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Current Distribution of Dipole Antenna for Different Lengths Using Different Types of Basis Functions Applying To Method of Moment

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ABSTRACT: In this paper, the generation of current distribution of a planar dipole antenna for different length of the antenna using different types of basic functions applying Method Of Moment (MOM) is discussed. Here, we give a brief description on MOM for generating the current distribution of the dipole antenna over the entire length of the antenna so that we became enable to calculate the self impedance of the dipole antenna. The calculation starts from the basic Pocklingtons' Integral Equation (PIE) of Greens' function. The basic functions are chosen as the rooftop function and the pulse function and the further progress has been done.

KEYWORDS: Current Distribution; Dipole Antenna; Length; Method Of Moments; Basic Function

I. INTRODUCTION

Numerical methods for solving electromagnetic problems are most common techniques which are being used during the last decades, specially with the speedy growing inventions of fast computers and powerful software. The Method Of Moments (MOM) is a powerful computational method in solving linear partial differential equations which are in the form of integral equations. This method gives the solution for the induced current which has been formulated as integral where the integrand is the current density(unknown).In this paper, a approach for fast efficient algorithm in solving the famous Hallens' and Pocklingtons' integral equations, regarding the current waveform distribution on a finite-length thin wire antenna is attempted. In order to solve this aim, the Method of Moments (MOM) which is a powerful computational technique for solving integral equations is applied. The aim of this paper is to solve the time harmonic HE and PE for unknown current matrix by applying Galerkin method and to find the waveform of the current over the thin wire structure.

II. RELATED WORK

In [1], the point matching method is introduced where the POCKLINGTON"s INTEGRAL EQUATION(PIE) & HALLEN"s INTEGRAL EQUATION(HIE) is made equal. We can modify the integral equation into the matrix form after formulating the problem in terms of Hallen"s integral equation and Pocklington's integral equations. For this purpose, N equal number of segmentation are done & applying the point matching form of MOM. In [4], for solution of matrix equations, Gaussian elimination, LU decomposition ,Condition Numbr method and Iterative Method has been used. The Iterative Method is applied in four different way namely, Conjugate Gradient, Bi-conjugate Gradient, Conjugate Gradient Squared, Bi-conjugate Gradient Stabilized and Stopping Criteria. In [3] , two types of current distribution is generated for Galerkins and for point matching solution. Galerkins method is used when a similar function is used for both the basis and weighting functions. Here, the Entire-domain cosine function has been used by the author for comparing the results for both PIE and HIE . The Dirac-delta function as the weighting function & the pulse function is used as basis function in order to simplify HIE and PIE for transforming them to the matrix formation which are very easy to implement in MATLAB. In [16] , Sub-sectional basis functions have been described. The most frequent choice is a method of moment with sub-sectional basis functions. The most commonly used shapes for the elementary cells are the triangle and rectangular. Even though the triangle shapes are most flexible, rectangular cells

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involve simpler calculations. Three types of test functions are described by the author, namely rooftop functions with Galerkin, rooftop function with razor testing and two- dimensional pulses and point matching.[2]

II. CURRENT DISTRIBUTION USING METHOD OF MOMENTS

The Pocklington's integral equation is stated as:

$$j/\omega\varepsilon \int_{-l/2}^{l/2}\left[k^2+\frac{\partial^2}{\partial z^2}\right] I_z(Z')G(Z,Z')dZ' = -\frac{V_i}{\Delta Z} \quad [8][4] \qquad (1)$$

where, G(Z,Z')is the Greens' function and is given by $\frac{e^{jkR}}{4\pi R}$ $R=\sqrt{4(a\sin\frac{\emptyset}{2})^2 + (Z-Z')^2}$ [4][8][10]

' a' is the radius. The current $I_z(z')$ is defined across the length of the antenna from Z' =- $l/2$ *to* $Z = l/2$.The kernel $[k^2+\delta^2/\delta z^2]$ represents the wave equation differential operator on the free space Greens' function . The constant 'k' denotes the free space wave number. 'ΔZ' is the feed gap and V_i is the excitation function. 'ϵ' is the permittivity and 'ω' is the angular frequency.

