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# LOCATION OF LIBRATION POINTS IN THE GENERALISED PHOTOGRAVITATIONAL ELLIPTIC RESTRICTED THREE BODY PROBLEM

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**Abstract**: In this paper the location of libration points, triangular as well as collinear libration points in the generalized photogravitational elliptic restricted three body problem has been studied. The problem is generalized in the sense that the bigger primary is taken as an oblate spheroid and the smaller primary is considered as a source of radiation. It has been found that triangular and collinear libration points are different from those in classical case.

**Key words***:* Libration / Generalised / Photogravitational / Restricted

## I. Introduction

Radzievskii [6] studied the location of the equilibrium points by taking more massive primary as the source of radiation in the restricted problem of three bodies. Danby [1] examined the stability of triangular points in the elliptic restricted problem of three bodies. Sharma [7] studied the photogravitational restricted three body problem aking smaller primary as an oblate spheroid. Haque and Ishwar [2] examined the non-linear stability in the perturbed photogravitational restricted three body problem. Markellos, Perdios and Labropoulou [4] studied the linear stability of the triangular equilibrium points in the photogravitational elliptic restricted problem. Khasan [3] discussed the librational solutions to the photogravitational restricted three body problem.

In the present paper we have studied the location of libration points triangular as well as libration points in the generalized photogravitational elliptic restricted three body problem. We have taken the bigger primary as an oblate spheroid and the smaller primary libration points contain eccentricity, oblateness and radiation factor which are different from those in classical case.

## II. Location of Triangular Libration Points:

THE EQUATIONS OF MOTION OF CONSIDERED PROBLEM ARE

$$\xi'' - 2\eta' = \frac{\partial \Omega^*}{\partial \xi}$$

$$\eta'' - 2\xi' = \frac{\partial \Omega^*}{\partial \eta}$$

$$\zeta'' = \frac{\partial \Omega^*}{\partial \zeta} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

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WHERE

$$\Omega^* = \frac{1}{\left(1-e^2\right)^{1/2}}\left[\frac{\xi^2+\eta^2}{2}+\frac{1}{n^2}\left\{\frac{(1-\mu)}{r_1}+\frac{(1-\mu)A_1}{2r_1^{\,3}}+\frac{\mu q_2}{r_2}\right\}\right] \quad \ldots\ldots\ldots\ldots (2)$$

$A_1$ IS THE OBLATENESS COEFFICIENT, $q_2$ THE RADITATION PRESSURE, E THE ECCENTRICITY OF THE ELLIPSE AND N THE MEAN ANGULAR VELOCITY GIVEN BY $n^2 = \frac{\left(1+\frac{3A_1}{2}\right)\left(1+e^2\right)^{1/2}}{a\left(1-e^2\right)}$ ; A BEING THE SEMI-MAJOR AXIS OF THE ELLIPSE.

MULTIPLYING THE ABOVE EQUATIONS BY 2 $\xi'$, 2 $\eta'$ & 2 $\zeta'$ RESPECTIVELY, ADDING AND INTEGRATING, WE GET

$\xi'^2 + \eta'^2 + \zeta'^2 = 2\Omega^* + C$, WHERE C IS THE JACOBI CONSTANT.

I.E. $F(\xi,\eta,\zeta) = 2\Omega^* + C$

WHERE

$$F(\xi,\eta,\zeta) = \frac{1}{\left(1-e^2\right)^{1/2}}\left[\xi^2+\eta^2+\frac{1}{n^2}\left\{\frac{2\mu q_2}{r_2}+\frac{2(1-\mu)}{r_1}+\frac{2(1-\mu)A_1}{2r_1^{\,3}}\right\}\right]+C \quad \ldots\ldots\ldots\ldots (3)$$

FOR EQUILIBRIUM POSITION, WE HAVE

$$\frac{\partial f}{\partial \xi} = \frac{\partial f}{\partial \eta} = \frac{\partial f}{\partial \zeta} = 0$$

