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Impact of Harvesting in Three Species Food Web Model With Two Distinct Functional Responses

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Abstract: The intuition with two-species models may be applied to community food web questions. The critical behavior to community function may arise only through the interaction of three or more species. In this paper we investigate the dynamical behavior of the system consisting of two preys with distinct functional responses and a predator. We also study the effect of harvesting on prey species. Harvesting is strong impact on the dynamics evaluation of population. To a certain extent it can control the long term stationary density of population efficiently. Finally the local and global stability analyses were carried out.

Keywords: Food web, Functional response, Harvesting, Local and Global stability

I. INTRODUCTION

Population Dynamics has been a prime branch of theoretical ecology. Lotka- Volterra model describes interaction between two species in an ecosystem, a predator and a prey. The model was developed independently by Lotka [9] and Volterra [14]. After that more realistic prey-predator models were introduced by Holling [7] suggesting three kinds of functional responses for different species to model the phenomena of predation. Lotka-Volterra prey-predator model is one of the fundamental population models describing the species competency interaction in a closed habitual. The food web models are more complex and intractable as compared to food chains as more complex multilevel interactions are possible in food webs. Relatively less attention has been given to the study of food webs and their rich complex dynamical behavior [1],[4],[5],[11]. The authors [6] examined the steady state condition stability analysis and other comparivitive studies of two logistic models such as exponential and quadratic in the population dynamics of host-parasite interaction they have been observed that in the presence of parasite affects the logistic growth dynamics in the host. In Ecology there are huge number of food chain models with three or more species with different functional responses such as Holling-Tanner type[12,13], Beddington-Deangelis type[10,15] and ratio dependent type[2,3]. But most of the models have same type of growth rate and same functional response. From biological point of view, it is unrealistic in nature. In fact in the real world, predators feed on different types of consumption ways. Hence in this paper we consider two types of functional responses, one the Holling type-II and other Bedding ton- De Angelis functional response.

II. MATHEMATICAL MODEL

Consider a food web, comprising of two preys and one predator species. One of the prey species is harvested. The two prey species growing logistically and direct competition is considered between them. But they are apparent competition through the shared predation. Indeed, this apparent competition appears, as both prey types are included in predator's diet. The mathematical model for the food web is given by the following system of equations,

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$$\begin{aligned}\frac{dX}{dt} &= r_1 X(1-\frac{X}{K_1}) - \frac{a_1 XZ}{1+b_1 X} - HX \\ \frac{dY}{dt} &= r_2 Y(1-\frac{Y}{K_2}) - \frac{cYZ}{1+d_1 Y + d_2 Z} \\ \frac{dZ}{dt} &= \frac{\lambda_1 a_1 XZ}{1+b_1 X} + \frac{\lambda_2 cYZ}{1+d_1 Y + d_2 Z} - eZ\end{aligned} \qquad (1)$$

Where r_1, r_2 capita intrinsic growth rate for preys X and Y, K_1, and K_2 are carrying capacities for preys X, Y. Also a_1 and c are capturing rates of predator Z on X and Y. ${b_1}/{a_1}$ and ${d_1}/{c}$ are the predators handling time on preys X and Y. H is constant effort harvesting rate of prey X, e is natural death rate of predator Z. d_2 Measures the effect of anti- predator behavior of prey Y. λ_1, and λ_2 are co-efficients which measures the predators efficiency to convert prey biomass of X and Y respectively.

We introduce the following dimensionless transformation

$x = b_1 X$, $y = d_1 Y$, $z = d_2 Z$, $\beta_1 = {1}/{K_1.b_1}$, $\beta_2 = {1}/{K_2.d_1}$, $g_1 = {a_1}/{r_1 d_2}$, $g_2 = {c}/{r_2 d_2}$, $h_1 = {\lambda_1 a_1}/{e b_1}$ $h_2 = {\lambda_2 c}/{e d_1}$ $M = {H}/{r_1}$

The dimensionless nonlinear system is obtained as

$$\begin{aligned}\frac{dx}{dt} &= r_1 x[(1-\beta_1 x) - \frac{g_1.z}{1+x} - M] \\ \frac{dy}{dt} &= r_2 y[(1-\beta_2 y) - \frac{g_2.z}{1+y+z}] \\ \frac{dz}{dt} &= ez[\frac{h_1 x}{1+x} + \frac{h_2 y}{1+y+z} - 1]\end{aligned} \qquad (2)$$

Lemma: The set $\Omega = \{(x.y,z): 0 \le x \le {1}/{\beta_1}, 0 \le y \le {1}/{\beta_2}\ 0 \le \xi_1 x + \xi_2 y + \xi_3 z \le \frac{\delta}{\eta}\}$ where $\xi_1 = \frac{e_1 h_1}{r_1 g_1}, \xi_2 = \frac{e_2 h_2}{r_2 g_2}, \delta = \frac{\xi_1 (r_1+\eta)}{\beta_1} + \frac{\xi_2 (r_2+\eta)}{\beta_2}, \eta \le e$ is the region of attraction for all solutions initiating in the interior of the positive octant.

