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COMMON UNIQUE FIXED POINT THEOREM FOR RANDOM OPERATORS IN HILBERT SPACE

Bijendra Singh[1], G.P.S Rathore[2], Priyanka Dubey[3], Naval Singh[4]

Professor & Dean, School of Studies in Mathematics,Vikram University, Ujjain(M.P), India[1]

Sr.Scientist, K.N.K Horticulture College, Mandsaur(M.P) India[2]

Asst.Professor, Bansal institute of Research &Technology, Bhopal(M.P) India[3]

Asst.Professor, Govt.Science and Commerce College,Benazeer,Bhopal,(M.P) India[4]

Abstract –: The object of this paper is to obtain a common fixed point theorem for four continuous random operators by considering a sequence of measurable functions satisfying certain contractive condition in separable Hilbert space.

Mathematics Subject Classification: 54H25,47H10.

Keywords : Separable Hilbert Space, random operators, common random fixed point, rational inequality.

I. INTRODUCTION

The study of the random fixed point theorems in abstract spaces is initiated by Spacek [1] and Hans [2] and are the stochastic generalizations of the classical fixed point theorems in separable Banach spaces.The research along this line gained momentum after the publication of the paper by Bharucha-Reid [3] and since then several random fixed point theorems have been proved in the literature.. Random operator theory is needed for the study of various classes of random equations. Now this theory has become the full fledged research area and various ideas associated with random fixed point theory are used to obtain the solution of non linear random system [4 ,5].The study of the random fixed point theory has attracted much attention in recent years[6,7,8]. These results extend the corresponding result in [9].
In this paper we construct a sequence of measurable functions and consider its convergence to the common unique random fixed point of four continuous random operators defined on a non-empty closed subset of a separable Hilbert space. For the purpose of obtaining the random fixed point of the four continuous random operator, we have used a rational inequality and the parallelogram law.

II. PRELIMINARIES

Throughout this paper (Ω,Σ) denotes a measurable space consisting of a set Ω and sigma algebra Σ of subsets of Ω .H stands for a separable Hilbert space, and C is a non-empty closed subset of H.

Definition 2.1: A function $f:\Omega\to C$ is said to be measurable if $f^{-1}(B\cap C)\in\Sigma$ for every Borel subset B of H.

Definition 2.2 **:** A function $F:\Omega\times C\to C$ is said to be random operator if $F(.,x):\Omega\to C$ is measurable for every $x\in C$.

Definition 2.3 **:** A measurable function $g:\Omega\to C$ is said to be random fixed point of the random operator $F:\Omega\times C\to C$ if $F(t,g(t))=g(t)$ for all $t\in\Omega$.

Definition 2.4 **:** A random operator $F:\Omega\times C\to C$ is said to be continuous if for fixed $t\in\Omega, F(t,.):C\to C$ is continuous.

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Condition (A).Four mappings $E, F, S, T, S: C \to C$, where C is a non-empty closed subset of a Hilbert space H ,is satisfy condition (A) if

$$ES = SE, FT = TF, E(H) \subset T(H) \text{ and } F(H) \subset S(H) \quad \ldots(1)$$

$$\|Ex - Fy\|^2 \le \beta_1 \frac{\|Sx - Ex\|^2[\|Ty - Fy\|^2 + \|Ex - Ty\|^2]}{\|Sx - Ty\|^2 + \|Ex - Ty\|^2} + \beta_2 \frac{\|Ex - Ty\|^2[\|Sx - Ex\|^2 + \|Ty - Fy\|^2]}{\|Sx - Ty\|^2 + \|Ex - Ty\|^2}$$
$$+ \frac{\beta_3 \|Sx - Ex\|^2 \|Ty - Fy\|^2}{\|Sx - Ty\|^2} + \beta_4 \frac{\|Ty - Fy\|^2[1 + \|Sx - Ex\|^2]}{1 + \|Sx - Ty\|^2}$$
$$+ \beta_5 \frac{\|Sx - Fy\|^2[\|Sx - Ex\|^2 + \|Ty - Fy\|^2]}{\|Sx - Fy\|^2 + \|Ex - Ty\|^2} + \beta_6 \frac{\|Sx - Ty\|^2[\|Sx - Ex\|^2 + \|Ty - Fy\|^2]}{\|Sx - Ty\|^2}$$
$$+ \beta_7[\|Sx - Ex\|^2 + \|Ty - Fy\|^2] + \beta_8 \|Sx - Ty\|^2 . \quad \ldots(2)$$

for each $x, y \in C$, with $Sx \ne Ty$ and $\|Sx - Ty\|^2 + \|Ex - Ty\|^2 \ne 0$. $\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6, \beta_7, \beta_8$ being positive real number and

$$\beta_1 + \beta_3 + \beta_4 + 2(\beta_5 + \beta_6 + \beta_7) + \beta_8 < 1 \quad \ldots(3)$$

