ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 2, Issue 10, October 2013

MHD Flow past an Accelerated Vertical Plate with Variable Heat and Mass Diffusion in the Presence of Rotation

R.Muthucumaraswamy[1], Tina Lal[2] and D.Ranganayakulu[3]

Professor & Head, Department of Applied Mathematics, Sri Venkateswara College of Engineering, Irungattukottai 602117, Sriperumbudur Taluk, India. E-mail : msamy@svce.ac.in[1]

Lecturer, Department of Mathematics, Central Polytechnic, Taramani, Chennai 600113, India[2]

Associate Professor, Department of Mathematics, SIVET College, Gowrivakkam, Chennai 600073, India[3]

Abstract: Laplace transform solution of MHD flow past an incompressible fluid past a uniformly accelerated infinite vertical plate with variable heat and mass transfer, in the presence of rotation has been studied. The plate temperature as well as cocentration level near the plate are raised linearly with time. The velocity profiles, temperature and concentration are studied for different physical parameters. It is observed that the velocity increases with decreasing magnetic field parameter.

Keywords: Rotation, accelerated, vertical plate, vertical plat, magnetic field.

I. INTRODUCTION

MHD plays an important role in agriculture, petroleum industries, geophysics and in astrophysics. MHD flow has application in metrology, solar physics and in motion of earth's core. Also it has applications in the field of stellar and planetary magnetospheres, aeronautics, chemical engineering and electronics. In the field of power generation, MHD is receiving considerable attention due to the possibilities it offers for much higher thermal efficiencies in power plants.

Gupta et al [2] studied free convection on flow past a linearly accelerated vertical plate in the presence of viscous dissipative heat using perturbation method. Kafousias and Raptis [3] extended the above problem to include mass transfer effects subjected to variable suction or injection. Free convection effects on flow past an accelerated vertical plate with variable suction and uniform heat flux in the presence of magnetic field was studied by Raptis et al [7].

MHD effects on flow past an infinite vertical plate for both the classes of impulse as well as accelerated motion of the plate was studied by Raptis and Singh[5]. Mass transfer effects on flow past an uniformly accelerated vertical plate was studied by Soundalgekar [10]. Again, mass transfer effects on flow past an accelerated vertical plate with uniform heat flux was analyzed by Singh and Singh [8]. Basant Kumar Jha and Ravindra Prasad[1] analyzed mass transfer effects on the flow past an accelerated infinite vertical plate with heat sources. Singh [9] studied MHD flow past an impulsively started vertical plate in a rotating fluid. Rotation effects on hydromagnetic free convective flow past an accelerated isothermal vertical plate was studied by Raptis and Singh [6]. Recently, Muthucumaraswamy et al [4] studied rotation on MHD flow past an accelerated isothermal vertical plate with uniform mass diffusion using Laplace transform technique.

Hence, it is proposed to study the effects of rotation on the hydromagnetic free convection flow of an incompressible viscous and electrically conducting fluid past a uniformly accelerated infinite vertical plate in the presence of variable heat and mass diffusion. The dimensionless governing equations are solved using the Laplace transform technique. The solutions are in terms of exponential and complementary error function. Such a study is found useful in magnetic control of molten iron flow in the steel industry, liquid metal cooling in nuclear reactors, magnetic suppression of molten semi-conducting materials and meteorology.

II. ANALYSIS

Consider the unsteady hydromagnetic flow of an electrically conducting fluid induced by viscous incompressible fluid past a uniformly accelerated motion of an infinite vertical plate when the fluid and the plate rotate as a rigid body with a uniform angular velocity Ω' about z′-axis in the presence of an imposed uniform magnetic field B_0 normal to the plate. Initially, the temperature of the plate and concentration near the plate are assumed to be T_∞ and C'_∞. At time $t' > 0$, the plate starts moving with a velocity $u = u_0 t'$ in its own plane and the temperature of the plate as well as wall concentration near the plate are raised linearly with time. Since the plate occupying the plane $z' = 0$ is of infinite extent, all the physical quantities depend only on z′ and t′. It is assumed that the induced magnetic field is negligible so that $\vec{B} = (0, 0, B_0)$. Then the unsteady flow is governed by free-convective flow of an electrically conducting fluid in a rotating system under the usual Boussinesq's approximation in dimensionless form are as follows:

$$\frac{\partial u}{\partial t'} - 2\Omega' V' = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu\frac{\partial^2 u}{\partial z^2} - \frac{\sigma B_0^{\,2}}{\rho}u \qquad (1)$$

$$\frac{\partial V'}{\partial t'} + 2\Omega' u = \frac{\partial^2 V'}{\partial z^2} - \frac{\sigma B_0^{\,2}}{\rho}V' \qquad (2)$$

$$\rho C_p \frac{\partial T}{\partial t'} = k\frac{\partial^2 T}{\partial z^2} \qquad (3)$$

$$\rho C_p \frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial z^2} \qquad (4)$$

