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Different Types of Formal Logical Systems in Mathematics

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Perspective

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ABOUT THE STUDY

Mathematical logic is the study of logical reasoning in mathematics. Model theory, proof theory, set theory, and recursion theory are significant subfields. The mathematical characteristics of formal systems of logic, such as their expressive or deductive capability, are frequently the subject of research in mathematical logic. However, it may also refer to the use of logic to define sound mathematical reasoning or establish the foundation for mathematics. Although categorical logic is studied in the mathematical area of category theory, which utilizes many rigorous axiomatic methods, category theory is not typically thought of as a part of mathematical logic.

Mathematicians, such as Saunders Mac Lane, have advocated category theory as a basic framework for mathematics that is independent of set theory due to its usefulness in a variety of areas of mathematics. Toposes, which resemble extended models of set theory, may use either classical or nonclassical logic.

Mathematical logic is primarily concerned with the formal logical expression of mathematical notions. These systems all have the trait of only taking into account expressions in a single fixed formal language, although having numerous other differences. Because of their relevance to the mathematical foundations and because of their desired proof-theoretic qualities, propositional logic and first-order logic are the systems that are now the subject of the greatest research. Along with non-classical logics like intuitionistic logic, stronger classical logics like second-order logic or infinitary logic are also examined.

First-order logic

A specific type of formal logic is first-order logic. Only finite expressions as well-formed formulae are used in its syntax, and all quantifiers are restricted to a single area of discourse in its semantics.

Other classical logics

In addition to first-order logic, other logics are examined. These include higher-order logics, which directly include elements of set theory into their semantics, and infinitary logics, which enable formulae to offer an endless amount of information.

Nonclassical and modal logic

Additional modal operators, such as one that asserts that a given formula is not only true but also necessary true, are included in modal logics. The characteristics of first-order provability and set-theoretic forcing have been studied using modal logic, despite the fact that it is not frequently employed to axiomatize mathematics.

Algebraic logic

Algebraic logic investigates the semantics of formal logics using abstract algebraic techniques. The use of Heyting algebras to describe truth values in intuitionistic propositional logic and the use of Boolean algebras to represent truth values in classical propositional logic are two key examples. In order to study stronger logics like first-order and higher-order logic, more intricate algebraic structures as cylindrical algebras are used.

There are three different types of theories, they are as follows:

- Set theory
- Model theory
- Recursion theory

The study of sets, which are indefinite collection of items, is known as set theory. Before formal axiomatizations of set theory were discovered, Cantor developed a number of fundamental concepts, such as ordinal and cardinal numbers, informally. The first such axiomatization, credited to Zermelo, was somewhat expanded to create Zermelo-Fraenkel set theory (ZF), the most popular fundamental theory for mathematics nowadays.

Model theory is the study of different formal theories' models. A model is a framework that provides a practical interpretation of the theory in this context, whereas a theory is a collection of formulae with a specific formal logic and signature. Although it shares many similarities with universal algebra and algebraic geometry, model theory places a greater emphasis on logical issues.

A theory's collection of all models is referred to as its elementary class, and classical model theory aims to ascertain the characteristics of each elementary class, or if certain classes of structures constitute elementary classes.

The study of computable functions and the Turing degrees, which group incomputable functions into sets with varying degrees of incomputability, is known as recursion theory, also known as computability theory. The study of generalized computability and definability is also a part of recursion theory. The computability of functions from the natural numbers to the natural numbers is the main emphasis of classical recursion theory.