WHICH GIVE

$$\frac{1}{\left(1-e^2\right)^{1/2}}\left[\xi-\frac{1}{n^2}\left\{\frac{\mu q_2(\xi-\xi_2)}{r_2^{\,3}}+\frac{(1-\mu)(\xi-\xi_1)}{r_1^{\,3}}+\frac{3(1-\mu)A_1(\xi-\xi_1)}{2r_1^{\,5}}\right\}\right]=0 \quad \ldots\ldots\ldots\ldots (4)$$

$$\frac{1}{\left(1-e^2\right)^{1/2}}\left[\eta-\frac{1}{n^2}\left\{\frac{\mu q_2\eta}{r_2^{\,3}}+\frac{(1-\mu)\eta}{r_1^{\,3}}+\frac{3(1-\mu)A_1\eta}{2r_1^{\,5}}\right\}\right]=0 \quad \ldots\ldots\ldots\ldots (5)$$

$$\& \; \zeta\left[\frac{\mu q_2}{r_2^{\,3}}+\frac{(1-\mu)}{r_1^{\,3}}+\frac{3(1-\mu)A_1}{2r_1^{\,5}}\right]=0 \quad \ldots\ldots\ldots\ldots (6)$$

IF $\zeta = 0$, WE HAVE PLANNAR LIBRATION POINTS, OTHERWISE WE WILL GET OUT OF PLANE LIBRATION POINTS.

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LET THE PLANNAR LIBRATION POINTS AWAY FROM X-AXIS BE DENOTED BY $L_4$ AND $L_5$ WHICH ARE CALLED TRIANGULAR LIBRATION POINTS.

WE HAVE $\eta \neq 0$ WHICH GIVES

$$1-\frac{1}{n^2}\left[\frac{\mu q_2}{r_2^{\ 3}}+\frac{(1-\mu)}{r_1^{\ 3}}+\frac{3(1-\mu)A_1}{2r_1^{\ 5}}\right]=0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

IT IS OBVIOUS THAT $\xi_1-\xi_2=1$, THE DISTANCE BETWEEN TWO PRIMARIES SO THAT $\xi_2=-\mu$, $\xi_1=1-\mu$.

WITH THE HELP OF (4), (5) & (7) AND PUTTING THE VALUES OF $\xi_1$ & $\xi_2$, WE GET

$$r_1^{\ 2}=a^{2/3}\left(1-e^2\right)+A_1$$

$$\& \ r_2^{\ 2}=\left(aq_2\right)^{2/3}\left(1-e^2-A_1\right)$$

USING THESE VALUES IN $r_1^{\ 2}-r_2^{\ 2}=2\xi+2\mu-1$, WE GET

$$\xi=\frac{1}{2}-\mu+\frac{1}{2}\left[\left(aq_2\right)^{2/3}\left(1-e^2-A_1\right)-a^{2/3}\left(1-e^2\right)-A_1\right] \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

ALSO FROM $\eta^2=r_1^{\ 2}-(\xi+\mu)^2$, WE GET

$$\eta=\pm\left[\left(aq_2\right)^{2/3}\left(1-e^2-A_1\right)\frac{1}{4}\left\{1+2\left(aq_2\right)^{2/3}\left(1-e^2-A_1\right)-2a^{2/3}\left(1-e^2\right)-2A_1\right\}\right]^{1/2} \ \ldots\ldots\ldots (9)$$

HENCE THE COORDINATE OF $L_4$ AND $L_5$ ARE $(\xi,\eta)$ & $(\xi,-\eta)$ RESPECTIVELY WHERE $\xi$ AND $\eta$ ARE GIVEN BY (8) AND (9).

IF WE PUT $e=A_1=0$ & $q_2=a=1$, THE COORDINATES OF $L_4$ ARE $\left(\frac{1}{2}-\mu,\pm\frac{\sqrt{3}}{2}\right)$ WHICH GIVES THE SAME RESULT OF MC CUSKEY [5].