Proof: From the first equation of (2) we note that $\frac{dx}{dt} \le r_1 x(1-\beta_1 x)$

This gives $x(t) \le \frac{\Gamma}{e^{-rt} + \Gamma \beta_1}$ where $\Gamma = \frac{x(0)}{1 - x(0)\beta_1}$

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As $t \to \infty$ we get $x(t) \le \frac{1}{\beta_1}$, Similarly from the second equation of (2), we get $y(t) \le \frac{1}{\beta_2}$. Let (x, y, z) be any solution with positive initial condition (x_0, y_0, z_0). Now consider the function $w(t) = \xi_1 x(t) + \xi_2 y(t) + z(t)$ so that

$$\frac{dw}{dt} + \eta w(t) = \xi_1 \frac{dx}{dt} + \xi_2 \frac{dy}{dt} + \frac{dz}{dt} + \eta(\xi_1 x(t) + \xi_2 y(t) + z(t)) \tag{3}$$

Substituting equations (2) in (3) and simplifying we get

$$w(t) \le \frac{\delta}{\eta}(1 - e^{-\eta t}) + w(0)e^{-\eta t}$$

As $t \to \infty$ we get $w(t) \le \frac{\delta}{\eta}$, this completes the proof.

III. EQUILIBRIUM POINTS

The system (2) has at most six equilibrium points which are

i) $E_0 = (0,0,0)$ is the origin, all species are extinct. ii) $E_1 = (\frac{1-M}{\beta_1}, 0, 0)$

iii) $E_2 = (0, 1/\beta_2, 0)$ (iv) $E_3(\frac{1-M}{\beta_1}, \frac{1}{\beta_2}, 0)$ v) $E_4 = (\frac{1}{h_1 - 1}, 0, \frac{h_1}{g_1(1-h_1)^2}[(h_1 - 1)(1-M) - \beta_1])$

vi) $E_5(0, y^*, z^*)$ Where

$$y^* = \frac{(h_2 + g_2 - g_2 h_2) + \sqrt{(h_2 + g_2 - g_2 h_2)^2 + 4h_2 g_2 \beta_2}}{2h_2 \beta_2}, z^* = y^*(h_2 - 1) - 1$$

The dynamical behavior of the fixed points of three- dimensional system (2) can be studied by the computation of the eigenvalues of Jacobian matrix of (2). The Jacobian matrix J at the state variable (x, y, z) has the form

$$J(x,y,z) = \begin{pmatrix} r_1 - 2\beta_1 r_1 x - \frac{g_1 r_1 z}{(1+x)^2} - M r_1 & 0 & \frac{-g_1 r_1 x}{1+x} \\ 0 & r_2 - 2\beta_2 r_2 y - \frac{g_2 r_2 z(1+z)}{(1+y+z)^2} & \frac{g_2 r_2 y(1+y)}{(1+y+z)^2} \\ \frac{h_1 e z}{(1+x)^2} & \frac{h_2 e z(1+z)}{(1+y+z)^2} & -e + \frac{h_1 e x}{(1+x)} + \frac{h_2 e y(1+y)}{(1+y+z)^2} \end{pmatrix}$$

Non-linear systems are much harder to analyze, since in most of the cases they do not possess quantitative solution even when explicit solutions are available and that they are often too complicated to provide much insight. One of the most useful techniques for analyzing non-linear system quantitavely is the linearized stability technique. The stability of the system is investigated by obtaining the eigenvalues of the Jacobin matrix is

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associated with fixed points. In order to study the stability of the fixed point model, we first see the following theorem.