We can express the exponential function in a series form like,

$$e^{jkR} = \sum_{q=0}^{\infty}\frac{(jkR)^q}{q!} = 1 + jkR - \frac{k^2.R^2}{2} + \frac{i.k^3.R^3}{6} - \cdots \quad [6][7]$$

So, the Greens' function can be written in the form

$$G(Z,Z') = \frac{1}{4\pi}\sum_{q=-1}^{\infty}\frac{(jk)^{q+1}.R^q}{(q+1)!} = \frac{1}{4\pi}\left(\frac{1}{R} + \frac{jk}{1} - \frac{k^2.R}{2} + \frac{i.k^2.R^2}{6} - \ldots.\right) \quad [6][11][7]$$

$$G(Z,Z') = \frac{1}{4.\pi}.\frac{(e^{jkR}-1)}{R} + \frac{k^2.R}{8.\pi} \qquad (2)$$

from equation (1),

$$\left[\frac{\partial^2}{\partial R^2} + k^2\right]\left(\frac{1}{4.\pi}.\frac{(e^{jkR}-1)}{R} + \frac{k^2.R}{8.\pi}\right)$$

$$= \left[\frac{\partial^2}{\partial R^2} + k^2\right]\left(\frac{1}{4.\pi}.\frac{(e^{jkR}-1)}{R}\right) + \left[\frac{\partial^2}{\partial R^2} + k^2\right]\left(\frac{k^2.R}{8.\pi}\right)$$

$$= \left[\frac{\partial^2}{\partial R^2} + k^2\right]\left(\frac{1}{4.\pi}.\frac{e^{jkR}}{R}\right) - \left[\frac{\partial^2}{\partial R^2} + k^2\right]\left(\frac{1}{4.\pi.R}\right) + \left[\frac{\partial^2}{\partial R^2} + k^2\right]\left(\frac{k^2.R}{8.\pi}\right)$$

$$= \frac{\partial^2}{\partial R^2}\left(\frac{1}{4.\pi}.\frac{e^{jkR}}{R}\right) + \left(\frac{k^2}{4.\pi}.\frac{e^{jkR}}{R}\right) - \frac{1}{2.\pi.R^3} + \frac{k^2}{4.\pi}.\frac{1}{R} + \frac{k^4.R}{8.\pi}$$

$$= \frac{\partial}{\partial R^2}\left[\frac{4.\pi.R.e^{jkR}.jk.-e^{jkR}.4.\pi}{(4\pi R)^2}\right]$$

$$= \frac{\partial}{\partial R}\left[\frac{e^{jkR}.jk}{4.\pi.R}\right] - \frac{\partial}{\partial R}\left[\frac{e^{jkR}}{4.\pi.R^2}\right]$$

$$= \frac{jk}{4\pi}\left[\frac{R.e^{jkR}.jk - e^{jkR}}{R^2}\right] + \frac{1}{4\pi}\left[\frac{R^2.jk\,e^{jkR} - 2R.e^{jkR}}{R^4}\right] + \left(\frac{k^2}{4.\pi}.\frac{e^{jkR}}{R}\right) - \frac{1}{2.\pi.R^3} + \frac{k^2}{4.\pi}.\frac{1}{R} + \frac{k^4.R}{8.\pi}$$

Now, neglecting the higher negative terms of R, we took $\left(\frac{k^2}{4.\pi}.\frac{1}{R} + \frac{k^4.R}{8.\pi}\right)$ as our integrating factor.

III. METHODOLOGY USING BASIS FUNCTIONS

i) For this particular, we choose rooftop function as the basis function

Now consider the current

$$I_z(Z') = \sum_{n=1}^{2N} b_n \wedge \qquad (3)$$

where, 'b_n' is an unknown constant & '∧' is the rooftop function.

'n' is the variable.

For the n[th] term, the rooftop function can be defined as below:

at centre: $Z'_{cn} = -\frac{l}{2} + \frac{ln}{2N+1}$ (4)

at low end: $Z'_{ln} = -\frac{l}{2} + \frac{l(n-1)}{2N+1}$ (5)

at upper end: $Z'_{un} = -\frac{l}{2} + \frac{l(n+1)}{2N+1}$ (6)

So, the integration is done between two segments:

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1) between lower end to centre i.e. $\frac{(Z'-Z'_{ln})}{l/(2N+1)}$; $Z'_{ln} \le Z' \le Z'_{cn}$

