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Lowener, Star and "θ " Partial Ordering of S-Unitary Matrices

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ABSTRACT: Some results relating to 'θ' partial ordering on s-unitary matrices are given. Theorems relating to Lowener , Star and 'θ' partial ordering on s-unitary matrices are obtained.. Further 'θ' partial ordering preserved under s-unitary similarity has been proved.

MATHEMATICS SUBJECT CLASSIFICATION: 15A45,15B57,15A09

KEYWORDS: Lowener partial ordering, Partial ordering, Star partial ordering, s-unitary matrix s-unitary similarity, ' θ' partial ordering.

I. INTRODUCTION

Some characterizations of the star partial ordering and rank subtractivity for matrices was discussed by Hartwig and Styan in [3]. Jurgen Grob observed some remarks on partial ordering of Hermitian Matrices in [5]. Xifu Liu and Hu yang discussed some results on the partial ordering of block matrices[8]. In [4],Jorma K.Merikoski and Xiaogi Liu have developed star partial ordering on Normal Matrices .Krishnamoorthy and Govindarasu introduced the concept of θ' partial ordering [7].

II. PRELIMINARIES AND NOTATIONS:

Let C_{nxn} be the space of nxn complex matrices of order n. For $A \in C_{nxn}$ let $A^T, \overline{A}, A^*, A^S, \overline{A}^S (= A^\theta)$ denote transpose, conjugate, conjugate transpose, secondary transpose, conjugate secondary transpose of a matrix A respectively. Anna Lee[1] has initiated the study of secondary symmetric matrices. Also she has shown that for a complex matrix A, the usual transpose A^T and secondary transpose A^S are related as $A^S = VA^TV$ where 'V'is the associated permutation matrix whose elements on the secondary diagonal are 1 and other elements are zero. .Also $\overline{A}^S$ denotes the conjugate secondary transpose of A i.e $\overline{A}^S$ = (c_{ij}) where

$c_{ij} = \overline{a_{n-j+1,\ n-i+1}}$ [2] and $\overline{A}^S = VA^*V = A^\theta$. Also 'V' satisfies the following properties. $V^T = V^\theta = \overline{V} = V^* = V$ and $V^2 = I$.

A matrix $A \in C_{nxn}$ is said to be s-unitary if $A^\theta A = AA^\theta = I$ [6].

Definition 2.1: [Lowener Partial Ordering]

For $A, B \in C_{nxn}$, we define $A \underset{L}{\geq} B$ if $A - B \geq 0$

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Definition 2.2: [Star Partial Ordering]

For $A, B \in C_{nxn}$, we define $A \underset{*}{\overset{>}{-}} B$ if $B^{*}B = B^{*}A$ and $BB^{*} = AB^{*}$

III.LOWENER,STAR AND " θ " PARTIAL ORDERING OF S-UNITARY MATRICES

Definition 3.1 ['θ' PARTIAL ORDERING] [7]

Let $A, B \in C_{nxn}$, $A \underset{\theta}{\overset{<}{-}} B$ iff $A^{\theta}A = A^{\theta}B$ and $AA^{\theta} = BA^{\theta}$.

Theorem 3.2

Let $VA \underset{\theta}{\overset{<}{-}} AV$ If A is s-unitary then A is unitary.

Proof:

Let $VA \underset{\theta}{\overset{<}{-}} AV$

$\Rightarrow (VA)^{\theta}VA = (VA)^{\theta}AV$ [By Definition 3.1]

$\Rightarrow A^{\theta}VVA = A^{\theta}VAV$

$\Rightarrow A^{\theta}A = A^{\theta}VAV$

$\Rightarrow I = A^{\theta}VAV = VA^{*}VVAV$ [Since A is s-unitary]

$\Rightarrow I = VA^{*}AV$

$\Rightarrow VIV = A^{*}A$

$\Rightarrow I = A^{*}A$ **.........(3.2.1)**

$VA \underset{\theta}{\overset{<}{-}} AV \Rightarrow VA(VA)^{\theta} = AV(VA)^{\theta}$ [By definition 4.3.1]

$\Rightarrow VAA^{\theta}V = AVA^{\theta}V$ [Since A is s-unitary]

$\Rightarrow VIV = AVVA^{*}VV$

$\Rightarrow I = AA^{*}$

Therefore $A^{*}A = I$ -----------------------**(3.2.2)**

From **(3.2.1)** and **(3.2.2)** we have $A^{*}A = AA^{*} = I$
Therefore A is unitary.