III. MAIN THEOREM

Let C be a non-empty closed subset of a separable Hilbert space H .Let E, F, S, T be four continuous random operators defined on C such that for $t \in \Omega, E(t,.), F(t,.), T(t,.), S(t,.): C \to C$ satisfy condition (A).Then E, F, T and S have a common unique random fixed point in C .

Proof: Let the function $g_0 : \Omega \to C$ be arbitrary measurable function. By (1),there exists a function $g_1 : \Omega \to C$ such that $T(t, g_1(t)) = E(t, g_0(t))$ for $t \in \Omega$ and for this function $g_1 : \Omega \to C$, we can choose another function $g_2 : \Omega \to C$ such that $F(t, g_1(t)) = S(t, g_2(t))$ for $t \in \Omega$,and so on. Inductively, we can define a sequence of functions for $t \in \Omega$, $\{y_n(t)\}$ such that

$$y_{2n}(t) = T(t, g_{2n+1}(t)) = E(t, g_{2n}(t)) \quad \text{and} \quad y_{2n+1}(t) = S(t, g_{2n+2}(t)) = F(t, g_{2n+1}(t)) \quad \ldots(4)$$

for $t \in \Omega$ and n = 0,1,2,3…

From condition (A),

Now consider for $t \in \Omega$.

$$\|y_{2n}(t) - y_{2n+1}(t)\|^2 = \|E(t, g_{2n}(t) - F(t, g_{2n+1}(t)\|^2$$
$$\le \beta_1 \frac{\|S(t, g_{2n}(t)) - E(t, g_{2n}(t))\|^2[\|T(t, g_{2n+1}(t)) - F(t, g_{2n+1}(t))\|^2 + \|E(t, g_{2n}(t)) - T(t, g_{2n+1}(t))\|^2]}{\|S(t, g_{2n}(t)) - T(t, g_{2n+1}(t))\|^2 + \|E(t, g_{2n}(t)) - T(t, g_{2n+1}(t))\|^2}$$
$$+ \beta_2 \frac{\|E(t, g_{2n}(t)) - T(t, g_{2n+1}(t))\|^2[\|S(t, g_{2n}(t)) - E(t, g_{2n}(t))\|^2 + \|T(t, g_{2n+1}(t)) - F(t, g_{2n+1}(t))\|^2]}{\|S(t, g_{2n}(t)) - T(t, g_{2n+1}(t))\|^2 + \|E(t, g_{2n}(t)) - T(t, g_{2n+1}(t))\|^2}$$

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$$+\beta_3 \frac{\|T(t,g_{2n+1}(t))-F(t,g_{2n+1}(t))\|^2 \|S(t,g_{2n}(t))-E(t,g_{2n}(t))\|^2}{\|S(t,g_{2n}(t))-T(t,g_{2n+1}(t))\|^2}$$

$$+\beta_4 \frac{\|T(t,g_{2n+1}(t))-F(t,g_{2n+1}(t))\|^2 [1+\|S(t,g_{2n}(t))-E(t,g_{2n}(t))\|^2]}{1+\|S(t,g_{2n}(t))-T(t,g_{2n+1}(t))\|^2}$$

$$+\beta_5 \frac{\|S(t,g_{2n}(t))-F(t,g_{2n+1}(t))\|^2 [\|S(t,g_{2n}(t))-E(t,g_{2n}(t))\|^2+\|T(t,g_{2n+1}(t))-F(t,g_{2n+1}(t))\|^2]}{\|S(t,g_{2n}(t))-F(t,g_{2n+1}(t))\|^2+\|E(t,g_{2n}(t))-T(t,g_{2n+1}(t))\|^2}$$