2) between centre to upper end i.e. $\frac{(-Z'+Z'_{un})}{l/(2N+1)}$; $Z'_{cn} \le Z' \le Z'_{un}$

Thus, for the 1st segment combining equations (4) & (5),we get;

$$\int_{Z'_{ln}}^{Z'_{cn}} \sum_{n=1}^{2N} b_n \frac{k^2}{4.\pi} . \frac{1}{R} + \frac{k^4.R}{8.\pi} \frac{(Z'-Z'_{ln})}{l/(2N+1)} dZ' = -j\omega\varepsilon E_z^i(\rho = a) \qquad (7)$$

Thus, for the 2nd segment combining equations (4) & (6),we get;

$$\int_{Z'_{cn}}^{Z'_{un}} \sum_{n=1}^{2N} b_n \frac{k^2}{4.\pi} . \frac{1}{R} + \frac{k^4.R}{8.\pi} \frac{(-Z+Z'_{un}{}')}{l/(2N+1)} dZ' = -j\omega\varepsilon E_z^i(\rho = a) \qquad (8)$$

Now, the equations (7) & (8) are again multiplied by the weighting function which is same as the basis function but with an another variable 'm'.

at centre: $Z'_{cm} = -\frac{l}{2} + \frac{lm}{2N+1}$ (9)

at low end: $Z'_{lm} = -\frac{l}{2} + \frac{l(m-1)}{2N+1}$ (10)

at upper end: $Z'_{um} = -\frac{l}{2} + \frac{l(m+1)}{2N+1}$ (11)

So, the integration is done between two segments: 1) $Z'_{lm} \le Z' \le Z'_{cm}$ & 2) $Z'_{cm} \le Z' \le Z'_{um}$

Now, we get the total equation comprising of 'm' & 'n' variables of weighting function & basis function. The total field equation is given here:

$$FIELD\,(m,n) = (A*n*MEN) + \left(\left(\frac{1}{4}\right)*A*MEN\right) + \left(\left(\frac{1}{2}\right)*A\right) - \left(\left(\frac{1}{2}\right)*D*n\right) - D + \left(\left(\frac{1}{2}\right)*A*B\right) - \left(A*n* MEN2*terms+1+ 13*D*MEN+ k42*pi+ I*XZ-C-KU+SG- G*XZ-C-KU+SG+ F*B*XZ-C-KU+SG+ A*L- (A*n*L) - (A*n*MEN) + \left(\left(\frac{1}{3}\right)*D*MEN\right) + \left(\left(\frac{1}{2}\right)*A*MEN\right) + \left(\left(\frac{1}{2}\right)*A*n*MEN\right) + (A*(n-1)*L) - \left(\left(\frac{1}{4}\right)*A*MEN\right) + (D*(n-1)*MEN) - \left(\left(\frac{1}{2}\right)*A*L\right) - \left(F*L*(E-XH-SG+H)\right) + \left(I*(E-XH-SG+H)\right) - \left(J*(E-XH-SG+H)\right) = -j\omega\text{€}E_z^i(\rho = a) \qquad (12)$$

where,

$A = (((k\text{^}4)*(length\text{^}2)) / ((8*pi)*((2*terms)+1)))$

$B = ((length*(m-terms)) / (((2*terms)+1)\text{^}2))$

$C = ((((length*(m))/((2*terms)+1)) - ((length*(n+1)) /((2*terms)+1))) * log2(((length*m) / ((2*terms)+1)) - ((length*(n+1))/((2*terms)+1))))$

$D = (((k\text{^}4) * (length\text{^}2)) / ((8*pi) * (((2*terms)+1)\text{^}2)))$

$KU = ((((length*(m+1)) / ((2*terms)+ 1)) - ((length*n) / ((2*terms)+ 1))) * log2(((length*(m+1) ((2*terms)+1)) - ((length*n) / ((2*terms)+1))))$

$H = ((((length*(m-1)) / ((2*terms)+ 1)) - ((length*(n)) / ((2*terms)+ 1))) * log2(((length*(m-1))/((2*terms)+1)) - ((length*n) / ((2*terms)+1))))$

$E = ((((length*m) / ((2*terms)+1)) - ((length*(n-1)) / ((2*terms)+ 1))) * log2(((length*m) / ((2*terms)+1)) -((length*(n-1)) / ((2*terms)+1))))$