Theorem: Let $p(\lambda)=\lambda^3+B\lambda^2+C\lambda+D$ be the roots of $p(\lambda)=0$.Then the following statements are true

a) If every root of the equation has absolute value less than one, then the fixed point of the System is locally asymptotically stable and fixed point is called a sink.
b) If at least one of the roots of equation has absolute value greater than one then the fixed point of the system is unstable and fixed point is called saddle.
c) If every root of the equation has absolute value greater than one then the system is a source.
d) The fixed point of the system is called hyperbolic if no root of the equation has absolute value equal to one. If there exists a root of equation with absolute value equal to one then the fixed point is called non-hyperbolic.

Lemma: 1 The boundary equilibrium point E_0 of the system (2) is stable fixed point when r_2 <1, otherwise unstable fixed point.

Proof: By linearizing system (2) at E_0, one obtain the Jacobian matrix,

$$J(E_0)=\begin{pmatrix} r_1-Mr_1 & 0 & 0 \\ 0 & r_2 & 0 \\ 0 & 0 & -e \end{pmatrix}$$

The eigenvalues of the Jacobian matrix at E_0 are r_1-Mr_1, r_2 and $-e$ .whenever $r_2<1$, Hence the equilibrium point E_0 is stable. Also the equilibrium point r_2 >1 is always positive and so $E_0(0,0,0)$ is unstable fixed point.

Lemma: 2 The boundary equilibrium point E_1 of the system (2) is locally asymptotically stable if satisfy the condition $M<1,\ r_2<1$ and $0<h_1<1$ otherwise unstable.

Proof: By linearizing system (2) at E_1, one obtain the Jacobian matrix

$$J(E_1)=\begin{pmatrix} r_1(M-1) & 0 & \dfrac{g_1r_1(M-1)}{\beta_1-(M-1)} \\ 0 & r_2 & 0 \\ 0 & 0 & e\dfrac{(M-1)(1-h_1)-\beta_1}{\beta_1-(M-1)} \end{pmatrix}$$

The roots of the matrix $J(E_1)$ are $r_1(M-1)$, r_2 and $e\dfrac{(M-1)(1-h_1)-\beta_1}{\beta_1-(M-1)}$ and they are negative if $M<1$, $r_2<1$ and $0<h_1<1$. Hence the equilibrium points are locally asymptotically stable. Moreover if $r_2>1$ is unstable.

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Lemma: 3 The boundary equilibrium point E_2 of the system (2) is locally asymptotically stable if satisfy the condition $M > 1,$ and $h_2 < \beta_2 + 1$.

Proof: By linearizing system (2) at E_2 ,one obtain the Jacobian

$$J(E_2) = \begin{pmatrix} r_1(1-M) & 0 & 0 \\ 0 & -r_2 & \dfrac{g_2 r_2}{1+\beta_2} \\ 0 & 0 & \dfrac{eh_2}{1+\beta_2} - e \end{pmatrix}$$

The eigenvalues of the matrix $J(E_2)$ are $r_1(1-M)$, $-r_2$ and $\dfrac{eh_2}{1+\beta_2} - e$. All the eigenvalues are negative if $M > 1,$ and $h_2 < \beta_2 + 1$. This shows the equilibrium point E_2 of the system is locally asymptotically stable.

Lemma: 4 The boundary equilibrium point E_3 of the system (2) is locally asymptotically stable if satisfy the condition $M > 1,$ and $h_2 < \beta_2 + 1$.

Proof: The Jacobian matrix evaluated at E_3 gives

$$J(E_3) = \begin{pmatrix} r_1(1-M) & 0 & \dfrac{g_1 r_1 (M-1)}{\beta_1 - (M-1)} \\ 0 & -r_2 & \dfrac{g_2 r_2}{1+\beta_2} \\ 0 & 0 & e\dfrac{(M-1)(1-h_1)-\beta_1}{\beta_1-(M-1)} + \dfrac{eh_2}{1+\beta_2} \end{pmatrix}$$

The eigenvalues of the matrix $J(E_3)$ are $r_1(1-M)$, $-r_2$ and $e\dfrac{(M-1)(1-h_1)-\beta_1}{\beta_1-(M-1)} + \dfrac{eh_2}{1+\beta_2}$. All the eigenvalues are negative if $M > 1, 0 < h_1 < 1$ and $h_2 < \beta_2 + 1$. This shows the equilibrium point E_3 of the system is locally asymptotically stable.

