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INTUITIONISTIC FUZZY MULTI-OBJECTIVE NON LINEAR PROGRAMMING PROBLEM (IFMONLPP) USING TAYLOR SERIES APPROACH

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ABSTRACT: This paper presents a method to solve intuitionistic fuzzy multi-objective non- linear programming problem (IFMONLPP) using a Taylor series. In the proposed approach, membership and non-membership functions associated with each objective of multi-objective non-linear programming problem (MONLPP) are transformed to a single objective linear programming problem by using a first order Taylor polynomial series. At the end, numerical examples are provided to illustrate the efficiency of the proposed method.

KEYWORDS: Intuitionistic fuzzy multi-objective non- linear programming problem (IFMONLPP), membership and non-membership functions, Taylor series.

I. INTRODUCTION

The concept of maximizing decision was initially proposed by Bellman and Zadeh [3], Zadeh [10 - 12]. By adopting this concept of fuzzy sets was applied in mathematical programs firstly by Zimmerman[12].In the past few years, many researchers have come to the realization that a variety of real world problems which have been previously solved by non-linear programming techniques are in fact more complicated. Frequently, these problems have multiple goals to be optimized rather than a single objective. Moreover, many practical problems cannot be represented by non-linear programming model. Therefore, attempts were made to develop more general mathematical programming methods and many significant advances have been made in the area of multi-objective non-linear programming. Several authors in the literature have studied the fuzzy multi- objective quadratic programming problems.

Out of several higher order fuzzy sets, intuitionistic fuzzy sets (IFS) [1,2] have been found to be highly useful to deal with vagueness. There are situations where due to insufficiency in the information available, the evaluation of membership values is not also always possible and consequently there remains a part indeterministic on which hesitation survives. Certainly fuzzy sets theory is not appropriate to deal with such problems; rather intuitionistic fuzzy sets (IFS) theory is more suitable**.** The Intuitionistic fuzzy set was introduced by Atanassov.K.T [1] in 1986.For the fuzzy multiple criteria decision making problems, the degree of satisfiability and non- satisfiability of each alternative with respect to a set of criteria is often represented by an intuitionistic fuzzy number (IFN). This Intuitionistic fuzzy mathematics is very little studied subject. In a recent review, Toksari [4] gave a Taylor series approach to fuzzy multi-objective fractional programming problem. A. Nagoorgani, R. Irene Hepzibah et al., [6] proposed a method to solve multi-objective fuzzy quadratic programming problem using Taylor series. However, many methods of solving multi-objective quadratic programming problems are available in the literature [4,7,8,13]. In this paper, membership and non-membership functions, which are associated with each objective of intuitionistic fuzzy multi-objective quadratic programming problem (IFMOQPP) are transformed by using first order Taylor polynomial series [4,8]. Then the I FMOQPP can be reduced to a single objective linear programming. The paper is organized as follows:

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The formulation of the problem is given in Section2 and Section3 deals with an algorithm for solving a intuitionistic fuzzy multi-objective quadratic programming problem. Finally, inSection4, the effectiveness of the proposed method is illustrated by means of an example. Some concluding remarks are provided in section5.

II. FORMATION OF THE PROBLEM

The multi-objective linear programming problem and the multi- objective intuitionistic fuzzy linear programming problem are described in this section.

A. Multi-objective quadratic programming problem (MOQPP)

A linear multi-objective optimization problem is stated as

Maximize or Minimize: $[z_1(x), z_2(x),\ldots\ldots, z_k(x)]$

Subject to $Ax \begin{pmatrix}\leq\\=\\\geq\end{pmatrix} b, x\geq 0$

where $z_j(x)$, j = 1, 2,… n is an N vector of cost coefficients, A an m x N – Coefficients matrix of constraints and b an m vector of demand (resource) availability.

B. Intuitionistic fuzzy multi-objective quadratic programming problem (IFMOQPP)

If an imprecise aspiration level is introduced to each of the objectives of MOQPP, then these intuitionistic fuzzy objectives are termed as intuitionistic fuzzy goals.