With the following initial and boundary conditions:

$u = 0, \quad T = T_\infty, \quad C' = C'_\infty \quad$ for all $y, t' \le 0$

$t' > 0: \; u = u_0 t', \; T = T'_\infty + (T'_w - T'_\infty)A\,t', \; C' = C'_\infty + (C'_w - C'_\infty)A\,t' \;$ at $y = 0$ $\qquad u \to 0, \quad T \to T_\infty, \quad C' \to C'_\infty$ as $y \to \infty \qquad (5)$

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 2, Issue 10, October 2013

On introducing the following non-dimensional quantities:

$$U = \frac{u}{(\nu u_0)^{1/3}},\ V = \frac{V'}{(\nu u_0)^{1/3}},\ t = t'\left(\frac{u_0^{\,2}}{\nu}\right)^{1/3},\ Z = z\left(\frac{u_0}{\nu^2}\right)^{1/3},$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty},\ Gr = \frac{g\beta(T_w - T_\infty)}{u_0},\ C = \frac{C' - C'_\infty}{C'_w - C'_\infty},\ Gc = \frac{g\beta^*(C'_w - C'_\infty)}{u_0} \qquad (6)$$

$$M = \frac{\sigma B_0^{\,2}}{\rho}\left(\frac{\nu}{u_0^{\,2}}\right)^{1/3},\ \Pr = \frac{\mu C_p}{k},\ Sc = \frac{\nu}{D},\ \mathrm{A} = \left(\frac{u_0^{\,2}}{\nu}\right)^{1/3}$$

we arrive at

$$\frac{\partial U}{\partial t} - 2\Omega V = Gr\theta + Gc\,C + \frac{\partial^2 U}{\partial Z^2} - MU \qquad (7)$$

$$\frac{\partial V}{\partial t} + 2\Omega U = \frac{\partial^2 V}{\partial Z^2} - MV \qquad (8)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial Z^2} \qquad (9)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Z^2} \qquad (10)$$

The initial and boundary conditions in non-dimensional quantities are

$$\begin{aligned} & q = 0, \quad \theta = 0, \quad C = 0 \quad \text{for all} \quad Z, t \le 0 \\ t > 0: \quad & q = t, \quad \theta = t, \quad C = t \quad \text{at} \quad Z = 0 \\ & q \to 0, \quad \theta \to 0, \quad C \to 0 \quad \text{as} \quad Z \to \infty \end{aligned} \qquad (11)$$

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 2, Issue 10, October 2013

is described by coupled partial differential equations (7) to (10) with the prescribed boundary conditions (11). To solve the equations (7) and (8), we introduce a complex velocity q = U + iV, equations (7) and (8) can be combined into a single equation:

$$\frac{\partial q}{\partial t} = G_r\theta + G_C\,C + \frac{\partial^2 q}{\partial Z^2} - mq \tag{12}$$

where m = M + 2iΩ.

Solution Procedure

The dimensionless governing equations (9, (10) and (12), subject to the initial and boundary conditions (11), are solved by the usual Laplace-transform technique and the solutions are derived as follows:

