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Common fixed point theorems for self maps on a fuzzy metric space involving integral type in equalities

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Abstract: Malhotra [7] proved a common fixed point theorem in fuzzy metric spaces for occasionally weakly compatible mappings with integral type in equality by reducing its minimum value. In this paper we prove a common fixed point theorem in fuzzy metric spaces for occasionally weakly compatible mappings with integral type inequality involving some special type of Lebesgue integrable functions.

Key words: Fuzzy metric space, occasionally weakly compatible (owc) mappings, common fixed point.

Mathematical Subject Classification (2010): 47H10, 54H25

I. INTRODUCTION

Fuzzy set was introduced by Zadeh [15].Kramosil and Michalek [6] introduced the notion of a fuzzy metric space, George and Veermani [4]modified the notion of fuzzy metric spaces with the help of continuous t-norms. Grabiec[3],Subramanyam[13], Vasuki[14], Pant and Jha [9]obtained some analogous results proved by Balasubramaniam [1] et al. Subsequently, it was developed extensively by many authors and used in various fields. Jungck [5] introduced the notion of compatible maps for a pair of self maps.George and Veermani [4], Sessa [11] initiated the tradition of improving commutative condition in fixed point theorems by introducing the notion of weak commuting property. Further Jungck and Rhoades [5] gave a more generalized condition defined as compatibility in metric spaces. Jungck and Rhoades [5] also introduced the concept of weakly compatible maps.Malhotra [7] proved a common fixed point theorem in fuzzy metric spaces for occasionally weakly compatible mappings with integral type in equality by reducing its minimum value. In this paper we prove a common fixed point theorem for selfmaps on fuzzy metric spaces for occasionally weakly compatible mappings with integral type inequality involving some special type of Lebesgue integrable functions (Definition [2.1]).

Definition 1.1 :(Zadeh.L.A. [14]) A fuzzy set A in a nonempty set X is a function with domain X and values in $[0,1]$.

Definition 1.2: (Schweizer.B. and Sklar. A. [9]) A function $*: [0,1] \times [0,1] \rightarrow [0,1]$ is said to be a continuous t-norm if $*$ satisfies the following conditions:

For $a, b, c, d \in [0,1]$

(i) $*$ is commutative and associative

(ii) $*$ is continuous

(iii) $a * 1 = a \text{ for all } a \in [0,1]$

(iv) $a * b \leq c * d \text{ whenever } a \leq c \text{ and } b \leq d$

Definition 1.3: (Kramosil. I. and Michalek. J. [5]) A triple $(X, M,*)$ is said to be a fuzzy metric space (FM space, briefly) if X is a nonempty set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times [0,\infty)$ satisfying the following conditions:

For $x, y, z \in X$ and $s, t > 0$.

(i) $M(x, y, t) > 0, M(x, y, 0) = 0$
(ii) $M(x, y, t) = 1$ *if and only if* $x = y$.
(iii) $M(x, y, t) = M(y, x, t)$
(iv) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$
(v) $M(x, y, \cdot): [0,\infty) \longrightarrow [0,1]$ is continuous.

Then M is called a fuzzy metric space on X.

The function $M(x, y, t)$ denotes the degree of nearness between x and y with respect to t.

Definition 1.4:(Jungck.G. and Rhoades.B.E. [4]) Two self mappings f and g of a fuzzy metric space $(X, M,*)$ are called compatible if $\lim_{n\to\infty} M(fgx_n, gfx_n, t) = 1$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = x$ for some $x \in X$.

Definition 1.5:(Cho.Y.J.[1]) Two self mappings f and g of a fuzzy metric space $(X, M,*)$ are called reciprocally continuous on x if $\lim_{n\to\infty} fx_n = fx$ and $\lim_{n\to\infty} gfx_n = gx$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = x$ for some $x \in X$.

Definition 1.6: (**George.A. and Veeramani.P. [3])** Let $(X, M,*)$ be a fuzzy metric space.

Then,

(i) A sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$ if $\lim_{n\to\infty} M(x_n, x, t) = 1 \ \forall \ t > 0$.
(ii) A sequence $\{x_n\}$ in X is called a Cauchy sequence if $\lim_{n\to\infty} M\left(x_{n+p}, x_n, t\right) = 1 \ \forall \ t > 0$ and $p = 1,2, \ldots$
(iii) An FM –space in which every Cauchy sequence is convergent is said to be complete.

Definition 1.7: (Malhotra.S.K.Naveen Verma and Ravindra Sen [6]) Two self maps f and g of a set $X$are occasionally weakly compatible (owc) iff there is a point x in X which is a coincidence point of f and g at which f and g commute.