Theorem 3.3

If A and B are s-unitary matrices and $AV = VA, VB = BV$ then $A \underset{\theta}{\overset{<}{-}} B \Rightarrow AV \underset{*}{\overset{<}{-}} BV$

Proof:

Given $A \underset{\theta}{\overset{<}{-}} B$

$A \underset{\theta}{\overset{<}{-}} B \Rightarrow A^{\theta}A = A^{\theta}B$ and $AA^{\theta} = BA^{\theta}$ [By Definition.3.1]

By definition $A^{\theta}A = A^{\theta}B$

$\Rightarrow VA^{*}VA = VA^{*}VB$ [Since A is s-unitary]

$\Rightarrow (AV)^{*}VA = (AV)^{*}VB$

$\Rightarrow (AV)^{*}AV = (AV)^{*}BV$ ------------**(3.3.1)** [Since $AV = VA, VB = BV$]

By definition $AA^{\theta} = BA^{\theta}$

$\Rightarrow AVA^{*}V = BVA^{*}V$ [Since A is s-unitary]

Premultiplying and postmultiplying the above equation by V we get,

$\Rightarrow VAVA^{*}VV = VBVA^{*}VV$

$\Rightarrow VAVA^{*} = VBVA^{*}$ [Since $V^{2} = I$]

$\Rightarrow VA(AV)^{*} = VB(AV)^{*}$

$\Rightarrow AV(AV)^{*} = BV(AV)^{*}$ ---------------**(3.3.2)** [Since $AV = VA, VB = BV$]

From **(3.3.1.)** and **(3.3.2)** we have $AV \underset{*}{\overset{<}{-}} BV$.

Therefore $A \underset{\theta}{\overset{<}{-}} B \Rightarrow AV \underset{*}{\overset{<}{-}} BV$.

Theorem 3.4

If A and B are s-unitary matrices and $AV = VA, VB = BV$ then $A \underset{*}{\overset{<}{-}} B \Rightarrow AV \underset{\theta}{\overset{<}{-}} BV$.

Proof: $A \underset{*}{\overset{<}{-}} B \Rightarrow A^{*}A = A^{*}B$ and $AA^{*} = BA^{*}$

Take $A^{*}A = A^{*}B$

$\Rightarrow (VA^{*}V)VA = (VA^{*}V)VB$

$\Rightarrow A^{\theta}VA = A^{\theta}VB$ [Since A is s-unitary]

$\Rightarrow VA^{\theta}VA = VA^{\theta}VB$

$\Rightarrow (AV)^{\theta}VA = (AV)^{\theta}VB$

$\Rightarrow (AV)^{\theta}AV = (AV)^{\theta}BV$ --------------------------**(3.4.1)**

Take $AA^{*} = BA^{*}$

$\Rightarrow AVVA^{*} = BVVA^{*}$

$\Rightarrow AV(VA^{*}V) = BV(VA^{*}V)$

$\Rightarrow AV(A^{\theta}) = BV(A^{\theta})$ [Since A is s-unitary]

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$$\Rightarrow AV(A^{\theta}V) = BV(A^{\theta}V)$$

$$\Rightarrow AV(VA)^{\theta} = BV(VA)^{\theta}$$

$$\Rightarrow AV(AV)^{\theta} = BV(AV)^{\theta} \text{-------------------}\mathbf{(3.4.2)} \quad [\text{Since } AV = VA, VB = BV]$$

From **(3.4.1)** and **(3.4.2)** we get $AV \underset{\theta}{\overset{<}{-}} BV$. Therefore $A \underset{*}{\overset{<}{-}} B \Rightarrow AV \underset{\theta}{\overset{<}{-}} BV$

Theorem 3.5

If $VA \underset{L}{\overset{<}{-}} VB$ and $VA \underset{\theta}{\overset{<}{-}} VB$ then $VA = VB$.