Let g_k^I be the aspiration level assigned to the k th objective $z_k(x)$.Then the intuitionistic fuzzy objectives appear as

(i) $Z_k(x) \gtrsim g_k{}^I$ (for maximizing $Z_k(x)$);

(ii) $Z_k(x) \lesssim g_k{}^I$ (for minimizing $Z_k(x)$);

where ≥ and ≤ indicate the fuzziness of the aspiration levels, and is to be understood as “essentially more than” and “essentially less than in the sense of Zimmerman[12].

Hence, the intuitionistic fuzzy multi-objective linear programming problem can be stated as follows:

Find X

so as to satisfy $Z_i(x) \lesssim g_i{}^I$, i=1,2,...............i_l, $Z_i(x) \gtrsim g_i{}^I$,i=$i_{l+1}, i_{l+2}, \ldots\ldots\ldots\ldots\ldots$,k

Subject to $x \in X$, $X \geq 0$.

Now, in the field of intuitionistic fuzzy programming, the intuitionistic fuzzy objectives are characterized by their associated membership functions and non-membership functions . They can be expressed as follows:

$$\text{If } Z_i(x) \gtrsim g_i{}^I,\ \mu_i{}^I(x) = \begin{cases} 1, & if\, z_i(x) \geq g_i \\ \frac{z_i(x)-\underline{t_i}}{g_i-\underline{t_i}}, & if\, \underline{t_i} \leq z_i(x) \leq g_i \\ 0, & if\, z_i(x) \leq \underline{t_i} \end{cases} \text{ and } \nu_i{}^I(x) = \begin{cases} 0, & if\, z_i(x) \geq g_i \\ \frac{z_i(x)-g_i}{\underline{t_i}-g_i}, & if\, \underline{t_i} \leq z_i(x) \leq g_i \\ 1, & if\, z_i(x) \leq \underline{t_i} \end{cases}$$

$$\text{If } Z_i(x) \lesssim g_i{}^I,\ \mu_i{}^I(x) = \begin{cases} 1, & if\ z_i(x) \leq g_i \\ \frac{\overline{t_i}-z_i(x)}{\overline{t_i}-g_i}, & if\, g_i \leq z_i(x) \leq \overline{t_i} \\ 0, & if\ z_i(x) \leq \overline{t_i} \end{cases} \text{ and } \nu_i{}^I(x) = \begin{cases} 0, & if\ z_i(x) \geq g_i \\ \frac{g_i-z_i(x)}{\underline{t_i}-g_i}, & if\, \underline{t_i} \leq z_i(x) \leq g_i \\ 1, & if\ z_i(x) \leq \overline{t_i} \end{cases}$$

where $\overline{t_i}$ and $\underline{t_i}$ are the upper tolerance limit and the lower tolerance limit respectively, for the i th intuitionistic fuzzy objective.

Now, in a intuitionistic fuzzy decision environment, the achievement of the objective goals to their aspired levels to the extent possible are actually represented by the possible achievement of their respective membership values and non-membership values to the highest degree. The relationship between constraints and the objective functions in the intuitionistic fuzzy environment is fully symmetric, that is, there is no longer a difference between the former and the latter . This guarantees the maximization of both objectives membership values and non-membership values

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simultaneously.

III. ALGORITHM FOR INTUITIONISTIC FUZZY MULTI-OBJECTIVE QUADRATIC PROGRAMMING PROBLEM

Toksari [4] proposed a Taylor series approach to fuzzy multi-objective linear fractional programming. Here, in the intuitionistic fuzzy multi-objective quadratic programming problem, membership functions and non-membership functions associated with each objective are transformed by using Taylor series at first and then a satisfactory value(s) for the variable(s) of the model is obtained by solving the intutionistic fuzzy model, which has a single objective function. Based on this idea, an algorithm for solving intuitionistic fuzzy multi-objective quadratic programming problem is developed here.

Step 1. Determine $x_i^*=(x_{i1}^*, x_{i2}^*, \ldots\ldots\ldots, x_{in}^*)$, that is used to maximize or minimize the i[th] membership function $\mu_i{}^I(x)$ and non-membership function $\nu_i{}^I(x)$ (i=1,2,……….k) where n is the number of variables.

