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Evaluation of Efficiency in DEA Models Using a Common Set of Weights

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ABSTRACT: Data Envelopment Analysis is a well-known OR technique for evaluating the relative efficiency of a set of similar decision making units (DMUs).In the classical DEA models, each DMU assigns weights to the factors so as to maximize efficiency. This is unacceptable, since the same factors have widely different weights for each DMU. So, in order to discriminate efficient and non-efficient DMUs properly, a common set of weights need to be used. In this paper, a multiobjective programming model is developed to derive a common set of weights. Solution methodology uses fuzzy programming. The method is illustrated through an example in which data sets are taken from the previous research on DEA's discriminating power and weight restriction.

KEYWORDS: DEA , set of common weights, multiobjective programming, fuzzy programming

I. INTRODUCTION

Data Envelopment Analysis is a well-known OR technique for evaluating the relative efficiency of a set of similar decision making units (DMUs). The number of applications of DEA is large covering fields as diverse as finance, health, education, manufacturing, transportation etc. Conventional DEA models are based on Linear Programming and consider continuous inputs and outputs.

Data Envelopment Analysis (DEA) was accorded this name because of the way it "envelops" observations in order to identify a "frontier" that is used to evaluate observations representing the performances of all of the entities that are to be evaluated. Uses of DEA have involved a wide range of different kinds of entities that include not only business firms but also government and non-profit agencies including schools, hospitals, military units, police forces and court and criminal justice systems as well as countries, regions, etc. The term "Decision Making Unit" (DMU) was therefore introduced to cover, in a flexible manner, any such entity, with each such entity to be evaluated as part of a collection that utilizes similar inputs to produce similar outputs. These evaluations result in a performance score that ranges between zero and unity and represents the "degree of efficiency" obtained by the thus evaluated entity. In arriving at these scores, DEA also identifies the sources and amounts of inefficiency in each input and output for every DMU. It also identifies the DMUs (located on the "efficiency frontier") that entered actively in arriving at these results. These evaluating entities are all efficient DMUs and hence can serve as benchmarks en route to effecting improvements in future performances of the thus evaluated DMUs. DEA measures the efficiency scores of DMUs which perform the same tasks by consuming the same inputs and producing the same outputs based on the idea of Farrell which is concerned with the estimation of technical efficiency and efficient frontiers. This approach first establishes an ‘‘efficient frontier’’ or a sort of “envelope “ formed by a set of decision making units (DMUs) that exhibit best practices and then assigns the efficiency level to other non-frontier units according to their distances to the efficient frontier. The basic idea has since generated a wide range of variations in measuring efficiency.

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The efficiency score of a unit is defined as the ratio of a weighted sum of its outputs to a weighted sum of its inputs and it is measured on a bounded ratio scale. The weights for inputs and outputs are estimated to the best advantage for each unit so as to maximize its relative efficiency. The underlying mathematical model is a linear program, which is given in either the multiplier form or in its dual form, the envelopment form. The former makes explicit use of the efficiency ratio while the latter provides an explicit representation of the envelope formed by the efficient frontier as well as the orientation with which the assessments are made (i.e. input or output oriented model). In terms of the multiplier form, an output multiplied by the corresponding weight is called virtual output. The sum of the virtual outputs over all the output dimensions, which forms the numerator of the efficiency ratio, is called total virtual output. Analogous are the definitions for inputs. The efficiency of a unit is thus obtained by the ratio of the total virtual output to the total virtual input.

The two basic DEA models are the CCR model [1] and the BCC model [2].These two models differentiate on the returns to scale assumed. The former assumes constant returns-to-scale whereas the latter assumes variable returns-to-scale.

The main characteristic of this model (known as the CCR or engineering ratio model)is the transformation, for each DMU, of the situation multiple output/multiple input into a ratio virtual output/virtual input. This ratio yields an efficiency measure that is a function of a set of multipliers, which are the decision variables of a fractional programming model (then transformed into a linear programming model). The extension of the CCR model for the case of variable returns to scale was proposed in the BCC model, in which an additional constraint guarantees that each DMU under evaluation is compared with a convex combination of the other DMUs.