$$\begin{aligned} q = {} & \left(\frac{t}{2} + c + d + (ac+bd)t\right)\left[\exp(2\eta\sqrt{mt})\,erfc(\eta+\sqrt{mt}) + \exp(-2\eta\sqrt{mt})\,erfc(\eta-\sqrt{mt})\right] \\ & - \eta\sqrt{\frac{t}{m}}\left[\frac{1}{2} + ac + bd\right]\left[\exp(-2\eta\sqrt{mt})\,erfc(\eta-\sqrt{mt}) - \exp(2\eta\sqrt{mt})\,erfc(\eta+\sqrt{mt})\right] \\ & - 2c\,erfc(\eta\sqrt{\Pr}) - c\exp(at)[\exp(2\eta\sqrt{(m+a)t})\,erfc(\eta+\sqrt{(m+a)t}) \\ & \qquad + \exp(-2\eta\sqrt{(m+a)t})\,erfc(\eta-\sqrt{(m+a)t})] \\ & - 2d\,erfc(\eta\sqrt{Sc}) + c\exp(at)[\exp(2\eta\sqrt{at\Pr})\,erfc(\eta\sqrt{\Pr}+\sqrt{at}) \\ & \qquad + \exp(-2\eta\sqrt{at\Pr})\,erfc(\eta\sqrt{\Pr}-\sqrt{at})] \\ & - d\exp(bt)[\exp(2\eta\sqrt{(m+b)t})\,erfc(\eta+\sqrt{(m+b)t}) \\ & \qquad + \exp(-2\eta\sqrt{(m+b)t})\,erfc(\eta-\sqrt{(m+b)t})] \\ & + d\exp(bt)[\exp(2\eta\sqrt{btSc})\,erfc(\eta\sqrt{Sc}+\sqrt{bt}) + \exp(-2\eta\sqrt{btSc})\,erfc(\eta\sqrt{Sc}-\sqrt{bt})] \\ & - 2act\left[\left(1+2\eta^2\Pr\right)erfc(\eta\sqrt{\Pr}) - \frac{2\eta\sqrt{\Pr}}{\sqrt{\pi}}\exp(-\eta^2\Pr)\right] \\ & - 2bdt\left[\left(1+2\eta^2 Sc\right)erfc(\eta\sqrt{Sc}) - \frac{2\eta\sqrt{Sc}}{\sqrt{\pi}}\exp(-\eta^2 Sc)\right], \end{aligned} \tag{13}$$

$$\theta = t\left\{\left(1+2\eta^2\Pr\right)erfc(\eta\sqrt{\Pr}) - \frac{2\eta\sqrt{\Pr}}{\sqrt{\pi}}\exp(-\eta^2\Pr)\right\} \tag{14}$$

$$C = t\left\{\left(1+2\eta^2 Sc\right)erfc(\eta\sqrt{Sc}) - \frac{2\eta\sqrt{Sc}}{\sqrt{\pi}}\exp(-\eta^2 Sc)\right\} \tag{15}$$

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 2, Issue 10, October 2013

where $a = \dfrac{m}{\Pr - 1}$, $b = \dfrac{m}{Sc - 1}$, $c = \dfrac{G_r}{2a^2(1-\Pr)}$, $d = \dfrac{G_C}{2b^2(1-Sc)}$ and $\eta = \dfrac{Z}{2\sqrt{t}}$

In order to get the physical insight into the problem, the numerical values of q have been computed from (13). While evaluating this expression, it is observed that the argument of the error function is complex and, hence, we have separated it into real and imaginary parts by using the following formula:

$$erf\,(a+ib) = erf\,(a) + \frac{\exp(-a^2)}{2a\pi}\left[1-\cos(2ab)+i\sin(2ab)\right] + \frac{2\exp(-a^2)}{\pi}\sum_{n=1}^{\infty}\frac{\exp(-n^2/4)}{n^2+4a^2}\left[f_n(a,b)+i\,g_n(a,b)\right] + \in(a,b)$$

where

$$f_n = 2a - 2a\cosh(nb)\cos(2ab) + n\sinh(nb)\sin(2ab)$$

and

$$g_n = 2a\cosh(nb)\sin(2ab) + n\sinh(nb)\cos(2ab)$$

$$\left|\in(a,b)\right| \approx 10^{-16}\left|erf\,(a+ib)\right|$$

III. RESULTS AND DISCUSSION

For physical understanding of the problem, numerical computations are carried out for different physical parameters Gr, Gc, Sc, Pr, m and t upon the nature of the flow and transport. The value of the Schmidt number Sc is taken to be 2.01 which correspond to Ethyl Benzene. Also, the values of Prandtl number Pr are chosen such that they represent air (Pr = 0.71) and water (Pr = 7.0). The numerical values of the velocity, temperature and concentration fields are computed for different physical parameters like Prandtl number, rotation parameter, magnetic field parameter, thermal Grashof number, mass Grashof number, Schmidt number and time.