Lemma: 1.8:(Mishra.S.N. and Sharma.N.[7]) Suppose $(X, M,*)$ is a fuzzy metric space.If for all $x, y \in X, t > 0$ and for a number $k \in (0,1), M(x, y, kt) \geq M(x, y, t)$ then $x = y$.

Lemma: 1.9:(Jungck.G. and Rhoades.B.E. [4]) Let X be a set and f, g owc self maps of X.If $f \& g$ have a unique point of coincidence,$w = fx = gx$, then w is the unique common fixed point of f and g.

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Vol. 2, Issue 6, June 2013

Notation 1.10: Let $\Phi = \{\phi; \phi: R^+ \to R\}$ such that (1) ϕ is Lebesgue integrable

$$(2)\ \epsilon > 0 \Longrightarrow \int_0^\epsilon \emptyset(t)dt > 0$$

Malhotra et.al.[7] proved a common fixed point theorem for four self maps on a fuzzy metric space which satisfy an inequality involving a function $\emptyset \in \Phi$.

Theorem 1.11: (Malhotra et.al.[7]) Let $(X, M,*)$ be a complete fuzzy metric space and let F, G, S and T are self–mapping of X. Let the pairs {F,S} and {G ,T} be owc . If there exists $q \in (0,1)$ such that

$$\int_0^{M(Fx,Gy,qt)} \varphi(t)dt \geq \int_0^{\min\{M(Sx,Ty,t),M(Sx,Fx,t),M(Gy,Ty,t),M(Fx,Ty,t),M(Gy,Sx,t)\}} \varphi(t)dt$$

for all $x, y \in X$ and for all $t > 0$, then there exists a unique point $w \in X$ such that $Fw = Sw = w$ and a unique point $z \in X$ such that $Gz = Tz = z$ Moreover , $z = w$, so that there is a unique common fixed point of F, G, S and T.

II. MAIN RESULT

In this section we introduce a special class of Lebesgue integrable function and use this notion to prove a fixed point theorem**.**

Definition 2.1: Let $\Psi = \{\psi: [0,1] \to [0,1]\}$ is Lebesgue integrable such that

$$\int_\alpha^\beta \psi(t)dt > 0 \text{ for } \alpha < \beta.$$

Theorem 2.2: Let $(X, M,*)$ be a complete fuzzy metric space and let F, G, S and T be self–mappings of X. Let the pairs (F, S) and (G , T) be owc . Suppose there exists $\psi \in \Psi$ and $q \in (0,1)$ such that

$$\int_0^{M(Fx,Gy,qt)} \psi(s)ds \geq \int_0^{\min\{M(Sx,Ty,t),M(Sx,Fx,t),M(Gy,Ty,t),M(Fx,Ty,t),M(Gy,Sx,t)\}} \psi(s)ds \quad (2.2.1)$$

for all $x, y \in X$ and for all $t > 0$.

Then F, G, S and T have a unique common fixed point.

Proof:
Since the pairs (F, S) and (G , T) are owc, there exist $x, y \in X$ such that $Fx = Sx$ and $Gy = Ty$.
Now we claim that $Fx = Gy$.

By (2.2.1)

$$\int_0^{M(Fx,Gy,qt)} \psi(s)ds \geq \int_0^{\min\{M(Sx,Ty,t),M(Sx,Fx,t),M(Gy,Ty,t),M(Fx,Ty,t),M(Gy,Sx,t)\}} \psi(s)ds$$

$$= \int_0^{\min\{M(Fx,Gy,t),M(Fx,Fx,t),M(Gy,Gy,t),M(Fx,Gy,t),M(Gy,Fx,t)\}} \psi(s)ds$$

$$= \int_0^{\min\{M(Fx,Gy,t),1,1,M(Fx,Gy,t),M(Gy,Fx,t)\}} \psi(s)ds$$

$$\Rightarrow \int_0^{M(Fx,Gy,qt)} \psi(s)ds \geq \int_0^{M(Fx,Gy,t)} \psi(s)ds$$

$$\Rightarrow \int_0^{M(Fx,Gy,t)} \psi(s)ds \geq \int_0^{M(Fx,Gy,qt)} \psi(s)ds \geq \int_0^{M(Fx,Gy,t)} \psi(s)ds$$

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International Journal of Innovative Research in Science, Engineering and Technology
Vol. 2, Issue 6, June 2013

$\Rightarrow \int_0^{M(Fx,Gy,qt)} \psi(s)ds = \int_0^{M(Fx,Gy,t)} \psi(s)ds = \int_0^{M(Fx,Gy,qt)} \psi(s)ds + \int_{M(Fx,Gy,qt)}^{M(Fx,Gy,t)} \psi(s)ds$

$\Rightarrow \int_{M(Fx,Gy,qt)}^{M(Fx,Gy,t)} \psi(s)ds = 0$

$\Rightarrow M(Fx,Gy,qt) = M(Fx,Gy,t)$ $(\because \psi \in \Psi)$

Hence it follows that $M(Fx,Gy,qt) = M(Fx,Gy,t)$ for every $t > 0$.