$$+\beta_6 \frac{\|S(t,g_{2n}(t))-T(t,g_{2n+1}(t))\|^2 [\|S(t,g_{2n}(t))-E(t,g_{2n}(t))\|^2+\|T(t,g_{2n+1}(t))-F(t,g_{2n+1}(t))\|^2]}{\|S(t,g_{2n}(t))-T(t,g_{2n+1}(t))\|^2}$$

$$+\beta_7 [\|S(t,g_{2n}(t))-E(t,g_{2n}(t))\|^2+\|T(t,g_{2n+1}(t))-F(t,g_{2n+1}(t))\|^2]$$

$$+\beta_8 \|S(t,g_{2n}(t))-T(t,g_{2n+1}(t))\|^2 \qquad \text{(by 2)}$$

$$\le \beta_1 \frac{\|y_{2n-1}(t)-y_{2n}(t)\|^2 [\|y_{2n}(t)-y_{2n+1}(t)\|^2+\|y_{2n}(t)-y_{2n}(t)\|^2]}{\|y_{2n-1}(t)-y_{2n}(t)\|^2+\|y_{2n}(t)-y_{2n}(t)\|^2}$$

$$+\beta_2 \frac{\|y_{2n}(t)-y_{2n}(t)\|^2 [\|y_{2n-1}(t)-y_{2n}(t)\|^2+\|y_{2n}(t)-y_{2n+1}(t)\|^2]}{\|y_{2n-1}(t)-y_{2n}(t)\|^2+\|y_{2n}(t)-y_{2n}(t)\|^2}$$

$$+\frac{\beta_3 \|y_{2n}(t)-y_{2n+1}(t)\|^2 \|y_{2n-1}(t)-y_{2n}(t)\|^2}{\|y_{2n-1}(t)-y_{2n}(t)\|^2}$$

$$+\beta_4 \frac{\|y_{2n}(t)-y_{2n+1}(t)\|^2 [1+\|y_{2n-1}(t)-y_{2n}(t)\|^2]}{1+\|y_{2n-1}(t)-y_{2n}(t)\|^2}$$

$$+\beta_5 \frac{\|y_{2n-1}(t)-y_{2n+1}(t)\|^2 [\|y_{2n-1}(t)-y_{2n}(t)\|^2+\|y_{2n}(t)-y_{2n+1}(t)\|^2]}{\|y_{2n-1}(t)-y_{2n+1}(t)\|^2+\|y_{2n}(t)-y_{2n}(t)\|^2}$$

$$+\beta_6 \frac{\|y_{2n-1}(t)-y_{2n}(t)\|^2 [\|y_{2n-1}(t)-y_{2n}(t)\|^2+\|y_{2n}(t)-y_{2n+1}(t)\|^2]}{\|y_{2n-1}(t)-y_{2n}(t)\|^2}$$

$$+\beta_7 [\|y_{2n-1}(t)-y_{2n}(t)\|^2+\|y_{2n}(t)-y_{2n+1}(t)\|^2]$$

$$+\beta_8 \|y_{2n-1}(t)-y_{2n}(t)\|^2. \qquad \text{(by 4)}$$

$$\|y_{2n}(t)-y_{2n+1}(t)\|^2 \le (\beta_1+\beta_3+\beta_4+\beta_5+\beta_6+\beta_7)\|y_{2n}(t)-y_{2n+1}(t)\|^2$$
$$(\beta_5+\beta_6+\beta_7+\beta_8)\|y_{2n-1}(t)-y_{2n}(t)\|^2$$

$$[1-(\beta_1+\beta_3+\beta_4+\beta_5+\beta_6+\beta_7)]\|y_{2n}(t)-y_{2n+1}(t)\|^2 \le (\beta_5+\beta_6+\beta_7+\beta_8)\|y_{2n-1}(t)-y_{2n}(t)\|^2$$

$$\|y_{2n}(t)-y_{2n+1}(t)\|^2 \le \frac{(\beta_5+\beta_6+\beta_7+\beta_8)}{[1-(\beta_1+\beta_3+\beta_4+\beta_5+\beta_6+\beta_7)]}\|y_{2n-1}(t)-y_{2n}(t)\|^2$$

$$\|y_{2n}(t)-y_{2n+1}(t)\| \le \left[\frac{(\beta_5+\beta_6+\beta_7+\beta_8)}{[1-(\beta_1+\beta_3+\beta_4+\beta_5+\beta_6+\beta_7)]}\right]^{1/2}\|y_{2n-1}(t)-y_{2n}(t)\|$$

$$\|y_{2n}(t)-y_{2n+1}(t)\| \le k\|y_{2n-1}(t)-y_{2n}(t)\|$$

Where $k=\left[\frac{(\beta_5+\beta_6+\beta_7+\beta_8)}{[1-(\beta_1+\beta_3+\beta_4+\beta_5+\beta_6+\beta_7)]}\right]^{1/2} < 1.$ (by 3)