Step 2. Transform membership and non-membership functions by using first-order Taylor polynomial series

$$\mu_i{}^I(x) \cong \widehat{\mu_i{}^I}(x) = \mu_i{}^I(x_i{}^*) + [(x_1 - x_{i1}{}^*)\frac{\partial \mu_i{}^I(x_i{}^*)}{\partial x_1} + (x_2 - x_{i2}{}^*)\frac{\partial \mu_i{}^I(x_i{}^*)}{\partial x_2} + \ldots\ldots + (x_n - x_{in}{}^*)\frac{\partial \mu_i{}^I(x_i{}^*)}{\partial x_n}]$$

$$\mu_i{}^I(x) \cong \widehat{\mu_i{}^I}(x) = \mu_i{}^I(x_i{}^*) + \sum_{j=1}^{n}(x_j - x_{ij}{}^*)\frac{\partial \mu_i{}^I(x_i{}^*)}{\partial x_j}$$

$$\nu_i{}^I(x) \cong \widehat{\nu_i{}^I}(x) = \nu_i{}^I(x_i{}^*) + [(x_1 - x_{i1}{}^*)\frac{\partial \nu_i{}^I(x_i{}^*)}{\partial x_1} + (x_2 - x_{i2}{}^*)\frac{\partial \nu_i{}^I(x_i{}^*)}{\partial x_2} + \ldots\ldots + (x_n - x_{in}{}^*)\frac{\partial \nu_i{}^I(x_i{}^*)}{\partial x_n}]$$

$$\nu_i{}^I(x) \cong \widehat{\nu_i{}^I}(x) = \nu_i{}^I(x_i{}^*) + \sum_{j=1}^{n}(x_j - x_{ij}{}^*)\frac{\partial \nu_i{}^I(x_i{}^*)}{\partial x_j}$$

Step 3. Find satisfactory $x^* = (x_1^*, x_2^*, \ldots\ldots\ldots, x_n^*)$ by solving the reduced problem to a single objective for membership function and non-membership function respectively.

$$p(x) = \sum_{i=1}^{k} \mu_i{}^I(x_i{}^*) + \sum_{j=1}^{n}(x_j - x_{ij}{}^*)\frac{\partial \mu_i{}^I(x_i{}^*)}{\partial x_j}$$

and $q(x) = \sum_{i=1}^{k} \nu_i{}^I(x_i{}^*) + \sum_{j=1}^{n}(x_j - x_{ij}{}^*)\frac{\partial \nu_i{}^I(x_i{}^*)}{\partial x_j}$

Thus IFMOQPP is converted into a new mathematical model and is given below:

Maximize or Minimize $\sum_{i=1}^{k} \mu_i{}^I(x_i{}^*) + \sum_{j=1}^{n}(x_j - x_{ij}{}^*)\frac{\partial \mu_i(x_i{}^*)}{\partial x_j}$ and

Maximize or Minimize $\sum_{i=1}^{k} \nu_i{}^I(x_i{}^*) + \sum_{j=1}^{n}(x_j - x_{ij}{}^*)\frac{\partial \nu_i{}^I(x_i{}^*)}{\partial x_j}$

where $\mu_i{}^I(x) = \begin{cases} 1, & if\, z_i(x) \geq g_i \\ \frac{z_i(x)-\underline{t_i}}{g_i-\underline{t_i}}, & if\, \underline{t_i} \leq z_i(x) \leq g_i \\ 0, & if\, z_i(x) \leq \underline{t_i} \end{cases}$ and $\nu_i{}^I(x) \begin{cases} 0, & if\, z_i(x) \geq g_i \\ \frac{z_i(x)-g_i}{\underline{t_i}-g_i}, & if\, \underline{t_i} \leq z_i(x) \leq g_i \\ 1, & if\, z_i(x) \leq \underline{t_i} \end{cases}$

and $\mu_i{}^I(x) = \begin{cases} 1, & if\, z_i(x) \leq g_i \\ \frac{\overline{t_i}-z_i(x)}{\overline{t_i}-g_i}, & if\, g_i \leq z_i(x) \leq \overline{t_i} \\ 0, & if\, z_i(x) \leq \overline{t_i} \end{cases}$ and $\nu_i{}^I(x) = \begin{cases} 0, & if\, z_i(x) \geq g_i \\ \frac{g_i-z_i(x)}{\underline{t_i}-g_i}, & if\, \underline{t_i} \leq z_i(x) \leq g_i \\ 1, & if\, z_i(x) \leq \overline{t_i} \end{cases}$

respectively.

