Generalized form of Vectorial Laws of Reflection and Refraction and Application to Meta Material

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Mini Review

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ABSTRACT

This communication presents the generalized laws of reflection and refraction in vectorial form with simple derivation. A few relations related to reflection and refraction in vectorial form have been included. The photon theory as well as the principle of conservation of momentum has been used for the derivation of these vectorial laws of reflection and refraction. The application of these laws in metamaterial has been discussed.

Keywords: Generalized laws; Reflection and refraction; Vector calculus; Momentum conservation; Metamaterial

INTRODUCTION

Geometrical optics is the branch of optics which is characterized by the limiting case of wavelength of light $\lambda \rightarrow 0$. In this approximation, the optical laws may be formulated in the Language of geometry [1]. Appropriate formulation of the laws of reflection and refraction is possible by considering the implications of Maxwell's equation when $\lambda \rightarrow 0$. Light rays have been defined as the orthogonal trajectories to the geometrical wave fronts S(x,y,z)=constant, and in a homogeneous medium the light rays have the form of a straight line $\left(\vec{r} = s\vec{a} + \vec{b}\right)$ where $\vec{a}$ and $\vec{b}$ being a constant vector and s is a constant scalar [1]. This equation is a vector equation of a straight line in the direction of the vector $\vec{a}$ passing through the point $\vec{r} = \vec{b}$. Hence in a homogeneous medium, the light ray has the form of a straight line.

LITERATURE REVIEW

Born and Wolf in the year 1959, first mentioned the vectorial form of laws of refraction as $n_2\left(\vec{n}_{12} \times \vec{s}_2\right) = n_1\left(\vec{n}_{12} \times \vec{s}_1\right)$ [1]. In a slightly different form, Herzberger also wrote the laws of refraction in vector form. After 46 years of Born and Wolf publication, P. R. Bhattacharjee, in the year 2005 first lucidly explained the laws of reflection and refraction in vector form [2-5]. Subsequently, he published many papers related to the vectorial representation of the laws of optics [6-15]. His publication rid the ambiguity from the traditional exposition of laws of optics. In this communication, we have discussed the laws of reflection and refraction in vectorial form in a slightly different way. We also included a few new relations related to reflection and refraction in vectorial form. For the derivation of these vectorial laws of reflection and refraction, we have used the photon theory as well as the principle of conservation of momentum.

Reflection-related laws in vector form

Traditional laws of reflection are i) The incident ray, the reflected ray and the normal at the point of incidence lie in the same plane. This plane is called the plane of incidence (or plane of reflection). ii) Angle of incidence is equal to the angle of reflection.

Now we explain these two laws in a generalized way in terms of vectors.

Incident ray, reflected ray and normal always lie in the same plane: We know that three vector lies in the same plane if the scalar triple vector of these vectors is zero. If we consider three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ then they will lie in the same plane when $\vec{a}\left(\vec{b} \times \vec{c}\right) = 0$. So if we consider

$\hat{e}_i$ = unit vector along the incidence ray,

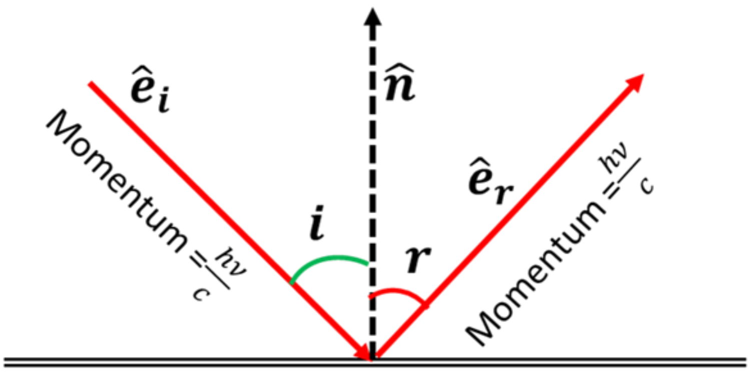
n = unit vector along the normal and

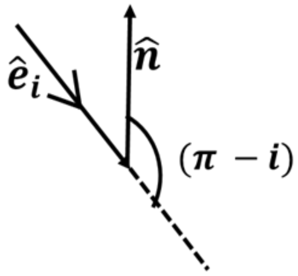
$\hat{e}_r$ = unit vector along the reflected ray, then in vector form first law becomes,

$$\left(\hat{e}_i \times n\right) \cdot \hat{e}_r = 0$$

This relation represents that the incident ray, the reflected ray and the normal at the point of incidence lie in the same plane (Figure 1).

