on direction-of-arrival estimation," *Proc. IEEE Trans. on Signal Processing*, vol. 37, no. 4, Apr. 1989.
- [24] Y. Huang, M. Panique "Performance analysis of a null steering algorithm," *Proc. IEEE*, No. 7, July 2007.
- [25] R. Compton, *Adaptive antennas concept and performance*, Prentice Hall Publication, 1995.
- [26] V. Madisetti, *Digital signal processing handbook*, 2nd ed. CRC Press, Dec. 2009, pp. 2-2–2-19.
- [27] L. Hui and W. Bo, "Coordinated beamforming technology in TD-LTE Advanced system," *International conference on communications and mobile computing*, 2010.