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IV. ILLUSTATIVE EXAMPLE

Consider the following MOQPP:

$$\left.\begin{array}{l} \text{Minimize } z_1(x) = -4x_1 + x_1^2 - 2x_1x_2 + 2x_2^2 \\ \text{Minimize } z_2(x) = -3x_1 + x_1^2 + 2x_1x_2 + 3x_2^2 \\ \text{Subject to the constraints } 2x_1+x_2 \le 6 \\ \qquad x_1+4x_2 \le 12 \text{ and } x_1, x_2 \ge 0. \end{array}\right\} \quad (4.1)$$

a) The membership and non-membership functions were considered to be intuitionistic triangular (see figure 1) [5]. When they depend on three scalar parameters (a_1,b_1,c_1). z_1 depends on intutionistic fuzzy aspiration levels (-113,-6.5,100) when z_2 depends intutionistic fuzzy aspiration levels (-240,5,250). The membership and non-membership functions of the goals are obtained as follows:

$$\mu_1^I(x) = \begin{cases} 0, & if\ z_1(x) \ge c_1 \\ \frac{c_1 - z_1(x)}{c_1 - b_1}, & if\ b_1 \le z_1(x) \le c_1 \\ \frac{z_1(x) - a_1}{b_1 - a_1}, & if\ a_1 \le z_1(x) \le b_1 \\ 0, & if\ z_1(x) \le a_1 \end{cases}$$

That implies

$$\mu_1^I(x) = \begin{cases} 0 & , if\ z_1(x) \ge 100 \\ \frac{100-(-4x_1+x_1^2-2x_1x_2+2x_2^2)}{100-(-6.5)} & , if\ -6.5 \le z_1(x) \le 100 \\ \frac{(-4x_1+x_1^2-2x_1x_2+2x_2^2)-(-113)}{-6.5-(-113)} & , if\ -113 \le z_1(x) \le -6.5 \\ 0 & , if\ z_1(x) \le -113 \end{cases}$$

In the similar way,

$$\mu_2^I(x) = \begin{cases} 0 & , if\ z_2(x) \ge 250 \\ \frac{250-(-3x_1+x_1^2+2x_1x_2+3x_2^2)}{250-5} & , if 5 \le z_2(x) \le 250 \\ \frac{(-3x_1+x_1^2+2x_1x_2+3x_2^2)-(-240)}{5-(-240)} & , if\ -240 \le z_2(x) \le 5 \\ 0 & , if\ z_2(x) \le -240 \end{cases}$$

If $\mu_1^I(x) = \max(\min(\frac{100-(-4x_1+x_1^2-2x_1x_2+2x_2^2)}{100-(-6.5)}, \frac{(-4x_1+x_1^2-2x_1x_2+2x_2^2)-(-113)}{-6.5-(-113)}), 0)$ and

$\mu_2^I(x) = \max(\min(\frac{250-(-3x_1+x_1^2+2x_1x_2+3x_2^2)}{250-5}, \frac{(-3x_1+x_1^2+2x_1x_2+3x_2^2)-(-240)}{5-(-240)}), 0)$ then $\mu_1^{I*}(2.46,1.08)$ and $\mu_2^{I*}(0.89,0.22)$.