Consequently $Sx = Fx = Gy = Ty$ (2.2.2)
Suppose that $\exists\, z \in X \ni Fz = Sz$. Then following the above argument we get that
$Fz = Sz = Gy = Ty$.
$\therefore Fx = Fz$
Write $w = Fx = Sx$.
Since (F,S) is owc, $Fx = Sx \Rightarrow FSx = SFx \Rightarrow Fw = Sw$
Now replacing x by w in (2.1.1) following the argument mentioned above and we get
$Fw = Sw = Gy = Ty$.
$\therefore Fw = Sw = Gy = Ty = Fx = Sx$.

$\therefore Fw = Fx = w = Sx = Sw$.
$\Rightarrow Fw = w = Sw$.
Hence w is a common fixed point of F and G.
From (2.2.2) we have $Gy = Ty = w$,followingthe above argument follows that $Gw = Tw = w$.
Thus w is a common fixed point of F, S, G and T.

Uniqueness : Let $w\&v$ be $\ni Fw = Sw = Gw = Tw = w$ and $Fv = Sv = Gv = Tv = v$.

By (2.2.1)

$$\int_0^{M(Fw,Gv,qt)} \psi(s)ds \geq \int_0^{\min\{M(Sw,Tv,t),M(Sw,Fw,t),M(Gv,Tv,t),M(Fw,Tv,t),M(Gv,Sw,t)\}} \psi(s)ds$$

$\int_0^{M(w,v,qt)} \psi(s)ds \geq \int_0^{\min\{M(w,v,t),1,1M(w,v,t),M(v,w,t)\}} \psi(s)ds = \int_0^{M(w,v,t)} \psi(s)ds$

$\Rightarrow \int_0^{M(w,v,qt)} \psi(s)ds \geq \int_0^{M(w,v,t)} \psi(s)ds$.

$\Rightarrow \int_0^{M(w,v,t)} \psi(s)ds \geq \int_0^{M(w,v,qt)} \psi(s)ds \geq \int_0^{M(w,v,t)} \psi(s)ds$.

$\Rightarrow \int_0^{M(w,v,t)} \psi(s)ds = \int_0^{M(w,v,qt)} \psi(s)ds$
$\Rightarrow M(w,v,qt) = M(w,v,t)\ \forall\, t > 0$.
$\Rightarrow w = v$.
Hence common fixed point of F, G, S and T is unique.

Theorem 2.3: Let $(X, M,*)$ be a complete fuzzy metric space and let F, G, S and T be self–mapping of X. Let the pairs (F,S) and $(G\,,T)$ be owc . If there exists $q \in (0,1)$ such that

$\int_0^{M(Fx,Gy,qt)} \psi(s)ds \geq \int_0^{\phi(\min\{M(Sx,Ty,t),M(Sx,Fx,t),M(Gy,Ty,t),M(Fx,Ty,t),M(Gy,Sx,t)\})} \psi(s)ds$ (2.3.1)

for all $x, y \in X$ and $\phi: [0,1] \to [0,1]$ such that $\phi(t) \geq t$ for all $0 < t < 1$.
Then F, G, S and T have a unique common fixed point.

Proof : By (2.3.1)

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology
Vol. 2, Issue 6, June 2013

$$\int_0^{M(Fx,Gy,qt)} \psi(s)ds \geq \int_0^{\phi(\min\{M(Sx,Ty,t),M(Sx,Fx,t),M(Gy,Ty,t),M(Fx,Ty,t),M(Gy,Sx,t)\})} \psi(s)ds$$

$$= \int_0^{\phi(\min\{M(Fx,Gy,t),M(Fx,Fx,t),M(Gy,Gy,t),M(Fx,Gy,t),M(Gy,Fx,t)\})} \psi(s)ds$$