$XZ = ((((length*(m+1)) / ((2*terms)+ 1)) -((length*(n+1)) / ((2*terms)+1))) * log2(((length*(m+1)) / ((2*terms)+1)) - ((length*(n+1)) / ((2*terms)+1))))$

$SG = ((((length*m) / ((2*terms)+1)) - ((length*(n)) / ((2*terms)+1))) * log2(((length*(m)) / ((2*terms)+1)) - ((length*n) / ((2*terms)+1))))$

$XH = ((((length*(m-1)) / ((2*terms)+1)) - ((length*(n-1)) / ((2*terms)+1))) * log2(((length*(m-1)) / ((2*terms)+ 1)) - ((length*(n-1)) / ((2*terms)+1))))$

$MEN = ((length) / ((2*terms)+1))$

$F = (((k\text{^}4) * ((2*terms)+1)) / (4*pi))$

$G = (((k\text{^}4) * (n+1)) / (4*pi))$

$I = (((k\text{^}4) * ((2*terms)+1)) / (8*pi))$

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$J = (((k\text{^}4) * (n-1)) / (4 * pi))$
$L = ((length * (m - terms - 1)) / (((2 * terms) + 1)\text{^}2))$

Now equations (12) can be formed as a matrix where the equation comprising of elements 'm' & 'n' is denoted as 'Z_{mn}'. The constant 'b_n' can be rewritten as a current function 'I_n'. The excitation function at the right hand side of equation (12) is denoted as 'V_m', *[Zmn] [In] = [Vm]* [8][12]

The unknown coefficients 'I_n' can be found by solving using matrix inversion techniques, or *$[In]=[Zmn]^{-1}[Vm]$* [10][4]

The excitation function V_m is nothing but -jω€V, Where, V is the feed voltage & has 2N number of elements. The values of N for the rooftop function & the pulse function are different. We assume the feed voltage be 1volt. The other parameters have their usual values. Among the 2N elements all elements will be '0' except at the middle position as the dipole has the maximum current at the middle. Here, we approximately taken two points as the middle positions for both the rooftop function & the pulse function.

For the rooftop function,

For $(-\frac{l}{2}) \leq Z' \leq -\frac{l}{2} + \frac{ln}{2N+1}$, current = *In (1,1) * ((Z' + l/2)) / ((l/(2N+1)))*

For $-\frac{l}{2} + \frac{2Nl}{2N+1} \leq Z' \leq \frac{l}{2}$, current = *In(2N,1) * ((-Z' + l/2)) / ((l/(2N+1)))*

For elsewhere, *i = ((Z' + l/2)) / ((l/(2N+1)))*

current = *In(i,1) * (upper function) + In(i+1,1) * (lower function)*

ii) For this particular, we choose pulse function as the basis function

Now consider the current

$$I_z(Z') = \sum_{n=1}^{2N} b_n \;\Pi \qquad (13)$$

where, $'b_n'$ is an unknown constant & ' п ' is the pulse function.

'n' is the variable. For the n^th term, the rooftop function can be defined as below:

for $Z' \geq -\frac{l}{2} + \frac{l(n-1)}{2N}$, п =1,

for $Z' \leq -\frac{l}{2} + \frac{l(n)}{2N}$, п =1 and

elsewhere п =0

So, the integration for the field can be defined as:

$$\int_{\frac{-l}{2}+\frac{l(n-1)}{2N}}^{-\frac{l}{2}+\frac{l(n)}{2N}} \sum_{n=1}^{2N} b_n \left(\frac{k^2}{4.\pi} . \frac{1}{R} + \frac{k^4.R}{8.\pi}\right) dZ' = -j\omega\varepsilon E_z^i(\rho = a) \qquad (14)$$

Now, the solution of equation (14) is integrated over the weighting function. The weighting function is same as the basis function & can be defined as:

for $Z' \geq -\frac{l}{2} + \frac{l(m-1)}{2N}$, п =1,

for $Z' \leq -\frac{l}{2} + \frac{l(m)}{2N}$, п =1 and

elsewhere п =0

Now, we get the total equation comprising of 'm' & 'n' variables of weighting function & basis function. The total field equation is given here:

$$FIELD\,(m,n) = ((A * MEN) - (D * 2 * MEN) + (A * B) + (D * MEN) - (F * (SG - H - E + XH))) = -j\omega€E_z^i(\rho = a)$$

where,

$A = (((k\text{^}4) * (length\text{^}2)) / ((8*pi) * (2* terms)))$

$B = ((length*((2*m)-1-(2*terms))) *terms)\text{^}2))$

$D = (((k\text{^}4) * (length\text{^}2)) / ((8*pi)*((2 *terms)\text{^}2)))$

$H = ((((length*(n)) / ((2*terms)+1 - ((length*(m-1)) / ((2*terms)+1))) * log2(((length*(n)) / ((2*terms)+1)) -((length*(m-1)) / ((2*terms)+1))))$