Replacing 2n by n

$$\|y_n(t)-y_{n+1}(t)\| \le k\|y_{n-1}(t)-y_n(t)\|$$

On further reducing

$$\|y_n(t)-y_{n+1}(t)\| \le k^n\|y_0(t)-y_1(t)\| \quad \text{for all } t\in\Omega. \qquad \ldots(5)$$

Now we shall prove for $t\in\Omega$, $\{y_n(t)\}$ is a Cauchy sequence. For this for every positive integer p we have, $t\in\Omega$.

$$\|y_n(t)-y_{n+p}(t)\| = \|y_n(t)-y_{n+1}(t)+y_{n+1}(t)-y_{n+2}(t)+y_{n+2}(t)-\ldots+y_{n+p-1}(t)-y_{n+p}(t)\|$$
$$\le \|y_n(t)-y_{n+1}(t)\|+\|y_{n+1}(t)-y_{n+2}(t)\|+\ldots+\|y_{n+p-1}(t)-y_{n+p}(t)\|$$
$$\le [k^n+k^{n+1}+\ldots+k^{n+p-1}]\|y_0(t)-y_1(t)\| \quad \text{by (5)}$$
$$= k^n[1+k+k^2+\ldots+k^{p-1}]\|y_0(t)-y_1(t)\|$$
$$\le \frac{k^n}{(1-k)}\|y_0(t)-y_1(t)\| \quad \text{for all } t\in\Omega.$$

$$\Rightarrow \|y_n(t)-y_{n+p}(t)\| \to 0 \text{ as } n\to\infty \text{ for } t\in\Omega. \qquad \ldots(6)$$

From eq (6) ,it follows that for $t\in\Omega$, $\{y_n(t)\}$ is a cauchy sequence and hence is convergent in closed subset C of Hilbert space H.

For $t\in\Omega$, let $\{y_n(t)\}\to y(t)$ as $n\to\infty$. …(7)

Since C is closed, g is a function from C to C .and consequently the subsequences $\{E(t,g_{2n}(t))\}$, $\{F(t,g_{2n+1}(t))\}$, $\{T(t,g_{2n+1}(t))\}$ and $\{S(t,g_{2n+2}(t))\}$ of $\{y_n(t)\}$ for $t\in\Omega$ also converges to the $\{y(t)\}$ and continuity of E,F,T,S gives

$$E(t,S(t,g_n(t))\to E(t,y(t))$$
$$S(t,E(t,g_n(t))\to S(t,y(t))$$
$$F(t,T(t,g_n(t))\to F(t,y(t))$$

and

$$T(t,S(t,g_n(t))\to T(t,y(t))$$

$$E(t,y(t))=S(t,y(t)),F(t,y(t))=T(t,y(t)) \quad \text{for } t\in\Omega \text{ ,from (1)} \qquad \ldots(8)$$

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IV. EXISTENCE OF RANDOM FIXED POINT

Consider for $t \in \Omega$.

$$\|E(t,y(t))-y(t)\|^2 = \|E(t,y(t))-y_{2n+1}(t)+y_{2n+1}(t)-y(t)\|^2$$

$$\le 2\|E(t,y(t))-y_{2n+1}(t))\|^2+2\|y_{2n+1}(t)-y(t)\|^2$$

$$\le 2\|E(t,y(t))-F(t,g_{2n+1}(t))\|^2+2\|y_{2n+1}(t)-y(t)\|^2 \quad ;\ \text{(by (4))}$$

[By Pareallelogram law $\|x+y\|^2 \le 2\|x\|^2+\|y\|^2$.]

