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LAPLACE TRANSFORM OF SOME WELL KNOWN SPECIAL FUNCTIONS

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Abstract: Integral Transforms draw the attention of many researchers and hence various types of Integral Transforms introduced time-to-time (Debnath [2], Snedon [4]). Integral Transforms play a key role in the field of Special Functions. In this paper, authors studied Laplace Transforms (this is one of the renowned Integral Transform) of several well-known Special Functions.

Keywords: Bessel's function, Hermite Polynomial, Hypergeometric function, Legendre Polynomials, Laguerre Polynomial, Laplace Transforms.

Introduction

In the present article, we define some well known Special Functions and derive Laplace transform of it. First of all we define the Laplace transform of f(t) as

$$L\{f(t)\}=\int_0^\infty e^{-st} f(t)dt,\ \text{Re}(s)>0 \tag{1}$$

The following well-known Special Functions defined as (Rainville[3], Srivastava and Manocha [5]). Define Pochhamer symbol $(\alpha)_n$ by the equation

$$(\alpha)_n = \alpha(\alpha+1)(\alpha+2)...(\alpha+n-1) \tag{2}$$

$$= \prod_{m=1}^{n} (\alpha+m-1),\ (n\geq 1)\ (\alpha)_0 = 1,\ \ \alpha\neq 0$$

It is clear that $(1)_n = n!$. The function $(\alpha)_n$ is called the factional function . The results given below are very useful in the study of Special Functions.

1. If n is a positive integer, then
$$\frac{\Gamma(\alpha+n)}{\Gamma(\alpha)}=(\alpha)_n,$$
where α is neither zero nor a negative integer.

2. If α is not an integer, then $\frac{\Gamma(1-\alpha-n)}{\Gamma(1-\alpha)}=\frac{(-1)^n}{(\alpha)_n}$

3. $$(1-z)^{-\alpha}=\sum_{n=0}^{\infty}\frac{(\alpha)_n z^n}{n!}$$

4. $$(\alpha)_{n-k}=\frac{(-1)^k(\alpha)_n}{(1-\alpha-n)_k},\ \ 0\leq k\leq n$$
In particular, when

$$\alpha=1,\ \ then\ \ (n-k)!=\frac{(-1)^k n!}{(-n)_k},\ \ \ 0\leq k\leq n$$

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5. $(\alpha)_{2n} = 2^{2n}\left(\frac{\alpha}{2}\right)_n \left(\frac{\alpha+1}{2}\right)_n$

The function F(a, b; c; z) is defined as

$$F(a,b;c;z) = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n z^n}{(c)_n n!} \tag{3}$$

where c is neither zero nor negative.

The function F(a, b; c; z) is also written as $F\left[\begin{matrix} a, & b; & \\ & & z \\ & c; & \end{matrix}\right]$

A Hypergeometric function $_pF_q$ is defined as

$$ {}_pF_q\left[\begin{matrix} a_1, a_2,...,a_p; & \\ & z \\ b_1,b_2,...,b_q; & \end{matrix}\right] = \sum_{n=0}^{\infty} \frac{\prod_{k=1}^{p} (a_k)_n}{\prod_{m=1}^{q} (b_m)_n} \frac{z^n}{n!} \tag{4}$$

The Bessel's function is define as

$$J_n(t) = \sum_{n=0}^{\infty} \frac{(-1)^k t^{2k+1}}{2^{2k+n} k!\Gamma(1+n+k)} \tag{5}$$

Also, the Legendre Polynomial is defined as

$$P_n(t) = \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{(-1)^k \left(\frac{1}{2}\right)_{n-k} (2t)^{n-2k}}{k!(n-2k)!} \tag{6}$$

The Hermite Polynomial is defined as

$$H_n(t) = \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{(-1)^k n!(2t)^{n-2k}}{k!(n-2k)!} \tag{7}$$

The Laguerre Polynomial is defined as

$$L_n^{(\alpha)}(t) = \sum_{k=0}^{n} \frac{(-1)^k (1+\alpha)_n t^k}{k!(n-k)!(1+\alpha)_k} \tag{8}$$

The class of Polynomial set $M_n^{(\alpha)}(t)$ is defined as per Khan M.A.[1],

$$M_n^{\alpha}(t) = \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{(-1)^k (2t)^{n-2k}}{k!(n-2k)!(\alpha)_{n-k}} \tag{9}$$

MAIN RESULTS

Throughout the discussion, we assume the validity of integration term by term in the summation.