The temperature profiles are calculated for water and air from equation (14) and these are shown in Figure 1. The effect of the Prandtl number plays an important role in temperature field. It is observed that the temperature increases with decreasing Prandtl number. This shows that the heat transfer is more in air than in water.

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 2, Issue 10, October 2013

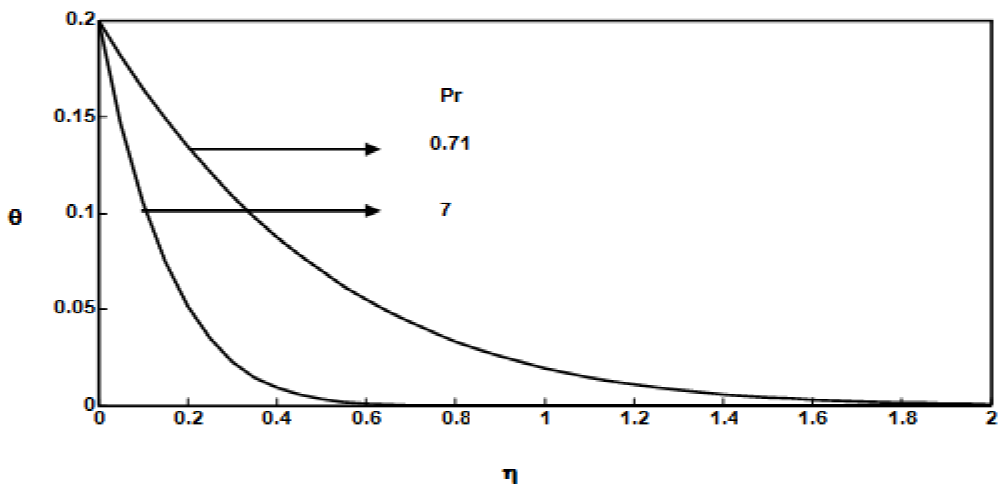

Figure 1 Temperature Profiles for different Pr

Figure 2 represents the effect of concentration profiles at time t = 0.2 for different Schmidt number (Sc = 0.16, 0.3, 0.6). The effect of concentration is important in concentration field. The profiles have the common feature that the concentration decreases in a monotone fashion from the surface to a zero value far away in the free stream. It is observed that the wall concentration increases with decreasing values of the Schmidt number.

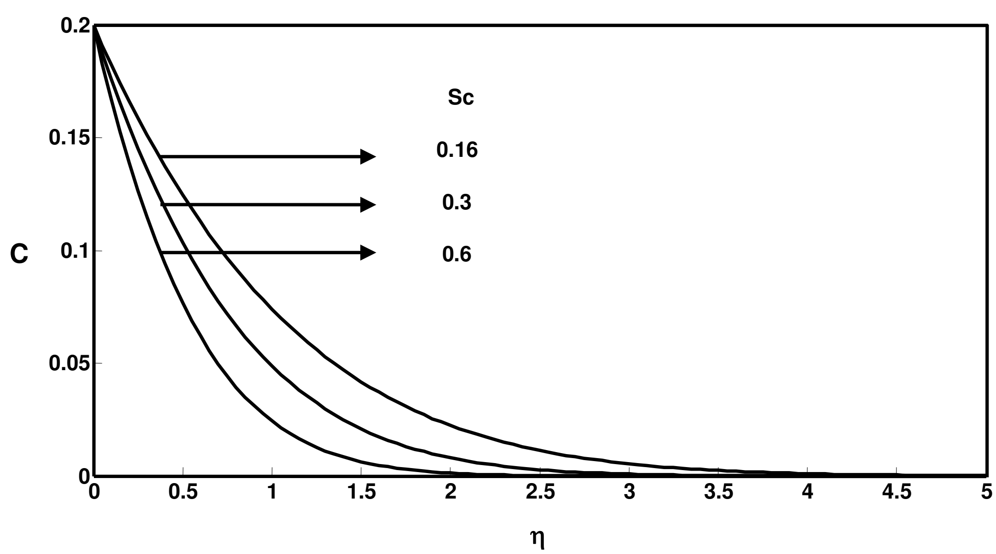

Figure 2 Concentration Profiles for different Sc

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 2, Issue 10, October 2013

Figure 3. demonstrates the effects of different thermal Grashof number (Gr =2, 5), mass Grashof number (Gc = 2, 5), M = 5, Ω = 0.5 and Pr = 7, on the primary velocity at time t = 0.4. It is observed that the velocity increases with increasing values of the thermal Grashof number or mass Grashof number.