$$\le 2\beta_1 \frac{\|S(t,y(t))-E(t,y(t))\|^2[\|T(t,g_{2n+1}(t))-F(t,g_{2n+1}(t))\|^2+\|E(t,y(t))-T(t,g_{2n+1}(t)\|^2]}{\|S(t,y(t))-T(t,g_{2n+1}(t)\|^2+\|E(t,y(t))-T(t,g_{2n+1}(t))\|^2}$$

$$+2\beta_2 \frac{\|E(t,y(t))-T(t,g_{2n+1}(t))\|^2[\|S(t,y(t))-E(t,y(t))\|^2+\|T(t,g_{2n+1}(t)-F(t,g_{2n+1}(t)\|^2]}{\|S(t,y(t))-T(t,g_{2n+1}(t)\|^2+\|E(t,y(t))-T(t,g_{2n+1}(t))\|^2}$$

$$+2\beta_3 \frac{\|T(t,g_{2n+1}(t))-F(t,g_{2n+1}(t))\|^2\|S(t,y(t))-E(t,y(t))\|^2}{\|S(t,y(t))-T(t,g_{2n+1}(t)\|^2}$$

$$+2\beta_4 \frac{\|T(t,g_{2n+1}(t))-F(t,g_{2n+1}(t))\|^2[1+\|S(t,y(t))-E(t,y(t))\|^2]}{1+\|S(t,y(t))-T(t,g_{2n+1}(t)\|^2}$$

$$+2\beta_5 \frac{\|S(t,y(t))-F(t,g_{2n+1}(t))\|^2[\|S(t,y(t))-E(t,y(t))\|^2+\|T(t,g_{2n+1}(t))-F(t,g_{2n+1}(t))\|^2]}{\|S(t,y(t))-F(t,g_{2n+1}(t))\|^2+\|E(t,y(t))-T(t,g_{2n+1}(t))\|^2}$$

$$+2\beta_6 \frac{\|S(t,y(t))-T(t,g_{2n+1}(t))\|^2[\|S(t,y(t))-E(t,y(t))\|^2+\|T(t,g_{2n+1}(t)-F(t,g_{2n+1}(t)\|^2]}{\|S(t,y(t))-T(t,g_{2n+1}(t)\|^2}$$

$$+2\beta_7[\|S(t,y(t))-E(t,y(t))\|^2+\|T(t,g_{2n+1}(t))-F(t,g_{2n+1}(t)\|^2$$

$$+2\beta_8\|S(t,y(t))-T(t,g_{2n+1}(t)\|^2+2\|y_{2n+1}(t)-y(t)\|^2 \quad \text{(by 2)}$$

$$\le 2\beta_1 \frac{\|S(t,y(t))-E(t,y(t))\|^2[\|y(t)-y(t)\|^2+\|E(t,y(t))-y(t)\|^2]}{\|S(t,y(t))-y(t)\|^2+\|E(t,y(t))-y(t))\|^2}$$

$$+2\beta_2 \frac{\|E(t,y(t))-y(t))\|^2[\|S(t,y(t))-E(t,y(t))\|^2+\|y(t)-y(t)\|^2]}{\|S(t,y(t))-y(t)\|^2+\|E(t,y(t))-y(t))\|^2}$$

$$+2\beta_3 \frac{\|y(t)-y(t)\|^2\|S(t,y(t))-E(t,y(t))\|^2}{\|S(t,y(t))-y(t)\|^2}$$

$$+2\beta_4 \frac{\|y(t)-y(t)\|^2[1+\|S(t,y(t))-E(t,y(t))\|^2]}{1+\|S(t,y(t))-y(t)\|^2}$$

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$$+2\beta_5 \frac{\|S(t,y(t))-y(t)\|^2[\|S(t,y(t))-E(t,y(t))\|^2+\|y(t)-y(t)\|^2]}{\|S(t,y(t)-y(t)\|^2+\|E(t,y(t))-y(t)\|^2}$$

$$+2\beta_6 \frac{\|S(t,y(t))-y(t)\|^2[\|S(t,y(t))-E(t,y(t))\|^2+\|y(t)-y(t)\|^2]}{\|S(t,y(t))-y(t)\|^2}$$