Lemma: 5 The boundary equilibrium point E_4 of the system (2) is locally asymptotically stable if satisfy the condition $M < 1,$ and $h_1 > 1$

Proof: The Jacobian matrix evaluated at E_4 gives

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$$J(E_4)=\begin{pmatrix} A_1^{\bullet} & 0 & \dfrac{-g_1r_1x_1^*}{(1+x_1^*)} \\ 0 & B_1^{\bullet} & 0 \\ \dfrac{h_1ez^*}{(1+x_1^*)^2} & \dfrac{h_2ez^*}{(1+z_1^*)} & 0 \end{pmatrix}$$

Where $A_1^* = r_1 - 2\beta_1 r_1 x^* - \dfrac{g_1r_1z^*}{(1+x^*)^2} - Mr_1$ and $B_1^* = r_2 - \dfrac{g_2r_2z^*}{(1+z^*)^2}$ and here $x_1^* = \dfrac{1}{h_1-1}$ and $z_1^* = \dfrac{h_1}{g_1(1-h_1)^2}[(h_1-1)(1-M)-\beta_1])$. The eigenvalues of the matrix $J(E_4)$ are negative real parts if A_1^* <0 and B_1^* <0. This clearly gives $\beta_1 < (1-M)(h_1-1) < 2\beta_1$ and $h_1 > 1$. Therefore the equilibrium point E_4 of the system is locally asymptotically stable.

Lemma: 6 The boundary equilibrium point E_5 of the system (2) is locally asymptotically stable if satisfy the condition $M > 1$, and $h_2 > 1$.

Proof: The Jacobian matrix evaluated at E_5 gives

$$J(E_5)=\begin{pmatrix} r_1 - g_1r_1z^* - Mr_1 & 0 & 0 \\ 0 & r_2 - 2\beta_2 r_2 y^* - \dfrac{g_2r_2z^*(1+z^*)}{(h_2y^*)^2} & \dfrac{-g_2r_2y^*(1+y^*)}{(h_2y^*)^2} \\ h_1ez^* & \dfrac{h_2ez^*(1+z^*)}{(hy^*)^2} & -e+\dfrac{e(1+y^*)}{(h_2y^*)^2} \end{pmatrix}$$ The matrix $J(E_5)$ will have eigenvalues with negative real part, if these conditions will hold if for $M > 1$, and $h_2 > 1$.

Theorem: The positive equilibrium point $E_6(x^*, y^*, z^*)$ will exist if $h_1 > 1$, the following Condition are satisfied $g_2 > \beta_2, x > 1, 0 < y < \dfrac{1}{\beta_2}$.

Proof: In order to find coexistence equilibrium point, we equate the equation to zero. From this we find two functions $f(x, y)$ and $g(x, y)$ which intersect at the equilibrium point $E_6(x^*, y^*, z^*)$. Equating the equation (2) to zero gives

$$r_1x[(1-\beta_1x) - \frac{g_1.z}{1+x} - M]=0$$

$$r_2y[(1-\beta_2y) - \frac{g_2.z}{1+y+z}]=0 \qquad (4)$$

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$$ez[\frac{h_1 x}{1+x}+\frac{h_2 y}{1+y+z}-1]=0$$

By applying elementary calculation

$$z^* = \frac{h_2 y^*(1+x^*)(1-\beta_2 y^*)}{g_2(1+x^*-x^* h_1)} \quad , \qquad f(x,y) = \frac{h_2 y^*(1+x^*)}{g_2(1+x^*-x^* h_1)} - \frac{(1+y^*)}{g_2-(1-\beta_2 y^*)}$$

and $g(x,y) = \dfrac{(1-\beta_1 x^* - M)}{g_1} - \dfrac{h_2 y^*(1-\beta_2 y^*)}{g_2(1+x^*-x^* h_1)}$

To prove the existence of $E_6(x^*, y^*, z^*)$ under which two function meet in the interior of the point (x, y) at a point (x^*, y^*) are found. Let $x \to 0$ and $y \to x_{2f}$ is given by

$x_{2f} = \dfrac{-q+\sqrt{q^2-4pr}}{2p}$ Where $p = h_2\beta_2$, $q = h_2 g_2 - g_2 - h_2$ and $r = -g_2$. It is clearly shows x_{2f} is positive and real $r < 0$.

Similarly Let $x \to 0$ and $y \to x_{2g}$ is given by $x_{2g} = \dfrac{-v+\sqrt{v^2-4uw}}{2u}$, Where $u = g_1 h_2 \beta_2$, $v = -h_2$ and $w = g_2(1-M)$ it shows that x_{2g} is positive and real if $M > 1$.