The membership and non-membership functions are transformed by using first-order Taylor polynomial series

$$\mu_1^I(x) \cong \widehat{\mu_1}^I(x) = \mu_1^I(2.46,1.08) + [(x_1 - 2.46)\frac{\partial \mu_1^I(2.46,1.08)}{\partial x_1} + (x_2 - 1.08)\frac{\partial \mu_1^I(2.46,1.08)}{\partial x_2}]$$

$$\mu_1^I(x) = 0.012x_1 - 0.006x_2 + 0.974 \qquad (4.2)$$

$$\mu_2^I(x) \cong \widehat{\mu_2}^I(x) = \mu_2^I(0.89,0.22) + [(x_1 - 0.89)\frac{\partial \mu_2^I(0.89,0.22)}{\partial x_1} + (x_2 - 0.22)\frac{\partial \mu_2^I(0.89,0.22)}{\partial x_2}]$$

$$\mu_2^I(x) = 0.003x_1 - 0.013x_2 + 0.974 \qquad (4.3)$$

Then the objective of the IFMOQPP is obtained by adding (4.2) and (4.3), that is

$p(x) = \widehat{\mu_1}^I(x) + \widehat{\mu_2}^I(x) = 0.015x_1 - 0.019x_2 + 1.948$

Subject to the constraints $2x_1+x_2 \le 6$

$x_1+4x_2 \le 12$

The problem is solved and the solution is obtained is as follows: x_1=2.67; x_2= 0.67; $z_1(x)$= 6.2231; $z_2(x)$= -4.0043 and the membership values are μ_1 = 0.881 and μ_2 = 0.963.The membership function values show that both goals z_1 and z_2 are satisfied with 88.1% and 96.3% respectively for the obtained solution which is x_1=2.67; x_2= 0.67.

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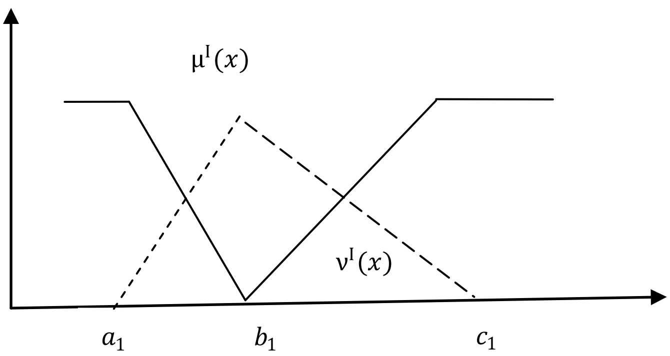

(a)

Fig. 1 Membership and non-membership functions defined as intuitionistic triangular (a)

The non-membership functions of the goals are as obtained as follows:

$$\nu_1^{I}(x) = \begin{cases} 1, if\ z_1(x) \geq c_1 \\ \frac{z_1(x)-b_1}{c_1-b_1}, if\ b_1 \leq z_1(x) \leq c_1 \\ \frac{b_1-z_1(x)}{b_1-a_1}, if\ a_1 \leq z_1(x) \leq b_1 \\ 1, if\ z_1(x) \leq a_1 \end{cases}$$

$$= \begin{cases} 1 & , if z_1(x) \geq 100 \\ \frac{(-4x_1+x_1^2-2x_1x_2+2x_2^2)-(-6.5)}{100-(-6.5)} & , if\ -6.5 \leq z_1(x) \leq 100 \\ \frac{(-6.5)-(-4x_1+x_1^2-2x_1x_2+2x_2^2)}{-6.5-(-113)} & , if\ -113 \leq z_1(x) \leq -6.5 \\ 1 & , if\ z_1(x) \leq -113 \end{cases}$$

In the similar way, $\nu_2^{I}(x) = \begin{cases} 1 & , if\ z_2(x) \geq 250 \\ \frac{(-3x_1+x_1^2+2x_1x_2+3x_2^2)-5}{250-5} & , if\ 5 \leq z_2(x) \leq 250 \\ \frac{5-(-3x_1+x_1^2+2x_1x_2+3x_2^2)}{5-(-240)} & , if\ -240 \leq z_2(x) \leq 5 \\ 1 & , if\ z_2(x) \leq -240 \end{cases}$

If $\nu_1^{I}((x)$ = max (min $(\frac{(-4x_1+x_1^2-2x_1x_2+2x_2^2)-(-6.5)}{100-(-6.5)}, \frac{(-6.5)-(-4x_1+x_1^2-2x_1x_2+2x_2^2)}{-6.5-(-113)})$, 1) and $\nu_2^{I}(x)$ = max (min $(\frac{(-3x_1+x_1^2+2x_1x_2+3x_2^2)-5}{250-5}, \frac{5-(-3x_1+x_1^2+2x_1x_2+3x_2^2)}{5-(-240)})$, 1), then $\nu_1^{I*}(2.46,1.08)$ and $\nu_2^{I*}(0.89,0.22)$.