$$+2\beta_7[\|S(t,y(t))-E(t,y(t))\|^2+\|y(t)-y(t)\|^2]$$

$$+2\beta_8\|S(t,y(t))-y(t)\|^2+2\|y(t)-y(t)\|^2$$

Therefore for $t\in\Omega$

$$\|E(t,y(t))-y(t)\|^2 \leq 2\beta_8\|S(t,y(t))-y(t)\|^2$$

$$(1-2\beta_8)\ \|E(t,y(t))-y(t)\|^2 \leq 0$$

$$\|E(t,y(t))-y(t)\|^2=0 \qquad (\because \beta_8<\frac{1}{2})$$

$$E(t,y(t))=y(t) \qquad \text{for } t\in\Omega. \qquad \ldots(9)$$

From eq (8) and (9)

$$E(t,y(t))=y(t)=S(t,y(t)) \qquad \ldots(10)$$

In an exactly similar way we can prove for all $t\in\Omega$.

$$F(t,y(t))=y(t)=T(t,y(t)) \qquad \ldots(11)$$

Again ,if $A:\Omega\times C\to C$ is a continuous random operator on a non-empty subset C of a seperable Hilbert space *H*, then for any measurable function $f:\Omega\to C$,the function $h(t)=A(t,f(t))$ is also measurable [10].

It follows from the construction of $\{y_n(t)\}$ (by(4)) and the above consideration that $\{y_n(t)\}$ is a sequence of measurable functions.From (7),it follows that $y(t)$ for $t\in\Omega$ is also a measurable function.This fact along with (10) and (11) shows that $g:\Omega\to C$ is a common random fixed point of E,F,S ,T .

V. UNIQUENESS

Let $h:\Omega\to C$ be another random fixed point common to E,F,T and S ,that is for $t\in\Omega$, $E(t,h(t))=h(t)$, $F(t,h(t))=h(t)$, $T(t,h(t))=h(t)$, $S(t,h(t))=h(t)$ …(12)

Then for $t\in\Omega$,

$$\|g(t)-h(t)\|^2=\|E(t,g(t)-F(t,h(t)\|^2$$

$$\leq \beta_1 \frac{\|S(t,g(t))-E(t,g(t))\|^2[\|T(t,h(t))-F(t,h(t))\|^2+\|E(t,g(t))-T(t,h(t))\|^2]}{\|S(t,g(t))-T(t,h(t))\|^2+\|E(t,g(t))-T(t,h(t))\|^2}$$

$$+\beta_2 \frac{\|E(t,g(t))-T(t,h(t))\|^2[\|S(t,g(t))-E(t,g(t))\|^2+\|T(t,h(t))-F(t,h(t))\|^2]}{\|S(t,g(t))-T(t,h(t))\|^2+\|E(t,g(t))-T(t,h(t))\|^2}$$

$$+\beta_3 \frac{\|S(t,g(t))-E(t,g(t))\|^2\|T(t,h(t))-F(t,h(t))\|^2}{\|S(t,g(t))-T(t,h(t))\|^2}$$

$$+\beta_4 \frac{\|T(t,h(t))-F(t,h(t))\|^2[1+\|S(t,g(t))-E(t,g(t))\|^2]}{1+\|S(t,g(t))-T(t,h(t))\|^2}$$

$$+\beta_5 \frac{\|S(t,g(t))-F(t,h(t))\|^2[\|S(t,g(t))-E(t,g(t))\|^2+\|T(t,h(t))-F(t,h(t))\|^2]}{\|S(t,g(t)-F(t,h(t))\|^2+\|E(t,g(t))-T(t,h(t))\|^2}$$

$$+\beta_6 \frac{\|S(t,g(t))-T(t,h(t))\|^2[\|S(t,g(t))-E(t,g(t))\|^2+\|T(t,h(t))-F(t,h(t))\|^2]}{\|S(t,g(t))-T(t,h(t))\|^2}$$

$$+\beta_7[\|S(t,g(t))-E(t,g(t))\|^2+\|T(t,h(t))-F(t,h(t))\|^2]$$

$$+\beta_8\|S(t,g(t))-T(t,h(t))\|^2$$

$$\Rightarrow \|g(t)-h(t)\|^2 \le \beta_8\|g(t)-h(t)\|^2 \quad \text{(by 12)}$$

$$\Rightarrow (1-\beta_8)\|g(t)-h(t)\|^2 \le 0$$

$$\because \beta_8 < 1$$

$$\Rightarrow g(t)=h(t) \text{ for all } t\in\Omega.$$

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