II. THE BASIC DEA MODEL

Consider a set of n decision making units(DMUs) of similar inputs and outputs. Let there be m inputs and s outputs. In the Classical DEA(CCR) model [3]. for evaluating the efficiency of a DMU, denoted by DMUo. Is as follows

$$Max\ \sum_{r=1}^{s} u_r y_{r_{j_o}}$$

Subject to constraints

$$\sum_{i=1}^{m} v_i x_{ij_o} = 1$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \quad j = 1,2,\ldots,n. \qquad (\text{I})$$

$$u_r, v_i \ge 0\ , \ for\ all\ r\ and\ i,$$

Where j is the DMU index, J=1,2,….,n, r the output index,r=1,2,…,s,I the input index, i=1,2,….,m, y_{rj} the value of the rth output for the jth DMU, x_{ij} the value of the ith input for the jth DMU, u_r the weight given to the rth output ,v_i the weight given to the ith input, and $\sum_{r=1}^{s} u_r y_{r_{j_o}}$ is the relative efficiency of DMUo, under evalution.

III.COMMON WEIGHT APPROACH IN DEA AND MULTIOBJECTIVE PROGRAMMING

Most of the DEA models must select a DMU among all DMUs, say DMUo , in order to compute the efficiency scores of each DMU. However, choosing different DMUo leads to various evaluation results. Each DMU generates its hyperplane for efficiency evaluation. The common weights approach [4] generates only one hyperplane for efficiency evaluation of all DMUs.

Many models for deriving common weights [5]. are available and continue to be explored as such models are interesting both from theoretical and practical view points. (a common set of weights means that only one frontier hyperplane generates a compromised solution; all DMUs lie beneath the hyperplane and agree with the final status.) Common weights derived by mulibojective linear programming (MOLP) [6-7] for a DEA model are theoretically supported by the concept of Pareto efficiency.DEA and MOLP both search for set non-inferior solutions. Thus characterizing the DEA model by multi objective programming is natural reasonable and appropriate . Li and Reeves [8] presented a multiobjective model that considers two additional efficiency measures: minimizing the sum of the DMU distances to frontier(minisum) and minimizing the sum of the largest distance (minimax), in addition to maximizing classical efficiency in DEA.

$$Max\ h_0 = \sum_{r=1}^{p} u_r y_{r0}$$

$$Min\ M$$

$$Min \sum_{k=1}^{n} d_k$$

Subject to the constraints (II)

$$\sum_{i=1}^{m} v_i x_{i0} = 1$$

$$\sum_{r=1}^{p} u_r y_{rk} - \sum_{i=1}^{m} v_i x_{ik} + d_k = 0 \qquad k = 1,2,\dots\dots,n.$$

$$M - d_k \geq 0$$

$$u_r, v_i, d_k \geq 0 \quad ,for\ all\ \ r,i,k$$

IV.METHODOLOGY

By solving model (II), a set of weights is obtained for inputs and outputs. The model suggested is a multiobjective programming problem which is developed as follows.

Let e_j be the efficiency of the jth DMU. That is, $e_j = \frac{\sum_{r=1}^{p} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}}$.

The proposed MOPP model is

$$Max\ e_j$$

where j =1,2,…,n

Subject to the constraints

$$l_r \leq u_r \leq t_r \quad , \quad r_k \leq v_k \leq s_k$$

Where l_r ,t_r , r_k and s_k are taken from the results of the three models suggested above as the maximum and minimum values of the factor weights.

To convert the above model onto a fuzzy programming model the following definition of fuzzy set [9] is considered.