The non-membership functions are transformed by using first-order Taylor polynomial series

$$\nu_1^{I}(x) \cong \widehat{\nu_1}^{I}(x) = \nu_1^{I}(2.46,1.08) + [(x_1 - 2.46)\frac{\partial \nu_1^{I}(2.46,1.08)}{\partial x_1} + (x_2 - 1.08)\frac{\partial \nu_1^{I}(2.46,1.08)}{\partial x_2}]$$

$\nu_1^{I}(x) = 0.012x_1-0.006x_2+0.977$ (4.4)

$$\nu_2^{I}(x) \cong \widehat{\nu_2}^{I}(x) = \nu_2(0.89,0.22) + [(x_1 - 0.89)\frac{\partial \nu_2^{I}(0.89,0.22)}{\partial x_1} + (x_2 - 0.22)\frac{\partial \nu_2^{I}(0.89,0.22)}{\partial x_2}]$$

$\nu_2^{I}(x) = 0.003x_1-0.013x_2+1$ (4.5)

Then the objective of the IFMOQPP is obtained by adding (4.4) and (4.5), that is

$q(x) = \widehat{\nu_1}^{I}(x) + \widehat{\nu_2}^{I}(x) = 0.015x_1-0.019x_2+1.977$

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Subject to the constraints $2x_1+x_2\leq 6$

$x_1+4x_2\leq 12$

The problem is solved and the solution is obtained is as follows: x_1=2.67; x_2= 0.67; $z_1(x)$= 6.2231; $z_2(x)$= -4.0043 and the non-membership values are ν_1= 0.119 and ν_2= 0.037.The non-membership function values show that both goals z_1 and z_2 are satisfied with 11.9% and 3.7% respectively for the obtained solution which is x_1=2.67; x_2= 0.67.

b) The membership and non-membership functions were considered to be intuitionistic trapezoidal (see figure 2) [5] when they depend on four scalar parameters (a_1,b_1,c_1,d_1). z_1 depends on intuitionistic fuzzy aspiration levels (5,6.5,8,9.5) when z_2 depends on intuitionistic fuzzy aspiration levels (-8.5,-4,0.5,5). The membership and non-membership functions of the goals are obtained as follows:

$$\mu_1{}^I(x) = \begin{cases} 0 & , if\ z_1(x) \geq d_1 \\ \frac{d_1-z_1(x)}{d_1-c_1} & , if\ c_1 \leq z_1(x) \leq d_1 \\ 1 & , if\ b_1 \leq z_1(x) \leq c_1 \\ \frac{z_1(x)-a_1}{b_1-a_1} & , if\ a_1 \leq z_1(x) \leq b_1 \\ 0 & , if\ z_1(x) \leq a_1 \end{cases} = \begin{cases} 0 & , if\ z_1(x) \geq 9.5 \\ \frac{9.5-(-4x_1+x_1^2-2x_1x_2+2x_2^2)}{9.5-8} & , if\ 8 \leq z_1(x) \leq 9.5 \\ 1 & , if\ 6.5 \leq z_1(x) \leq 8 \\ \frac{(-4x_1+x_1^2-2x_1x_2+2x_2^2)-(5)}{6.5-5} & , if 5 \leq z_1(x) \leq 6.5 \\ 0 & , if\ z_1(x) \leq 5 \end{cases}$$

In the similar way,

$$\mu_2{}^I(x) = \begin{cases} 0 & , if\ z_1(x) \geq 5 \\ \frac{5-(-3x_1+x_1^2+2x_1x_2+3x_2^2)}{5-0.5} & , if\ 0.5 \leq z_1(x) \leq 5 \\ 1 & , if\ -4 \leq z_1(x) \leq 0.5 \\ \frac{(-3x_1+x_1^2+2x_1x_2+3x_2^2)-(-8.5)}{4-(-8.5)} & , if\ -8.5 \leq z_1(x) \leq -4 \\ 0 & , if\ z_1(x) \leq -8.5 \end{cases}$$