Proof:

Given $VA \underset{\theta}{\overset{<}{-}} VB$

$VA \underset{\theta}{\overset{<}{-}} VB \Rightarrow (VA)^{\theta}VA = (VA)^{\theta}VB$, and $VA(VA)^{\theta} = VB(VA)^{\theta}$,

Given $VA \underset{\theta}{\overset{<}{-}} VB \Rightarrow (VA)^{\theta}VA = (VA)^{\theta}VB$

$\Rightarrow (VA)^{\theta}(VB - VA) = 0$ ------------**(3.5.1)**

Taking conjugate secondary transpose on both sides of **(3.5.1)** we get

$(VB - VA)^{\theta}(VA) = 0$

Since $VA \underset{L}{\overset{<}{-}} VB$ we have $(VB - VA)^{*}VB = V(VB - VA)V$

(3.5.1) $\Rightarrow VV(VB - VA)^{*}VVA = 0$

$\Rightarrow VV(VB - VA)VVVA = 0$

$\Rightarrow I(VB - VA)IVA = 0$ [Since $V^2 = I$]

$\Rightarrow VBVA - (VA)^2 = 0$

$\Rightarrow (VB - VA)VA = 0$

$\Rightarrow VBVA = (VA)^2$

$\Rightarrow VB = VA$ Hence the theorem, is proved by this paper.

Theorem 3.6

For $A, B \in C_{nxn}$ and V is the permutation matrix with units in the secondary diagonal.

$$A \underset{\theta}{\overset{<}{-}} B \Leftrightarrow VA \underset{\theta}{\overset{<}{-}} VB \Leftrightarrow AV \underset{\theta}{\overset{<}{-}} BV$$

Proof: $A \underset{\theta}{\overset{<}{-}} B \Leftrightarrow A^{\theta}A = A^{\theta}B$ and $AA^{\theta} = BA^{\theta}$ [By Definition 3.1]

\

$$\Leftrightarrow A^{\theta}VVA = A^{\theta}VVB \text{ and } VAA^{\theta}V = VBA^{\theta}V$$

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$$\Leftrightarrow (VA)^{\theta} VA = (VA)^{\theta} VB \text{ and } VA(VA)^{\theta} = VB(VA)^{\theta}$$

Therefore $A \underset{\theta}{\overset{<}{-}} B \Leftrightarrow VA \underset{\theta}{\overset{<}{-}} VB$

Similarly we can prove $A \underset{\theta}{\overset{<}{-}} B \Leftrightarrow AV \underset{\theta}{\overset{<}{-}} BV$

Hence $A \underset{\theta}{\overset{<}{-}} B \Leftrightarrow VA \underset{\theta}{\overset{<}{-}} VB \Leftrightarrow AV \underset{\theta}{\overset{<}{-}} BV$

Theorem 3.7

Let $A, B \in C_{nxn}$.. ' θ 'partial ordering is preserved under s-unitary similarity.

Proof: It is enough to prove $A \underset{\theta}{\overset{<}{-}} B \Leftrightarrow VP^{-1}VAP \underset{\theta}{\overset{<}{-}} VP^{-1}VBP$ for some s-unitary matrix P

$$A \underset{\theta}{\overset{<}{-}} B \Rightarrow VA \underset{\theta}{\overset{<}{-}} VB$$

$$\Leftrightarrow P^{\theta} VAP \underset{\theta}{\overset{<}{-}} P^{\theta} VBP$$

$$\Leftrightarrow VP^{\theta} VAP \underset{\theta}{\overset{<}{-}} VP^{\theta} VBP$$

$$\Leftrightarrow (VP^{-1}V)AP \underset{\theta}{\overset{<}{-}} (VP^{-1}V)BP$$

$$\Leftrightarrow C \underset{\theta}{\overset{<}{-}} D \text{ where } C = (VP^{-1}V)AP \text{ and } D = (VP^{-1}V)BP$$

Threrefore ' θ 'partial ordering is preserved under s-unitary similarity

IV.CONCLUSION

Results relating to 'θ' partial ordering on s-unitary matrices are discussed.Relation between Lowener , Star and 'θ' partial ordering on s-unitary matrices are derived. Also shown 'θ' partial ordering preserved under s-unitary similarity.This concept may be applied to any other matrices.

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