Let X be a non-empty set. A fuzzy set A in X is characterized by its membership function

$\mu_A : X \rightarrow [0,1]$

and $\mu_A(x)$ is interpreted as the degree of membership of element of element x in fuzzy set A for each $x \in X$.

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The MOPP can be converted to fuzzy programming model by defining membership functions for the efficiencies of different DMUs.

Let $p_j = min\{efficiency\ values\ of\ DMUs\}$ and $q_j - max\{efficiency\ values\ of\ DMUs\}$,
where efficiency values are obtained by solving model(II). Define membership function for efficiencies of the DMUs

$$\mu_{D_1}(e_j) = \begin{cases} \frac{e_j - p_j}{q_j - p_j} & , p_j < e_j < q_j \\ 0 & , e_j < p_j \\ 1 & , e_j > q_j \end{cases}$$

which is converted to the following fuzzy programming model

$$Max\ Min\ \{\mu_{D_1}, \mu_{D_2}, \dots\dots, \mu_{D_n}\}$$

Subject to the constraints

$$l_r \le u_r \le t_r \quad , \quad r = 1,2, \dots, s$$
$$r_k \le v_{ik} \le s_k \quad , \quad k = 1,2, \dots\dots, m$$

Let $z = \min\{\mu_{D_1}, \mu_{D_2}, \mu_{D_3}, \dots\dots, \mu_{D_n}\}$

Then the model becomes

$$Max\ Z$$

Subject to the constraints

$$\mu_{D_1} \ge Z$$
$$\mu_{D_2} \ge Z$$
$$\dots\dots\dots\dots$$
$$\mu_{D_n} \ge Z$$

$l_r \le u_r \le t_r$ and $r_k \le v_k \le s_k$ where r=1,2,…,s ; k= 1,2,….,m.

This model can be rewritten as

$$Max\ Z$$

Subject to the constraints

$\sum_{r=1}^{s} u_r y_{rj} \ge [z(q_j - p_j) + p_j] \sum_{i=1}^{m} v_i x_{ij}$, j= 1,2,….,n.

$l_r \le u_r \le t_r$ and $r_k \le v_k \le s_k$ where r=1,2,…,s ; k= 1,2,….,m.

In order to facilitate the solution procedure of this nonlinear programming problem, it is divided into the following two problems by fixing minimum value for $\sum_{i=1}^{m} v_i x_{ij}$ as well as maximum value for $\sum_{r=1}^{s} u_r y_{rj}$

Problem I

$$Max\ Z$$

Subject to the constraints

$\sum_{r=1}^{s} u_r y_{rj} \ge [z(q_j - p_j) + p_j]\ (min \sum_{i=1}^{m} v_i x_{ij})$, j= 1,2,….,n.

$l_r \le u_r \le t_r$, where r =1,2,…,s

Problem II

$$Max\ Z$$

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Subject to the constraints

$$\sum_{i=1}^{m} v_i x_{ij} \leq \frac{1}{z(q_j - p_j) + p_j} \left(max \sum_{r=1}^{s} u_r y_{rj} \right), \quad j= 1,2,\ldots,n.$$

$$r_i \leq v_i \leq s_i \quad \text{where } i= 1,2,\ldots,m.$$

Solving Problem I and Problem II , a set of common weight for the efficiencies can be obtained.

V.EXAMPLE

(Efficiency evaluation of six nursing homes). [10](Sexton 1986) considered a case of six nursing homes whose input and output data for a given year are described in Table 1, where the input and output variables are defined as follows:

StHr (x1) staff hours per day, including nurses, physicians.etc.
Supp (x2) supplies per day, measured in thousands of rupees
MCPD (y1) total Medicare-plus Medicaid-reimbursed patient days (0,000)
PPPD (y2) total privately paid patient days(0,000)

Table 1 Data of Example 1

DMU	INPUT		OUTPUT	
	StHr (x1)	Supp (x2)	MCPD (y1)	PPPD (y2)
A	1.50	0.2	1.40	0.35
B	4.00	.07	1.40	2.10
C	3.20	1.2	4.20	1.05
D	5.20	2.0	2.80	4.20
E	3.50	1.2	1.90	2.50
F	3.20	0.7	1.40	1.50

The results of classical DEA method, solved by LP, are given in Table 2. Using model (II), we obtained the results shown in Tables 3,4,5, corresponding to the three criteria, respectively.