1. *Laplace Transform of Hypergeometric Function*

From equations (1) and (4), we can write

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$$L\left\{{}_2F_1\left[\begin{matrix}a, & b; & t\\ & 1; & \end{matrix}\right]\right\}=L\left\{\sum_{n=0}^{\infty}\frac{(a)_n\,(b)_n}{(1)_n\,n!}\,t^n\right\}$$

$$=\int_0^{\infty} e^{-st}\sum_{n=0}^{\infty}\frac{(a)_n\,(b)_n}{(1)_n\,n!}\,t^n\,dt$$

$$=\sum_{n=0}^{\infty}\frac{(a)_n\,(b)_n}{(1)_n\,n!}\int_0^{\infty} e^{-st}\,t^n\,dt$$

$$=\sum_{n=0}^{\infty}\frac{(a)_n\,(b)_n}{(1)_n\,n!}\,L\left\{t^n\right\}$$

On the simplification, we arrived at

$$L\left\{{}_2F_1\left[\begin{matrix}a, & b; & t\\ & 1; & \end{matrix}\right]\right\}$$

$$=\sum_{n=0}^{\infty}\frac{(a)_n\,(b)_n}{(1)_n\,n!}\,\frac{\Gamma(n+1)}{s^{n+1}}$$

$$=\frac{1}{s}\,{}_2F_0\left[\begin{matrix}a, & b; & \frac{1}{s}\\ & -; & \end{matrix}\right]$$

2. *Laplace Transform of confluent Hypergeometric functions*

From equations (1) and (4), we can write

$$L\left\{{}_1F_1\left[\begin{matrix}a; & t\\ 1; & \end{matrix}\right]\right\}=L\left\{\sum_{n=0}^{\infty}\frac{(a)_n}{(1)_n}\,\frac{t^n}{n!}\right\}$$

$$=\int_0^{\infty} e^{-st}\sum_{n=0}^{\infty}\frac{(a)_n}{(1)_n}\,\frac{t^n}{n!}\,dt$$

This immediately gives,

$$L\left\{{}_1F_1\left[\begin{matrix}a; & t\\ 1; & \end{matrix}\right]\right\}$$

$$=\sum_{n=0}^{\infty}\frac{(a)_n}{(1)_n\,n!}\int_0^{\infty} e^{-st}\,t^n\,dt$$

$$=\frac{1}{s}\,{}_1F_0\left[\begin{matrix}a; & \frac{1}{s}\\ -; & \end{matrix}\right]$$

3. *Laplace Transform of Generalised Hypergeometric Functions*

$$L\left\{ {}_pF_q\left[\begin{matrix} a_1,\dots,a_p; & t \\ b_1,\dots,b_{q-1},1; & \end{matrix}\right]\right\}$$

$$=\int_0^\infty e^{-st}\,dt \sum_{n=0}^{\infty} \frac{\prod_{k=1}^{p}(a_k)_n}{\prod_{m=1}^{q-1}(b_m)_n(1)_n} \frac{t^n}{n!}$$

$$=\sum_{n=0}^{\infty} \frac{\prod_{k=1}^{p}(a_k)_n}{\prod_{m=1}^{q-1}(b_m)_n(1)_n n!} \int_0^\infty e^{-st} t^n\,dt$$

$$=\sum_{n=0}^{\infty} \frac{\prod_{k=1}^{p}(a_k)_n}{\prod_{m=1}^{q-1}(b_m)_n(1)_n n!} L\left\{t^n\right\}$$

$$=\sum_{n=0}^{\infty} \frac{\prod_{k=1}^{p}(a_k)_n}{\prod_{m=1}^{q-1}(b_m)_n n!n!} \frac{\Gamma(n+1)}{s^{n+1}}$$

$$=\frac{1}{s}\left\{ {}_pF_{q-1}\left[\begin{matrix} a_1,\dots,a_p; & \frac{1}{s} \\ b_1,\dots,b_{q-1}; & \end{matrix}\right]\right\}$$