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$E = ((((length*(n-1)) / ((2*terms)+1)) - ((length*(m)) / ((2*terms)+1))) * log2(((length*(n-1)) / ((2*terms)+1)) -((length*(m)) / ((2*terms)+1))))$

$SG = ((((length*n) / (2*terms)) - ((length *(m)) / (2*terms))) * log2(((length *(n)) / (2*terms)) -((length*m) / (2*terms))))$

$XH = ((((length*(n-1)) / ((2*terms)+1)) - ((length*(m-1)) / ((2*terms)+1))) * log2(((length*(n-1)) / (2*terms +1)) - ((length*(m-1)) / ((2 *terms)+1))))$

$MEN = ((length) / (2*terms))$

$F = ((k^4) / (4*pi))$

Same as previous, from the obtained matrix of variable 'm'& 'n' which equals to some excitation voltage, the unknown current matrix elements can be found. For the pulse function, we divide the dipole antenna into (2N) number of segments. As the value of pulse function is '1' in its operating region, so there will be (2N) positions along with their current. The dipole antenna length ranges from *(-l/2) to (l/2)* [assuming the length of dipole antenna is 'l']. So, for the first segment i.e from $(-l/2) \leq Z' \leq -\frac{l}{2}+\frac{ln}{2N}$,the current will be the first value of coefficient matrix. Similarly, from $-\frac{l}{2}+\frac{ln}{2N} \leq Z' \leq -\frac{l}{2}+\frac{2ln}{2N}$, the current will be the second value of the coefficient matrix & so on.

IV. SIMULATION RESULTS

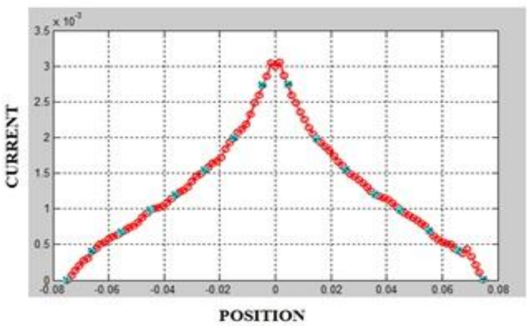

Fig 1. Current waveform for rooftop function for (λ/2)

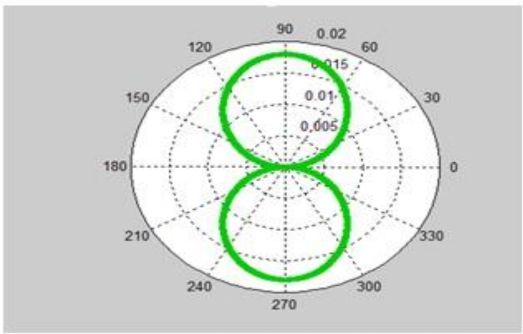

Fig 2. Radiation pattern rooftop function for (λ/2)

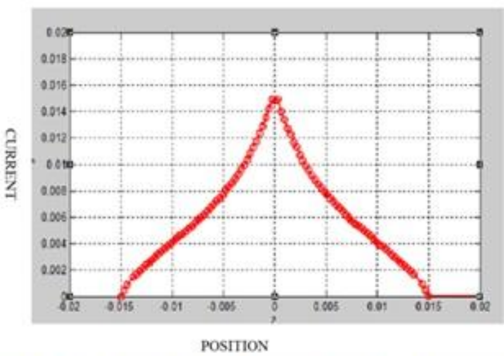

Fig 3. Current waveform for rooftop function for (λ/10)