Also find the derivative of function x_{2f} with respect to x, y ,we notice that $\dfrac{\partial f}{\partial x} > 0, \dfrac{\partial f}{\partial y} < 0$, it requires

$g_2 > \beta_2, x > 1$, $h_1 > 1$

Similarly the derivative of function x_{2g} with respect to x, y, we notice that $\dfrac{\partial g}{\partial x} < 0, \dfrac{\partial g}{\partial y} < 0$, it requires

$g_2 > \beta_2, x > 1$, $0 < y < \dfrac{1}{\beta_2}$ and $h_1 > 1$

Theorem: The co-existence equilibrium point $E_6(x^*, y^*, z^*)$ is globally asymptotically stable with respect to all solutions initiating in the interior of Ω satisfy the following conditions

$$z^* < \min(\frac{\beta_2 R_2}{g_2}, \frac{\beta_1 R_1}{g_1}), x^* = \frac{h_1 - g_1}{g_1}, y^* < \frac{h_2(g_1+\beta_1 R_1) - g_1 g_2}{g_1 g_2}, h_1 > g_1, h_2 > g_2$$

Proof: The proof can be reached by using Lypunov stability theorem which gives sufficient condition .Now let us consider a positive definite function $V(x, y, z)$

$$V(x,y,z) = \frac{1}{r_1}V_1(x,y,z) + \frac{1}{r_2}V_2(x,y,z) + \frac{1}{e}V_3(x,y,z) \qquad (5)$$

in the interior of the positive octant, Where

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$$V_1(x,y,z)=(x-x^*)-x^*\ln(\frac{x}{x^*}) \quad ; \quad V_1(x,y,z)=(y-y^*)-y^*\ln(\frac{y}{y^*})$$

and $$V_1(x,y,z)=(z-z^*)-z^*\ln(\frac{z}{z^*})$$

Note that

$$\frac{dV_1}{dt}=(x-x^*)(1-\beta_1 x)-\frac{g_1.z}{1+x}-M)$$
$$\frac{dV_2}{dt}=(y-y^*)(1-\beta_2 y)-\frac{g_2.z}{1+y+z}) \qquad (6)$$
$$\frac{dV_3}{dt}=(z-z^*)(\frac{h_1 x}{1+x}+\frac{h_2 y}{1+y+z}-1),$$

Differentiate (5) with respect to time t and substitute (6) in that expression which simplifies

$$\frac{dV}{dt}=-x^T Ax \quad \text{where} \quad x^T=(x-x^*,y-y^*,z-z^*)$$

and A is symmetric matrix given by

$$A=\begin{pmatrix} \beta_1-\frac{g_1 z^*}{(1+x)(1+x^*)} & 0 & \frac{g_1(1+x^*)-h_1}{2(1+x)(1+x^*)} \\ 0 & \frac{-g_2 z^*}{(1+y+z)(1+y^*+z*)}+\beta_2 & \frac{g_2(1+y^*)-h_2(1+z^*)}{2(1+y+z)(1+y^*+z*)} \\ \frac{g_1(1+x^*)-h_1}{2(1+x)(1+x^*)} & \frac{g_2(1+y^*)-h_2(1+z^*)}{2((1+y+z)(1+y^*+z*)} & \frac{h_2 y^*}{(1+y+z)(1+y^*+z*)} \end{pmatrix}$$

We notice that $\frac{dV}{dt}<0$ if the matrix A is positive definite and A satisfy the condition

$$z^*<\min(\frac{\beta_2 R_2}{g_2},\frac{\beta_1 R_1}{g_1}), x^*=\frac{h_1-g_1}{g_1}, y^*<\frac{h_2(g_1+\beta_1 R_1)-g_1 g_2}{g_1 g_2}, h_1>g_1, h_2>g_2$$

This completes the proof.

IV. CONCLUSION

The use of harvesting efforts as to control to obtain strategies for the control of prey-predator system with non- monotonic functional responses and it shows a method how to control a prey- predator system and drive the state either to equilibrium points or to a limit cycle [8]. In paper we analyzed the dynamical behavior of three species food web with two distinct functional responses of Beddington- Deangelis and Holling type-II functional responses. We also obtain the condition for existence of different equilibrium and discussed their stabilities in local and global manners by using stability theory of differential equations. Finally we investigated the dynamical behavior of the system consisting of two preys with different functional responses and a predator. We also studied the effect of harvesting on prey species. We concluded that harvesting is strong impact on the dynamics evaluation of population. To a certain extent it can control the long term stationary density of population efficiently. The local and global analyses were carried out at the end.

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