By putting p=2 and q=1 in above equation, we obtain result 1, while by putting p=1 and q=1 we get result 2 of section 2.

4. *Laplace Transform of Bessels functions*

From equations (1) and (5), we obtain

$$L\left\{J_n(t)\right\}$$

$$=L\left\{\sum_{k=0}^{\infty} \frac{(-1)^k t^{2k+n}}{2^{2k+n} k!\Gamma(1+n+k)}\right\}$$

$$=\int_0^\infty e^{-st} \sum_{k=0}^{\infty} \frac{(-1)^k t^{2k+n}}{2^{2k+n} k!\Gamma(1+n+k)}\,dt$$

$$=\sum_{k=0}^{\infty} \frac{(-1)^k}{2^{2k+n} k!\,\Gamma(1+n+k)} \int_0^\infty e^{-st} t^{2k+n}\,dt$$

$$=\sum_{k=0}^{\infty}\frac{(-1)^k}{2^{2k+n}\,k!\,\Gamma(1+n+k)}\frac{\Gamma(2k+n+1)}{s^{2k+n+1}}$$

$$=\frac{1}{2^n}\sum_{k=0}^{\infty}\frac{(-1)^k}{2^{2k}\,k!}\frac{\Gamma(n+1)}{\Gamma(n+k+1)}\frac{\Gamma(2k+n+1)}{\Gamma(n+1)}\frac{1}{s^{2k}}$$

From results 1 and 5 of section1, this can be written as

$$L\{J_n(t)\}$$

$$=\frac{1}{2^n}\sum_{k=0}^{\infty}\frac{(-1)^k}{2^{2k}\,k!}\frac{(1+n)_{2k}}{(1+n)_k}\frac{1}{s^{2k}}$$

$$=\frac{1}{2^n}\sum_{k=0}^{\infty}\frac{(-1)^k\,2^{2k}\left(\frac{1+n}{2}\right)_k\left(1+\frac{n}{2}\right)_k}{2^{2k}\,k!\,(1+n)_k}\frac{1}{s^{2k}}$$

$$=\frac{1}{2^n}\;{}_2F_1\left[\begin{matrix}\frac{1+n}{2},\,1+\frac{n}{2}; & -\frac{1}{s^2}\\ 1+n; & \end{matrix}\right]$$

5. *Laplace Transform of LegendrePolynomials*

Equations (1) and (6), Yields

$$L\{t^{\beta}P_n(t)\}=\int_0^{\infty}e^{-st}\,t^{\beta}P_n(t)\,dt$$

$$=\int_0^{\infty}e^{-st}\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^k\left(\frac{1}{2}\right)_{n-k}(2t)^{n-2k}\,t^{\beta}}{k!(n-2k)!}\,dt$$

Simplification of above equation by using results 2,4 and 5 of section 1, we get

$$L\{t^{\beta}P_n(t)\}$$

$$=2^n\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^k}{k!}\frac{(-n)_{2k}}{(-1)^{2k}\,n!}\frac{(-1)^k\left(\frac{1}{2}\right)_n}{\left(1-\frac{1}{2}-n\right)_k 2^{2k}}\int_0^{\infty}e^{-st}\,t^{\beta+n-2k}\,dt$$

$$=\frac{2^n\left(\frac{1}{2}\right)_n}{n!}\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-n)_{2k}}{k!\left(\frac{1}{2}-n\right)_k}\frac{1}{2^{2k}}\frac{\Gamma(1+n-2k+\beta)}{s^{1+n-2k+\beta}}$$