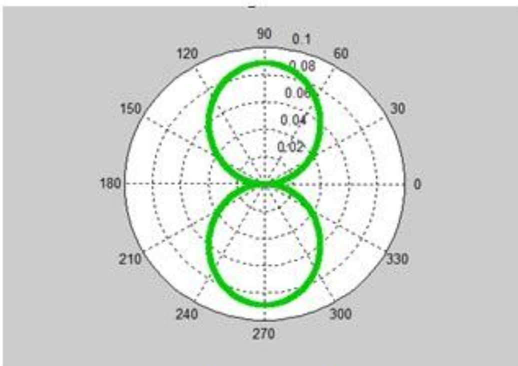

Fig 4. Radiation pattern rooftop function for (λ/10)

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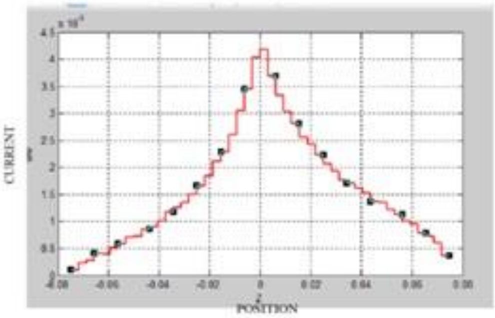

Fig 5. Current waveform for pulse function for (λ/2)

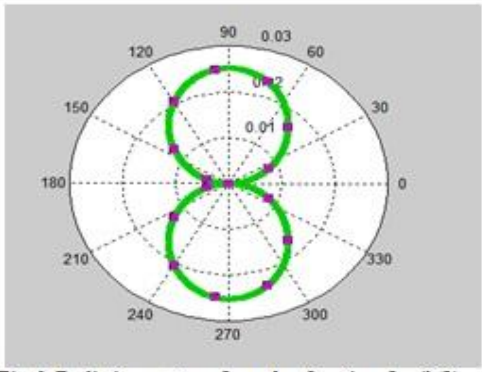

Fig 6. Radiation pattern for pulse function for (λ/2)

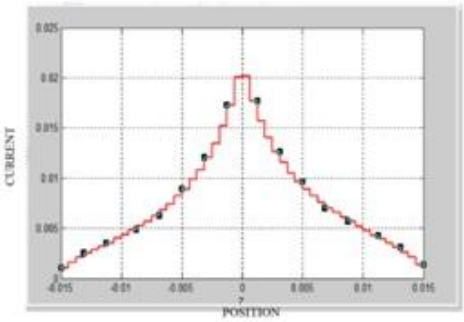

Fig 7. Current waveform for pulse function for (λ/10)

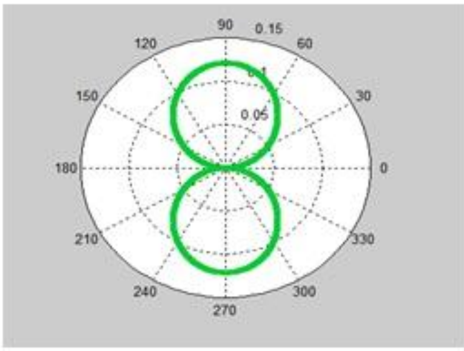

Fig 8. Radiation pattern for pulse function for (λ/10)

Figure(1) & Figure(3) shows the current waveform of the dipole antenna for length (λ/2) & (λ/10) using the rooftop function, whereas figure(5) & figure(7) shows the current waveform for the respective lengths using pulse function. Figure (2) & figure(4) are the radiation pattern for lengths (λ/2) &(λ/10) using the rooftop function, whereas Figure (6) & figure(8) are the radiation pattern for lengths (λ/2) &(λ/10) using the pulse function.

IV. CONCLUSION

Performances of two types of basic functions i.e. rooftop function & pulse function for different lengths of dipole antenna using method of moment is described. Results for different lengths are distinguishable. Radiation pattern for the dipole antenna for different lengths are also shown. For the matlab coding, we assume that the speed of light to be (3*10^8 m/s),frequency to be 1GHz,number of segments to be 48. The current distribution waveform is not suitable for 24 number of segments. The self impedance obtained for the pulse function for length λ/2 is approximately 63 ohm and for λ/10 it is approximately 8.6 ohm. It is found that some logarithmic terms are arrived in our formulation, but they are dissolved using particular mathematical formula. Also, singularities have come, but using some singularity removal techniques i.e. few proper expansion of Greens' function, we get rid of it.

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