$$= \int_0^{\phi(\min\{M(Fx,Gy,t),1,1,M(Fx,Gy,t),M(Gy,Fx,t)\})} \psi(s)ds$$

$$= \int_0^{\phi(M(Fx,Gy,t))} \psi(s)ds$$

$$\Rightarrow \int_0^{M(Fx,Gy,qt)} \psi(s)ds \geq \int_0^{\phi(M(Fx,Gy,t))} \psi(s)ds \geq \int_0^{(M(Fx,Gy,t))} \psi(s)ds$$

$$\Rightarrow \int_0^{M(Fx,Gy,t)} \psi(s)ds \geq \int_0^{M(Fx,Gy,qt)} \psi(s)ds \geq \int_0^{M(Fx,Gy,t)} \psi(s)ds$$

$$\Rightarrow \int_0^{M(Fx,Gy,t)} \psi(s)ds = \int_0^{M(Fx,Gy,qt)} \psi(s)ds$$

$\Rightarrow M(Fx, Gy, qt) = M(Fx, Gy, t)$

$\therefore Fx = Gy$

Hence result follows from theorem (2.2)

Theorem 2.4: Let $(X, M,*)$ be a complete fuzzy metric space and let F, G, S and T be self–mapping of X. Let the pairs (F, S) and (G, T) be owc . If there exists $q \in (0,1)$ such that

$$\int_0^{M(Fx,Gy,qt)} \psi(s)ds \geq \int_0^{\phi(M(Sx,Ty,t),M(Sx,Fx,t),M(Gy,Ty,t),M(Fx,Ty,t),M(Gy,Sx,t))} \psi(s)ds \quad (2.4.1)$$

for all $x, y \in X$ and $\phi: [0,1]^5 \to [0,1]$ such that $\phi(t, 1,1, t, t) > t$ for all $0 < t < 1$.
Then F, G, S and T have a unique common fixed point.

Proof : By (2.4.1)

$$\int_0^{M(Fx,Gy,qt)} \psi(s)ds \geq \int_0^{\phi(M(Sx,Ty,t),M(Sx,Fx,t),M(Gy,Ty,t),M(Fx,Ty,t),M(Gy,Sx,t))} \psi(s)ds$$

$$= \int_0^{\phi(M(Fx,Gy,t),1,1,M(Fx,Gy,t),M(Gy,Fx,t))} \psi(s)ds$$

$$> \int_0^{M(Fx,Gy,t)} \psi(s)ds$$

$$\Rightarrow \int_0^{M(Fx,Gy,t)} \psi(s)ds \geq \int_0^{M(Fx,Gy,qt)} \psi(s)ds \geq \int_0^{M(Fx,Gy,t)} \psi(s)ds$$

$$\Rightarrow \int_0^{M(Fx,Gy,t)} \psi(s)ds = \int_0^{M(Fx,Gy,qt)} \psi(s)ds$$

$\Rightarrow M(Fx, Gy, qt) = M(Fx, Gy, t)$

$\therefore Fx = Gy$

Hence result follows from theorem (2.2)

Theorem 2.5: Let $(X, M,*)$ be a complete fuzzy metric space and let F, G, S and T are self–mapping of X. Let the pairs (F, S) and (G, T) be owc . Suppose there exists $q \in (0,1)$ such that

$$\int_0^{M(Fx,Gy,qt)} \psi(s)ds \geq \int_0^{\min\{M(Sx,Ty,t)*M(Fx,Sx,t),M(Gy,Ty,t)*M(Fx,Ty,t)\}} \psi(s)ds \quad (2.5.1)$$

for all $x, y \in X$. Then F, G, S and T have a unique common fixed point.

Proof: $\int_0^{M(Fx,Gy,qt)} \psi(s)ds \geq \int_0^{\min\{M(Sx,Ty,t)*M(Fx,Sx,t),M(Gy,Ty,t)*M(Fx,Ty,t)\}} \psi(s)ds$

$$= \int_0^{\min\{M(Fx,Gy,t)*1,1*M(Fx,Gy,t)\}} \psi(s)ds$$

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Vol. 2, Issue 6, June 2013

$$= \int_0^{M(Fx,Gy,t)} \psi(s)ds$$

$\Rightarrow \int_0^{M(Fx,Gy,qt)} \psi(s)ds \geq \int_0^{M(Fx,Gy,t)} \psi(s)ds$
$\Rightarrow M(Fx,Gy,qt) = M(Fx,Gy,t)$
$\therefore Fx = Gy$
Hence result follows from theorem (2.2)
Theorem 2.6: Let $(X,M,*)$ be a complete fuzzy metric space where $*$ is the min $t-$ norm and let F,G,S and T be self–mapping of X. Let the pairs (F,S) and (G,T) be owc . Suppose there exists $q \in (0,1)$ such that

$$\int_0^{M(Fx,Gy,qt)} \psi(s)ds \geq \int_0^{\min\{M(Sx,Ty,t)*M(Fx,Sx,t),M(Gy,Ty,t)*M(Gy,Sx,2t)*M(Fx,Ty,t)\}} \psi(s)ds \quad (2.6.1)$$

for all $x,y \in X$. Then F,G,S and T have a unique common fixed point.