$$=\frac{2^n\left(\frac{1}{2}\right)_n\Gamma(1+\beta+n)}{n!\,s^{1+\beta+n}}\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{2^{2k}\left(-\frac{n}{2}\right)_k\left(-\frac{n}{2}+\frac{1}{2}\right)_k}{k!\left(\frac{1}{2}-n\right)_k 2^{2k}}\frac{\Gamma(1+n-2k+\beta)}{\Gamma(1+n+\beta)}s^{2k}$$

$$=\frac{2^n\left(\frac{1}{2}\right)_n\Gamma(1+\beta+n)}{n!\,s^{1+\beta+n}}\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{\left(-\frac{n}{2}\right)\left(-\frac{n}{2}+\frac{1}{2}\right)_k}{k!\left(\frac{1}{2}-n\right)_k}\frac{(-1)^{2k}}{(-\beta-n)_{2k}}s^{2k}$$

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$$=\frac{2^n\left(\frac{1}{2}\right)_n\Gamma(1+\beta+n)}{n!\,s^{1+\beta+n}}\ {}_2F_3\left[\begin{matrix}-\frac{n}{2},-\frac{n}{2}+\frac{1}{2}; & -\frac{s^2}{4}\\ \frac{1}{2}-n,\frac{1}{2}(-\beta-n),\frac{1}{2}(1-\beta-n); & \end{matrix}\right]$$

6. *Laplace Transform of Hermite Polynomials*

From equations (1) and (7), we get

$$L\left\{t^{\beta}H_n(t)\right\}=\int_0^{\infty}e^{-st}\,t^{\beta}H_n(t)\,dt$$

$$=\int_0^{\infty}e^{-st}\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^k\,n!(2t)^{n-2k}\,t^{\beta}}{k!(n-2k)!}\,dt$$

$$=\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^k\,n!2^n}{k!(n-2k)!2^{2k}}\int_0^{\infty}e^{-st}\,t^{\beta+n-2k}\,dt$$

By using results 2, 4 and 5 of section 1, one can easily show that

$$L\left\{t^{\beta}H_n(t)\right\}=2^n\,n!\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^k(-n)_{2k}}{k!(-1)^{2k}n!}\frac{1}{2^{2k}}\frac{\Gamma(1+\beta+n-2k)}{s^{1+\beta+n-2k}}$$

$$=2^n\,n!\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^k2^{2k}\left(-\frac{n}{2}\right)_k\left(-\frac{n}{2}+\frac{1}{2}\right)_k}{k!(-1)^{2k}n!\,2^{2k}}\frac{\Gamma(1+\beta+n-2k)}{s^{1+\beta+n-2k}}$$

$$=2^n\frac{\Gamma(1+\beta+n)}{s^{1+\beta+n}}\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^k\left(-\frac{n}{2}\right)_k\left(-\frac{n}{2}+\frac{1}{2}\right)_k}{k!}\frac{(-1)^{2k}}{(-\beta-n)_{2k}}s^{2k}$$

$$=2^n\frac{\Gamma(1+\beta+n)}{s^{1+\beta+n}}\ {}_2F_2\left[\begin{matrix}-\frac{n}{2},-\frac{n}{2}+\frac{1}{2}; & \frac{s^2}{4}\\ \frac{-\beta-n}{2},\frac{1-\beta-n}{2}; & \end{matrix}\right]$$

7. *Laplace Transform of Laguerre Polynomials*

From equations (1) and (8), we get

$$L\left\{\mathrm{L}_n^{(\alpha)}(t)\right\}=\int_0^{\infty}e^{-st}\,\mathrm{L}_n^{(\alpha)}(t)\,dt$$

$$=\sum_{k=0}^{n}\frac{(-1)^k(1+\alpha)_n}{k!(n-k)!(1+\alpha)_k}\int_0^{\infty}e^{-st}\,t^k\,dt$$