Figure 1. Diagram showing reflection of a ray of light by a plane mirror.





Generalized form of the angle of incidence is equal to the angle of reflection: If we consider

$\hat{e}_i$ =unit vector along incidence ray,

n =unit vector along the normal and

$\hat{e}_r$ =unit vector along the reflected ray,

$\hat{a}$ is a unit vector, which is normal to the plane in which incident ray, normal at the point of incidence and reflected ray lie and in upward direction.

Here

$$\hat{e}_i \times \hat{n} = |\hat{e}_i||\hat{n}|\sin(\pi - i)\hat{a}$$

$$= 1.1. sin(\pi - i)\ \hat{\boldsymbol{a}}$$

$$= sini\ \hat{\boldsymbol{a}}$$

$$\hat{e}_r \times \hat{n} = |\hat{e}_r||\hat{n}|\sin(r)\hat{a}$$

$$= 1 \cdot 1 \cdot \sin(r)\hat{a}$$

$$= \sin r\hat{a}$$

Now we use the concept of photon theory, according to which all photons of light of a particular frequency v, have the same energy E (=hv), whatever the intensity of the radiation may be. By increasing the intensity of light of a given wavelength, there is only an increase in the number of photons per second crossing a given area, with each photon having the same energy. Thus, photon energy is independent of the intensity of radiation.

Again, we know that the frequency of radiation is independent of the medium so the energy of a photon is independent of the medium but the momentum of a photon $\left(p=\frac{hv}{c}\right)$ depends on the medium. In the case of reflection, as incident and reflected ray lie in the same medium, so momentum remains same in case of incident and reflected ray.

Using the conservation law of momentum of photon along the horizontal axis we get

$$\frac{hv}{c}\sin i+0=\frac{hv}{c}\sin r+0$$

$$\sin i=\sin r$$

$$\hat{a}\sin i=\sin r\hat{a}$$

$$\hat{e}_i\times\hat{n}=\hat{e}_r\times\hat{n}$$

This relation represents the law of reflection in vector form (Figure2).

Direction of reflected ray in vector form: Vector form of reflected ray is;

$$\hat{e}_r=\hat{e}_i-2\left(\hat{e}_i\cdot\hat{n}\right)\hat{n}$$

Let

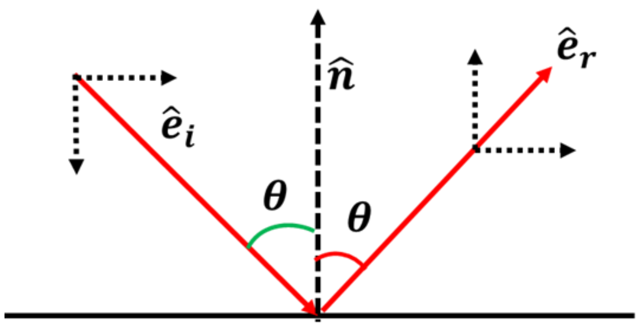
$\hat{e}_i$ =unit vector along the incident ray

$\hat{e}_r$ =unit vector along the reflected ray

$\hat{n}$ = unit vector outside normal

$\hat{t}$ = unit vector along tangential direction

Figure 2. Diagram showing unit vector incident ray and reflected ray with components along horizontal and vertical directions.