Proof: By(2.6.1)

$$\int_0^{M(Fx,Gy,qt)} \psi(s)ds \geq \int_0^{\min\{M(Sx,Ty,t)*M(Fx,Sx,t),M(Gy,Ty,t)*M(Gy,Sx,2t)*M(Fx,Ty,t)\}} \psi(s)ds$$

$$= \int_0^{\min\{M(Fx,Gy,t)*M(Fx,Sx,t),M(Gy,Ty,t)*M(Sx,Ty,t)*M(Ty,Gy,t)*M(Fx,Ty,t)\}} \psi(s)ds$$

$$= \int_0^{\min\{M(Fx,Gy,t)*1,1*M(Fx,Gy,t)*1*M(Fx,Gy,t)\}} \psi(s)ds$$

$$\geq \int_0^{M(Fx,Gy,t)} \psi(s)ds$$

$\Rightarrow \int_0^{M(Fx,Gy,qt)} \psi(s)ds \geq \int_0^{M(Fx,Gy,t)} \psi(s)ds$

$\Rightarrow \int_0^{M(Fx,Gy,t)} \psi(s)ds \geq \int_0^{M(Fx,Gy,qt)} \psi(s)ds \geq \int_0^{M(Fx,Gy,t)} \psi(s)ds$

$\Rightarrow \int_0^{M(Fx,Gy,t)} \psi(s)ds = \int_0^{M(Fx,Gy,qt)} \psi(s)ds$
$\Rightarrow M(Fx,Gy,qt) = M(Fx,Gy,t)$
$\therefore Fx = Gy$
Hence result follows from theorem (2.2)

Theorem 2.7: Let $(X,M,*)$ be a complete fuzzy metric space and let F,G,S and T be self–mappings of X. Let the pairs (F,S) and (G,T) be owc . If there exists $q \in (0,1)$ such that

$$\int_0^{M(Sx,Sy,qt)} \psi(s)ds \geq \int_0^{\alpha M(Fx,Fy,t)+\beta \min\{M(Fx,Fy,t),M(Sx,Fx,t),M(Sy,Fy,t)\}} \psi(s)ds \quad (2.7.1)$$

for all $x,y \in X$ and $t > 0$ where $\alpha,\beta > 0, \alpha+\beta \geq 1$, then F,G,S and T have a unique common fixed point.
Proof : Let the pair (F,S) be owc.

$\Rightarrow \exists\, x \in X \ni Fx = Sx.$
Suppose $\exists\, y \in X \ni Fy = Sy.$

$$\int_0^{M(Sx,Sy,qt)} \psi(s)ds \geq \int_0^{\alpha M(Fx,Fy,t)+\beta \min\{M(Fx,Fy,t),M(Sx,Fx,t),M(Sy,Fy,t)\}} \psi(s)ds$$

$$= \int_0^{\alpha M(Sx,Sy,t)+\beta \min\{M(Sx,Sy,t),1,M(Sx,Sy,t)\}} \psi(s)ds$$

$$= \int_0^{\alpha M(Sx,Sy,t)+\beta M(Sx,Sy,t)} \psi(s)ds$$

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International Journal of Innovative Research in Science, Engineering and Technology
Vol. 2, Issue 6, June 2013

$$= \int_0^{(\alpha+\beta)M(Sx,Sy,t)} \psi(s)ds$$

$$\int_0^{M(Sx,Sy,qt)} \psi(s)ds \geq \int_0^{(\alpha+\beta)M(Sx,Sy,t)} \psi(s)ds > \int_0^{(\alpha+\beta)M(Sx,Sy,t)} \psi(s)ds$$

$\Rightarrow \int_0^{M(Sx,Sy,qt)} \psi(s)ds \geq \int_0^{M(Sx,Sy,t)} \psi(s)ds$
$\Rightarrow M(Sx,Sy,qt) = M(Sx,Sy,t)$
$\therefore Sx = Sy$
$\therefore Fx = Fy$
$\Rightarrow F$ and S have a unique common fixed point.

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