Result 4 of section 1, immediately gives us,

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$$L\left\{L_n^{(\alpha)}(t)\right\} = (1+\alpha)_n \sum_{k=0}^{n} \frac{(-1)^k}{k!} \frac{(-n)_k}{(-1)^k n!} \frac{\Gamma(k+1)}{s^{k+1}} \frac{1}{(1+\alpha)_k}$$

$$= \frac{(1+\alpha)_n}{n!\, s} {}_1F_1\left[\begin{matrix} -n,1; & \frac{1}{s} \\ 1+\alpha; & \end{matrix}\right]$$

8. *Laplace Transform of Hypergeomtric finctions*

$$F\left[\begin{matrix} a, \quad b; & z\,(1-e^{-t}) \\ 1; & \end{matrix}\right]$$

From equations (1) and (3), we obtain

$$L\left\{F\left[\begin{matrix} a, \quad b; & z\,(1-e^{-t}) \\ 1; & \end{matrix}\right]\right\}$$

$$= L\left\{\sum_{n=0}^{\infty} \frac{(a)_n\,(b)_n}{(1)_n} z^n \frac{(1-e^{-t})^n}{n!}\right\}$$

$$= \sum_{n=0}^{\infty} \frac{(a)_n\,(b)_n}{(1)_n\, n!} z^n L\{(1-e^{-t})^n\}$$

$$= \sum_{n=0}^{\infty} \frac{(a)_n\,(b)_n}{(1)_n\, n!} z^n \int_0^{\infty} e^{-st}(1-e^{-t})^n dt$$

Taking

$$e^{-t} = y \quad \Rightarrow \quad dt = -\frac{1}{y} dy$$

Therefore,

$$\int_0^{\infty} e^{-st}(1-e^{-t})^n dt = \int_1^0 y^s (1-y)^n \left(-\frac{1}{y} dy\right)$$

$$= \int_0^1 y^{s-1}(1-y)^{(n+1)-1} dy = B(s,n+1) = \frac{\Gamma(s)\Gamma(n+1)}{\Gamma(s+n+1)}$$

Hence,

$$L\left\{F\left[\begin{matrix} a, & b; & \\ & & z(1-e^{-t}) \\ & 1; & \end{matrix}\right]\right\}$$

$$= \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(1)_n n!} z^n \frac{\Gamma(s)\Gamma(n+1)}{\Gamma(s+n+1)}$$

$$= \sum_{n=0}^{\infty} \frac{(a)_n (b)_n z^n}{s(s+1)_n n!}$$

$$= \frac{1}{s}\, {}_2F_1\left[\begin{matrix} a, & b; & \\ & & z \\ & s+1; & \end{matrix}\right]$$

9.

$$L\{t^n \sin at\} = \frac{a\Gamma(n+2)}{s^{n+2}} F\left[\begin{matrix} 1+\frac{n}{2}, & \frac{3}{2}+\frac{n}{2}; & \\ & & \frac{-a^2}{s^2} \\ & \frac{3}{2}; & \end{matrix}\right]$$

It is clear that

$$t^n \sin at$$

$$= t^n\left[at - \frac{(at)^3}{3!} + \frac{(at)^5}{5!} - \frac{(at)^7}{7!} + \ldots\ldots\ldots\right]$$

$$= at^{n+1} - a^3\frac{t^{n+3}}{3!} + a^5\frac{t^{n+5}}{5!} - \ldots\ldots\ldots$$

Therefore,

$$L(t^n \sin at)$$

$$= \frac{a\Gamma(n+2)}{s^{n+2}} + \frac{a^3\Gamma(n+4)}{s^{n+4}3!} - \frac{a^5\Gamma(n+6)}{s^{n+6}5!} + \ldots\ldots\ldots$$

$$= \frac{a\Gamma(n+2)}{s^{n+2}} + \frac{a^3\Gamma(n+4)}{s^{n+4}3!} - \frac{a^5\Gamma(n+6)}{s^{n+6}5!} + \ldots\ldots$$

$$= \frac{a\Gamma(n+2)}{s^{n+2}}\left[1 - \frac{a^2(n+2)(n+3)}{s^2 3!} + \frac{a^4(n+2)(n+3)(n+4)(n+5)}{s^4 5!} - \ldots\ldots\right]$$