Now $\hat{e}_i=\sin\theta\hat{t}-\cos\theta\hat{n}$

$$\hat{e}_r=\sin\theta q\hat{t}+\cos\theta\hat{n}$$

Now $\hat{e}_r - \hat{e}_i = 2\cos\theta\hat{n}$

Or

$$\hat{e}_r - \hat{e}_i = -2\left(\hat{e}_i \cdot \hat{n}\right)\hat{n}$$

$$\left[Here \hat{e}_i \cdot \hat{n} = \cos\left(\pi - \theta\right) = -\cos\theta\right]$$

So

$$\hat{e}_r = \hat{e}_i - 2\left(\hat{e}_i - \hat{n}\right)\hat{n}$$

This relation helps to solve many problems related to reflection by a mirror. If light rays incident on a mirror normally, then $\theta = 0$ and $\hat{e}_i = -\hat{n}$ and $\hat{e}_i \cdot \hat{n} = \cos 180 = -1$ so $\hat{e}_r = -\hat{n} + 2\hat{n} = \hat{n}$. So, we can easily conclude that if light ray's incident on a mirror normally, it reflected back normally in opposite direction.

DISCUSSION

Laws of refraction in vector form in plane surface

Traditional laws of reflection are i) The incident ray, the refracted ray and the normal at the point of incidence lie in the same plane. This plane is called the plane of incidence (or plane of refraction). ii) Snell's law of refraction relates the sines of the angles of incidence and refraction at an interface between two optical media to the indexes of refraction of the two media: $\mu_1 \sin i = \mu_2 \sin r$, where μ_1 and μ_2 are the absolute index of refraction in the incident and in the refracting medium, respectively, while i and r are the angles of incidence and of refraction, respectively.

Now we explain these two laws in a generalized way in terms of vectors.

Incident ray, refracted ray and normal always lie in the same plane: We know that three vector lies in the same plane if the scalar triple vector of these vectors is zero. If we consider three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ then they will lay in the same plane when

$$\vec{a} \cdot \left(\vec{b} \times \vec{c}\right) = 0$$

So if we consider

$\hat{e}_i$ =unit vector along the incidence ray,

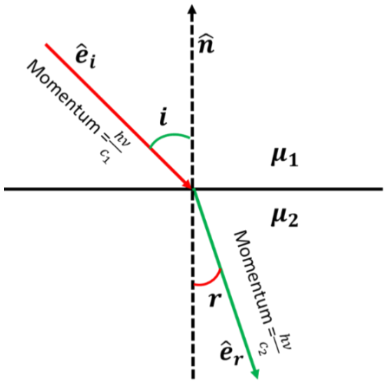
$\hat{n}$ = unit vector along the normal and

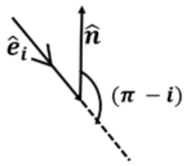
$\hat{e}_r$ = unit vector along the refracted ray

In vector from $\left(\hat{e}_i \times \hat{n}\right) \cdot \hat{e}_r = 0$

Snell's law in generalised vector form: Snell's law describes the refraction of light as it passes through a boundary between two different media. In vector form, Snell's law relates the incident, refracted, and normal vectors at the interface, incorporating the refractive indices of the media involved. Using vector calculus, we derive Snell's law of refraction in vector form, which describes the change in direction and speed of light when it crosses the boundary between two different media. This representation is indispensable in various applications, such as designing lenses and analyzing light propagation through optical fibers etc. (Figure 3).

Figure 3. Diagram showing refraction with momentum of incident and refracted ray.





If we consider

$\hat{e}_i$ =unit vector along incidence ray,

$\hat{n}$ =unit vector along the normal and

$\hat{e}_r$ =unit vector along the refracted ray,

$\hat{a}$ is a unit vector, which is normal to the plane in which incident ray, normal at the point of incidence and refracted ray lies and in upward direction.

$$\hat{e}_i \times \hat{n} = |\hat{e}_i||\hat{n}|\sin(\pi - i)\hat{a}$$

$$= 1 \cdot 1 \cdot \sin(\pi - i)\hat{a}$$

$$= \sin i\hat{a}$$

$$\hat{e}_r \times \hat{n} = |\hat{e}_r||\hat{n}|\sin(\pi - r)\hat{a}$$

$$= 1 \cdot 1 \cdot \sin(\pi - r)\hat{a}$$

$$= \sin r\hat{a}$$

Here momentum of the photon in the first medium is $\frac{hv}{c_1}$ and in the second medium $\frac{hv}{c_2}$. [Here c_1 and c_2 are velocities of light in the first medium and second medium respectively]. If μ_1 is the absolute refractive index of the first medium and μ_2 is the absolute refractive index of the second medium, then using conservation law of momentum of photon along horizontal direction we get (Figure 4).