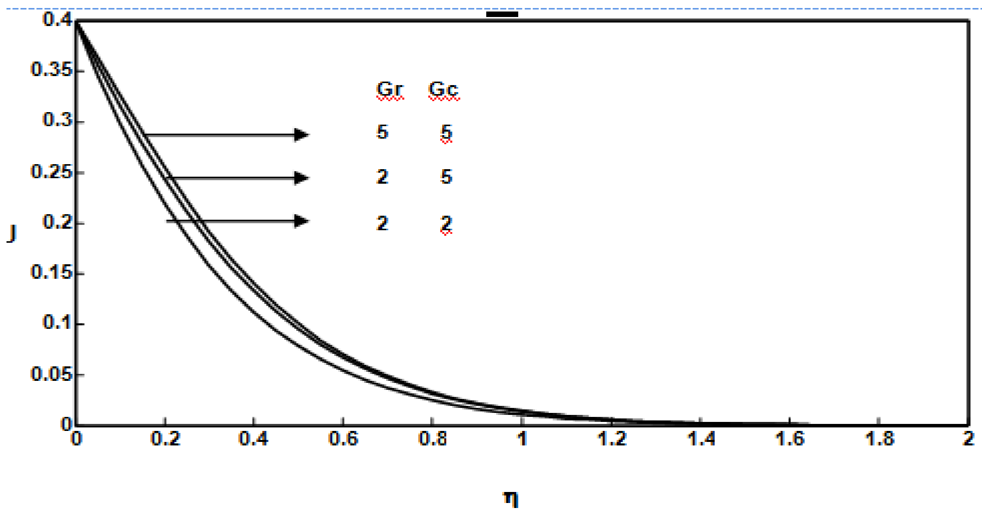

Figure 3 Primary Velocity Profiles for different Gr and Gc

Figure 4. illustrates the effects of the magnetic field parameter on the velocity when (M = 0, 5, 10), Gr = Gc = 5, Ω = 0.5, Pr = 7 and t = 0.4. It is observed that the velocity increases with decreasing values of the magnetic field parameter. This shows that the increase in the magnetic field parameter leads to a fall in the velocity. This agrees with the expectations, since the magnetic field exerts a retarding force on the free convective flow. The secondary velocity profiles for different

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 2, Issue 10, October 2013

magnetic field parameter (M=0,5,10) are presented in figure 5. The trend is just reversed with respect to magnetic field