If $\mu_1{}^I(x)$ = max (min $(\frac{9.5-(-4x_1+x_1^2-2x_1x_2+2x_2^2)}{9.5-8}, \frac{(-4x_1+x_1^2-2x_1x_2+2x_2^2)-(5)}{6.5-5})$, 0) and $\mu_2{}^I(x)$ = max (min $(\frac{5-(-3x_1+x_1^2+2x_1x_2+3x_2^2)}{5-0.5}, \frac{(-3x_1+x_1^2+2x_1x_2+3x_2^2)-(-8.5)}{4-(-8.5)})$, 0), then $\mu_1{}^{I*}(2.46,1.08)$ and $\mu_2{}^{I*}(0.89,0.22)$.

The membership and non-membership functions are transformed by using first-order Taylor polynomial series

$$\mu_1{}^I(x) \cong \widehat{\mu_1{}^I}(x) = \mu_1(2.46,1.08) + [(x_1-2.46)\frac{\partial \mu_1{}^I(2.46,1.08)}{\partial x_1} + (x_2-1.08)\frac{\partial \mu_1{}^I(2.46,1.08)}{\partial x_2}]$$

$$\mu_1{}^I(x) = 0.83x_1-0.4x_2-0.61 \qquad (4.6)$$

$$\mu_2{}^I(x) \cong \widehat{\mu_2{}^I}(x) = \mu_2(0.89,0.22) + [(x_1-0.89)\frac{\partial \mu_2{}^I(0.89,0.22)}{\partial x_1} + (x_2-0.22)\frac{\partial \mu_2{}^I(0.89,0.22)}{\partial x_2}]$$

$$\mu_2{}^I(x) = 0.71x_1-0.69x_2+1 \qquad (4.7)$$

Then the objective of the FMOLPP is obtained by adding (4.6) and (4.7), that is

$p(x) = \widehat{\mu_1{}^I}(x) + \widehat{\mu_2{}^I}(x) = x_1-1.09x_2+0.39$

Subject to the constraints $2x_1+x_2\leq 6$

$x_1+4x_2\leq 12$

The problem is solved and the solution is obtained as follows: x_1=2.67; x_2= 0.67; $z_1(x)$= 6.2231; $z_2(x)$= -4.0043 and the membership values are μ_1 = 0.8154 and μ_2 = 1. The membership function values show that both goals z_1 and z_2 are satisfied with 81.54% and 100% respectively for the obtained solution which is x_1=2.67; x_2= 0.67.

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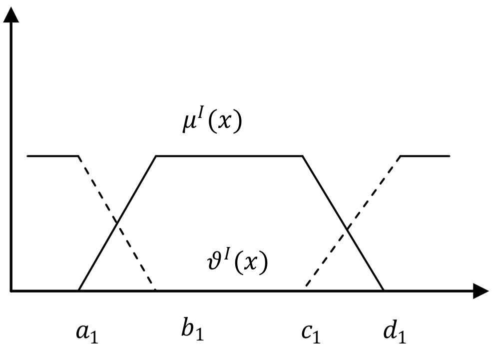

(b)

Fig. 2 Membership and non-membership functions defined as intuitionistic trapezoidal (b)

The non-membership functions of the goals are as obtained as follows:

$$\nu_1{}^I(x) = \begin{cases} 1, & if\ z_1(x) \ge d_1 \\ \frac{z_1(x)-c_1}{d_1-c_1}, & if\ c_1 \le z_1(x) \le d_1 \\ 0, & if\ b_1 \le z_1(x) \le c_1 \\ \frac{b_1-z_1(x)}{b_1-a_1}, & if\ a_1 \le z_1(x) \le b_1 \\ 1, & if\ z_1(x) \le a_1 \end{cases} = \begin{cases} 1, & if\ z_1(x) \ge 9.5 \\ \frac{(-4x_1+x_1^2-2x_1x_2+2x_2^2)-8}{9.5-8}, & if\ 8 \le z_1(x) \le 9.5 \\ 0, & if\ 6.5 \le z_1(x) \le 8 \\ \frac{6.5-(-4x_1+x_1^2-2x_1x_2+2x_2^2)}{6.5-5}, & if 5 \le z_1(x) \le 6.5 \\ 1, & if\ z_1(x) \le 5 \end{cases}$$