## III. LOCATION OF COLLINEAR LIBRATION POINTS:

TO FIND THE LOCATION OF COLLINEAR LIBRATION POINTS ON THE X-AXIS WE SUBSTITUTE $\eta=0$ & $\zeta=0$ IN $\frac{\partial f}{\partial \xi}=0$ AND WE GET

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$$\frac{1}{\left(1-e^2\right)^{1/2}}\left[\xi-\frac{1}{n^2}\left\{\frac{\mu q_2(\xi-\xi_2)}{r_2{}^3}+\frac{(1-\mu)(\xi-\xi_1)}{r_1{}^3}+\frac{3(1-\mu)A_1(\xi-\xi_1)}{2r_1{}^5}\right\}\right]=0 \quad \text{..........(10)}$$

WHERE $r_1{}^2=(\xi-\xi_1)^2+\eta^2+\zeta^2$

$$r_2{}^2=(\xi-\xi_2)^2+\eta^2+\zeta^2$$

WE CONSIDER $\xi-\xi_1=\rho$ SO THAT $\xi-\xi_2=1+\rho$

SUBSTITUTING THESE VALUES IN (10), WE GET

$$2n^2(1+\rho-\mu)(1+\rho)^2\rho^4-2\mu q_2\rho^4-2(1-\mu)(1+\rho)^2\rho^2-3(1-\mu)A_1(1+\rho)^2=0$$

FINALLY WE GET THE SOLUTION FOR THE LOCATION OF COLLINEAR LIBRATION POINT $L_1$ AS

$$\rho=\gamma_1+U_1X_1+(U_1Y_!+V_1X_1)\rho^2+2X_1(\mu-1)\gamma_1{}^3(V_1+4U_1X_1)\beta_2$$
$$+\{(U_1Y_1+V_1X_1+3X_1)(1-\mu)(1+\lambda_1)(1+\lambda_1+2U_1X_1)\}A_1$$

WHERE $\gamma_1$ IS THE VALUE OF $\rho$, THE DISTANCE BETWEEN $L_1$ AND THE SMALLER PRIMARY IN CLASSICAL CASE.

$$U_1=-2\left\{\frac{\gamma_1{}^7}{a}+(3-\mu)\frac{\gamma_1{}^6}{a}+(3-2\mu)\frac{\gamma_1{}^5}{a}-\gamma_1{}^4-2(1-\mu)\gamma_1{}^3-(1-\mu)\gamma_1{}^2\right\}$$

$$X_1=\left[14\frac{\gamma_1{}^6}{a}+12(3-\mu)\frac{\gamma_1{}^5}{a}+10(3-2\mu)\frac{\gamma_1{}^4}{a}+8\mu\frac{\gamma_1{}^3}{a}-8\gamma_1{}^3-12(1-\mu)\gamma_1{}^2-4(1-\mu)\gamma_1\right]^{-1}$$

$$Y_1=-\frac{21\frac{\gamma_1{}^6}{a}+18(3-\mu)\frac{\gamma_1{}^5}{a}+15(3-2\mu)\frac{\gamma_1{}^4}{a}+12\mu\frac{\gamma_1{}^3}{a}}{14\frac{\gamma_1{}^6}{a}+12(3-\mu)\frac{\gamma_1{}^5}{a}+10(3-2\mu)\frac{\beta\gamma_1{}^4}{a}+8\mu\frac{\gamma_1{}^3}{a}-8\gamma_1{}^3-12(1-\mu)\gamma_1{}^2-4(1-\mu)\gamma_1}$$

$$V_1=-\frac{3}{a}\left\{\gamma_1{}^7+(3-\mu)\gamma_1{}^6+(3-2\mu)\gamma_1{}^5+\mu\gamma_1{}^4\right\}$$

$$\beta_2=1-q_2$$

IF WE PUT $\rho=A_1=\beta_2=0$, A=1, WE GET THE SAME RESULT IN A CLASSICAL CASE OF MC CUSKEY [5].

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SIMILARLY FROM $\frac{\partial f}{\partial \eta}$ AND $\frac{\partial f}{\partial \zeta}$ WE GET THE SOLUTION FOR LOCATION FOR COLLINEAR LIBRATION POINTS $L_2$ & $L_3$ WHICH ARE ALSO AFFECTED BY ECCENTRICITY, OBLATENESS & RADIATION FACTOR.

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