Table 2 Classical DEA results of Example

DMU	Efficiency	Input Weights		Outp[ut Weights	
		v_1	v_2	u_1	u_2
A	1	0	5.000	0.714	0
B	1	0	1.429	0	0.476
C	1	0.172	0.374	0.238	0
D	1	0.069	0.321	0	0.238
E	0.977	0.110	0.513	0.115	0.304
F	0.867	0.155	0.722	0.162	0.427

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Table 3: Minimizing d0 DEA results of Example

DMU	Efficiency	Input Weights		Outp[ut Weights	
		v_1	v_2	u_1	u_2
A	1	0.517	1.121	0.505	0.837
B	1	0.138	0.642	0.144	0.380
C	1	0.172	0.374	0.082	0.279
D	1	0.080	0.292	0.115	0.184
E	0.977	0.110	0.513	0.115	0.304
F	0.867	0.155	0.722	0.162	0.427

Table 4 : Minimax DEA results of Example

DMU	Efficiency	Input Weights		Outp[ut Weights	
		v_1	v_2	u_1	u_2
A	1	0.449	1.634	0.457	1.029
B	0.953	0.153	0.556	0.156	0.350
C	0.883	0.132	0.481	0.135	0.303
D	1	0.080	0.292	0.082	0.184
E	0.974	0.127	0.463	0.129	0.291
F	0.846	0.174	0.633	0.177	0.399

Table 5: Minsum DEA results of Example

DMU	Efficiency	Input Weights		Outp[ut Weights	
		v_1	v_2	u_1	u_2
A	1	0.517	1.121	0.505	0.837
B	0.864	0.181	0.393	0.177	0.293
C	0.830	0.114	0.530	0.119	0.314
D	1	0.069	0.321	0.072	0.190
E	0.977	0.110	0.513	0.115	0.304
F	0.867	0.155	0.722	0.162	0.427

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Table 6 :Maximum and Minimum efficiencies of 6 DMUs(obtained using minimum and maximum weights of Tables 3,4 & 5)

DMU	Minimum Efficiency (p)	Maximum Efficiency (q)
A	0.85	.97
B	0.70	0.89
C	0.67	0.87
D	0.92	0.97
E	0.89	0.95
F	0.78	0.84

Then the model is

$$Max\ Min\ \{\mu_A, \mu_B, \ldots, \mu_F\}$$

Subject to the constraints

$$0.072 \le u_1 \le 0.505$$
$$0.184 \le u_2 \le 1.029$$
$$0.069 \le v_1 \le 0.517$$
$$0.292 \le v_2 \le 1.634$$

Solution obtained is $\mu_A = \mu_B = \mu_C = \cdots .. = \mu_F = 1$ with u_1 =0.0896, u_2 =0.1857, v_1 =0.0690 and v_2=0.2920.This implies, the efficiency values of the DMUs' A, B, ….,F are respectively the maximum values of efficiencies. Thus the suggested model was able to provide a set of common weights u_1 = 0.0896 , u_2 = 0.1857 , v_1 =0.0690 and v_2 = 0.2920 corresponding to which each of the DMUs attains their maximum efficiencies.

VI. Conclusion

In this paper , a multiobjective programming model is developed for deriving a set of commom weights for the inputs and outputs. Unlike other DEA models, this model not only evaluates the efficiency scores of the DMUs but suggests common sets of weights which gives these efficiency scores. In practical situations, this is useful as it provides an insight into the importance of inputs as well as outputs as far as the performance of the organizations are concerned.

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