In the similar way,

$$\nu_2{}^I(x) = \begin{cases} 1, & if\ z_1(x) \ge 5 \\ \frac{(-3x_1+x_1^2+2x_1x_2+3x_2^2)-0.5}{5-0.5}, & if\ 0.5 \le z_1(x) \le 5 \\ 0, & if\ -4 \le z_1(x) \le 0.5 \\ \frac{-4-(-3x_1+x_1^2+2x_1x_2+3x_2^2)}{4-(-8.5)}, & if\ -8.5 \le z_1(x) \le -4 \\ 1, & if\ z_1(x) \le -8.5 \end{cases}$$

If $\nu_1{}^I(x)$ = max (min ($\frac{(-4x_1+x_1^2-2x_1x_2+2x_2^2)-8}{9.5-8}$, $\frac{6.5-(-4x_1+x_1^2-2x_1x_2+2x_2^2)}{6.5-5}$), 1) and $\nu_2{}^I(x)$=max(min($\frac{(-3x_1+x_1^2+2x_1x_2+3x_2^2)-0.5}{5-0.5}$, $\frac{-4-(-3x_1+x_1^2+2x_1x_2+3x_2^2)}{4-(-8.5)}$), 1), then $\nu_1{}^{I*}(2.46,1.08)$ and $\nu_2{}^{I*}(0.89,0.22)$.

The non-membership functions are transformed by using first-order Taylor polynomial series

$$\nu_1{}^I(x) \cong \widehat{\nu_1}^I(x) = \nu_1{}^I(2.46,1.08) + [(x_1-2.46)\frac{\partial \nu_1{}^I(2.46,1.08)}{\partial x_1} + (x_2-1.08)\frac{\partial \nu_1{}^I(2.46,1.08)}{\partial x_2}]$$

$$\nu_1{}^I(x) = 0.83x_1-0.4x_2-1.61 \quad (4.8)$$

$$\nu_2{}^I(x) \cong \widehat{\nu_2}^I(x) = \nu_2{}^I(0.89,0.22) + [(x_1-0.89)\frac{\partial \nu_2{}^I(0.89,0.22)}{\partial x_1} + (x_2-0.22)\frac{\partial \nu_2{}^I(0.89,0.22)}{\partial x_2}]$$

$$\nu_2{}^I(x) = 0.17x_1-0.69x_2 \quad (4.9)$$

Then the objective of the FMOLPP is obtained by adding (4.6) and (4.7), that is

$q(x) = \widehat{\nu_1}^I(x) + \widehat{\nu_2}^I(x)) = x_1-1.09x_2-1.61$

Subject to the constraints $2x_1+x_2\le 6$

$x_1+4x_2\le 12$

The problem is solved and the solution is obtained as follows: x_1=2.67, x_2= 0.67; z_1(x)= 6.2231; z_2(x)= -4.0043 and the non-membership values are ν_1 = 0.1846 and ν_2 =0.The non-membership function values show that both goals z_1 and z_2 are satisfied with 18.46% and 0% respectively for the obtained solution which is x_1=2.67; x_2= 0.67.

V. CONCLUSION

In this paper, a powerful and robust method which is based on Taylor series is proposed to solve intuitionistic fuzzy multi-objective quadratic programming problems (IFMOQPP). Membership function and non-membership functions associated with each objective of the problem are transformed by using the first order Taylor polynomial series and the intuitionistic fuzzy multi-objective linear programming problem (IFMOLPP) is reduced to an equivalent multi-objective linear programming problem (MOLPP) by the proposed method. The obtained MOLPP problem is solved by assuming that the weights of the objectives are equal. The proposed solution method will be useful to more number of variables, objectives and applied to a numerical example of distinct types (triangular and trapezoidal) to test the effectiveness with respect to distinct definitions of objectives and changes of the parameters which facilitates computation to reduce the complexity in problem solving.

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