Now,

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$$F\left[\begin{matrix} 1+\frac{n}{2}, & \frac{3}{2}+\frac{n}{2}; & \\ & & \frac{-a^2}{s^2} \\ & \frac{3}{2}; & \end{matrix}\right] = \sum_{k=0}^{\infty} \frac{\left(1+\frac{n}{2}\right)_k \left(\frac{3}{2}+\frac{n}{2}\right)_k}{\left(\frac{3}{2}\right)_k} \frac{\left(-\frac{a^2}{s^2}\right)^k}{k!}$$

$$=1-\frac{\left(1+\frac{n}{2}\right)\left(\frac{3}{2}+\frac{n}{2}\right)}{\left(\frac{3}{2}\right)}\frac{\left(\frac{a^2}{s^2}\right)}{1!}+\frac{\left(1+\frac{n}{2}\right)\left(2+\frac{n}{2}\right)\left(\frac{3}{2}+\frac{n}{2}\right)\left(\frac{5}{2}+\frac{n}{2}\right)}{\frac{3}{2}\frac{5}{2}}\frac{\left(-\frac{a^2}{s^2}\right)^2}{2!}+\ldots\ldots$$

$$=1-\frac{a^2(n+2)(n+3)}{3!s^2}+\frac{a^4(n+2)(n+3)(n+4)(n+5)}{5!s^2}+\ldots\ldots$$

Hence the result holds.

10.

$$L\left\{t^c\,{}_pF_q\left[\begin{matrix} a_1,a_2,\ldots\ldots a_p; & \\ b_1,b_2,\ldots\ldots b_q; & zt \end{matrix}\right]\right\}=\frac{\Gamma(1+c)}{s^{1+c}}\,{}_{p+1}F_q\left[\begin{matrix} 1+c,a_1,a_2,\ldots\ldots a_p; & \\ b_1,b_2,\ldots\ldots b_q; & \frac{z}{c} \end{matrix}\right]$$

We have

$$L\left\{t^c\,{}_pF_q\left[\begin{matrix} a_1,a_2,\ldots\ldots a_p; & \\ b_1,b_2,\ldots\ldots b_q; & zt \end{matrix}\right]\right\}=\int_0^{\infty} e^{-st}t^c\,{}_pF_q\left[\begin{matrix} a_1,a_2,\ldots\ldots a_p; & \\ b_1,b_2,\ldots\ldots b_q; & zt \end{matrix}\right]dt$$

$$=\int_0^{\infty} e^{-st}t^c \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{p}(a_i)_n}{\prod_{j=1}^{p}(b_j)_n}\frac{z^n t^n}{n!}dt$$

$$=\sum_{n=0}^{\infty} \frac{\prod_{i=1}^{p}(a_i)_n}{\prod_{j=1}^{p}(b_j)_n}\frac{z^n}{n!}\int_0^{\infty} e^{-st}t^{c+n}dt$$

$$=\sum_{n=0}^{\infty} \frac{\prod_{i=1}^{p}(a_i)_n}{\prod_{j=1}^{p}(b_j)_n}\frac{z^n}{n!}\frac{\Gamma(c+n+1)}{s^{c+n+1}}$$

$$=\frac{\Gamma(1+c)}{s^{1+c}}\,{}_{p+1}F_q\left[\begin{matrix} 1+c,a_1,a_2,\ldots\ldots a_p; & \\ b_1,b_2,\ldots\ldots b_q; & \frac{z}{c} \end{matrix}\right]$$