$$\frac{hv}{c_1}\sin i+0=\frac{hv}{c_2}\sin r+0$$

$$\frac{1}{c_1}\sin i=\frac{1}{c_2}\text{sinr}$$

Multiplying both sides by c, we get

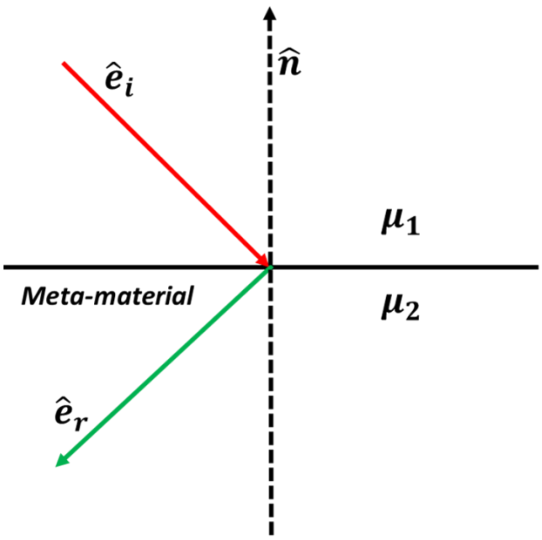
$$\frac{c}{c_1}\sin i=\frac{c}{c_2}\sin r$$

$$\mu_1\sin i=\mu_2\sin r$$

$$\mu_1\hat{a}\sin i=\mu_2\sin r\hat{a}$$

$$\mu_1\left(\hat{e}_i\times\hat{n}\right)=\mu_2\left(\hat{e}_r\times\hat{n}\right)$$

Figure 4. Diagram showing refraction in case of meta material.



Applications in meta material

The phenomenon of negative refraction is an active area of contemporary research. Most materials have the absolute refractive index, μ >1. So, when a light ray from air enters a naturally occurring material, then by Snell's law.

$$\mu_1\left(\hat{e}_i\times\hat{n}\right)=\mu_2\left(\hat{e}_r\times\hat{n}\right)\rightarrow(1)$$

It is understood that the refracted ray bends towards the normal in the opposite side. But it never emerges on the same side of the normal as the incident ray. According to electromagnetism, the refractive index of the medium is given by the relation,

$$\mu=\frac{c}{v}=\sqrt{\mu_r\varepsilon_r}\rightarrow(2)$$

where c is the speed of the electromagnetic waves in vacuum, v its speed in the medium, μ_r is the relative permeability and ε_r is the relative permittivity.

Now for negative values of μ_r and ε_r we may write them as $-\mu_r$ and $-\varepsilon_r$, then we may write:

$$\mu = \sqrt{(-\mu_r)(-\varepsilon_r)} = \sqrt{-1}\sqrt{-1}\sqrt{\mu_r\varepsilon_r} = (i)^2\sqrt{\mu_r\varepsilon_r} = -\sqrt{\mu_r\varepsilon_r}$$

Such negative refractive index materials have now already been prepared artificially and are called meta-materials [16-19]. They exhibit significantly different optical behavior, without violating any physical laws. Since μ is negative, it results in a change in the direction of propagation of the refracted light. However, similar to normal materials, the frequency of light remains unchanged upon refraction even in meta-materials.

The left-hand side of equation (1) is always positive. Now for a meta-material if μ_2 is negative then the term $(\hat{e}_r \times \hat{n})$ in the right-hand side must be negative as the left-hand side is always positive. For right side to be negative, the angle between $\hat{n}$ and $\hat{e}_r$ must be greater than 1800 so the refracted rays must lie on the same side to the normal as shown in Figure 4.

CONCLUSION

This communication, from the view point of academic interest, will enhance the theoretical foundation of the generalized vectorial laws of reflection and refraction. At the same time, it will enrich the traditional laws of physics as well. For deriving these laws, simple formula of vector calculus and conservation of momentum were used.

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