11. *Laplace Transform of the class of polynomial set* $M_n^{(\alpha)}(t)$

From equations (1) and (9), we obtain

$$L\left\{t^{\beta} M_n^{(\alpha)}(t):\ s\right\} = \int_0^{\infty} e^{-st} t^{\beta} M_n^{(\alpha)}(t)\,dt$$

$$= \int_0^{\infty} e^{-st} t^{\beta} \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{(-1)^k (2t)^{n-2k}}{k!(n-2k)!(\alpha)_{n-k}}\,dt$$

On the multiple use of results 2 and 5 of section 1, we can write

$$L\left\{t^{\beta} M_n^{(\alpha)}(t):\ s\right\}$$

$$= \int_0^{\infty} e^{-st} t^{\beta} \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{(-1)^k}{k!} \frac{(2t)^n}{\frac{(-1)^{2k} n!}{(-n)_{2k}}(n-2k)!} \frac{1}{\frac{(-1)^k (\alpha)_n}{(1-\alpha-n)_k}} \frac{1}{(2t)^{2k}}\,dt$$

$$= \frac{2^n}{n!} \int_0^{\infty} e^{-st} t^{\beta} \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{(2)^{2k} \left(-\frac{n}{2}\right)_k \left(-\frac{n}{2}+\frac{1}{2}\right)_k (1-\alpha-n)_k t^n}{k!(\alpha)_n 2^{2k} t^{2k}}\,dt$$

$$= \frac{2^n}{n!(\alpha)_n} \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{\left(-\frac{n}{2}\right)_k \left(-\frac{n}{2}+\frac{1}{2}\right)_k (1-\alpha-n)_k}{k!} \int_0^{\infty} e^{-st} t^{\beta+n-2k}\,dt$$

$$= \frac{2^n}{n!(\alpha)_n} \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{\left(-\frac{n}{2}\right)_k \left(-\frac{n}{2}+\frac{1}{2}\right)_k (1-\alpha-n)_k}{k!} \frac{\Gamma(1+\beta+n-2k)}{s^{1+\beta+n-2k}}$$

$$= \frac{2^n \Gamma(1+\beta+n)}{n!(\alpha)_n s^{1+\beta+n}} \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{\left(-\frac{n}{2}\right)_k \left(-\frac{n}{2}+\frac{1}{2}\right)_k (1-\alpha-n)_k}{k!} \frac{(-1)^{2k}}{(\beta-n)_{2k}} s^{2k}$$

$$= \frac{2^n \Gamma(1+\beta+n)}{n!(\alpha)_n s^{1+\beta+n}} \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{\left(-\frac{n}{2}\right)_k \left(-\frac{n}{2}+\frac{1}{2}\right)_k (1-\alpha-n)_k}{k!\left(\frac{-\beta-n}{2}\right)_k \left(\frac{1-\beta-n}{2}\right)_k} \left(\frac{s^2}{4}\right)_k$$

$$= \frac{2^n \Gamma(1+\beta+n)}{n!(\alpha)_n s^{1+\beta+n}} {}_3F_2\left[\begin{matrix} -\frac{n}{2}, & -\frac{n}{2}+\frac{1}{2}; & 1-\alpha-n \\ & & & \frac{s^2}{4} \\ \frac{1}{2}(-\beta-n) & \frac{1}{2}(1-\beta-n); & \end{matrix}\right]$$

REFERENCES

[1] Khan M.A.:A note on a polynomial set generated by G(2xt-t^2) for the choice G(u) =$_0F_1$(-;α; u), Acta Ciencia Indica 28; 117-126; 2002.
[2] Lokenath Debnath: Intrgral Transforms and their applications, second edition, Chapman and Hall/CRC Taylor and Francis Group, 2007.
[3] Rainville E.D. Special Functions, Macmillan, New York, 1960.
[4] Sneddon,I.N., The Use of Integral transforms, McGraw-Hill Book Company, New York, 1972.
[5] Srivastava H.M. and manocha H.L.: A treatise on generating functions, John Wiley and Sons, New York, Ellis Horwood, Chichester, 1984.