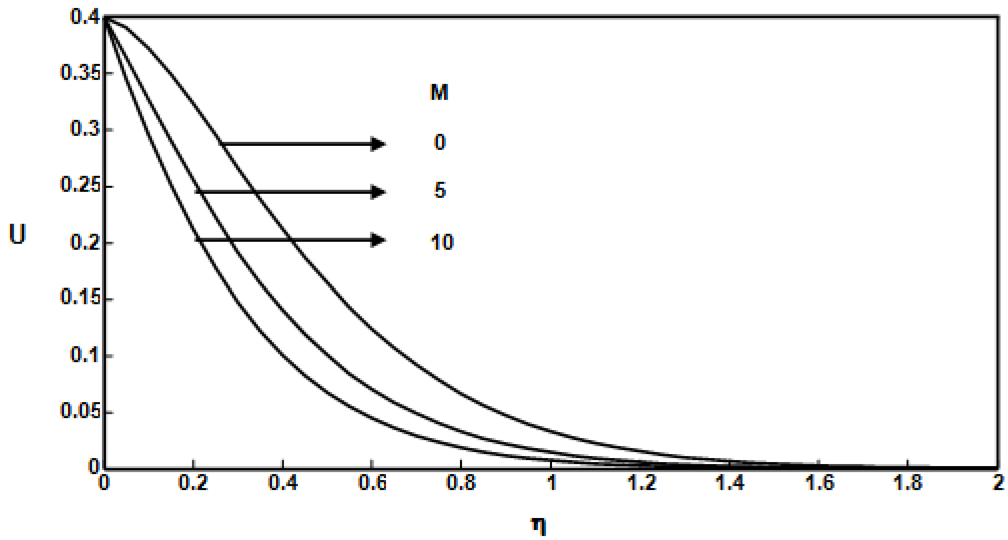

Figure 4 Primary Velocity Profiles for different M

parameter

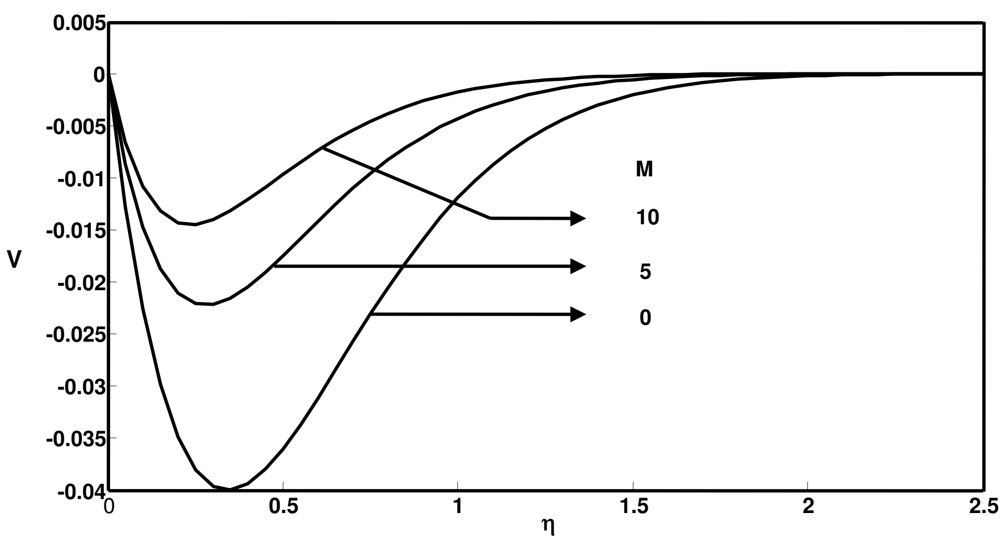

Figure 5 Secondary Velocity Profiles for different M

The secondary velocity profiles for different rotation parameter (Ω = 0.5, 1), Gr = Gc = 5, Pr = 7, M = 10 and t = 0.4 are shown in figure 6. It is observed that the velocity increases with decreasing values of the rotation parameter. The secondary velocity profiles for different thermal Grashof number (Gr = 2, 5), mass Grashof number (Gc = 2, 5), Ω = 0.5, M = 5, Pr = 7

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 2, Issue 10, October 2013

and t = 0.4 are presented in figure 7. The trend shows that the velocity increases with increasing values of thermal Grashof number or mass Grashof number.

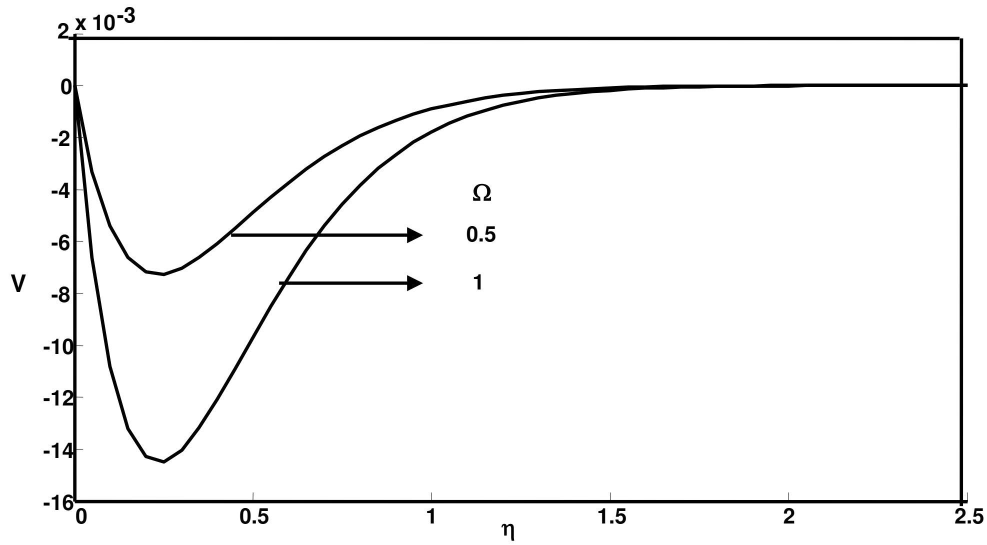

Figure 6 Secondary Velocity Profiles for different Ω

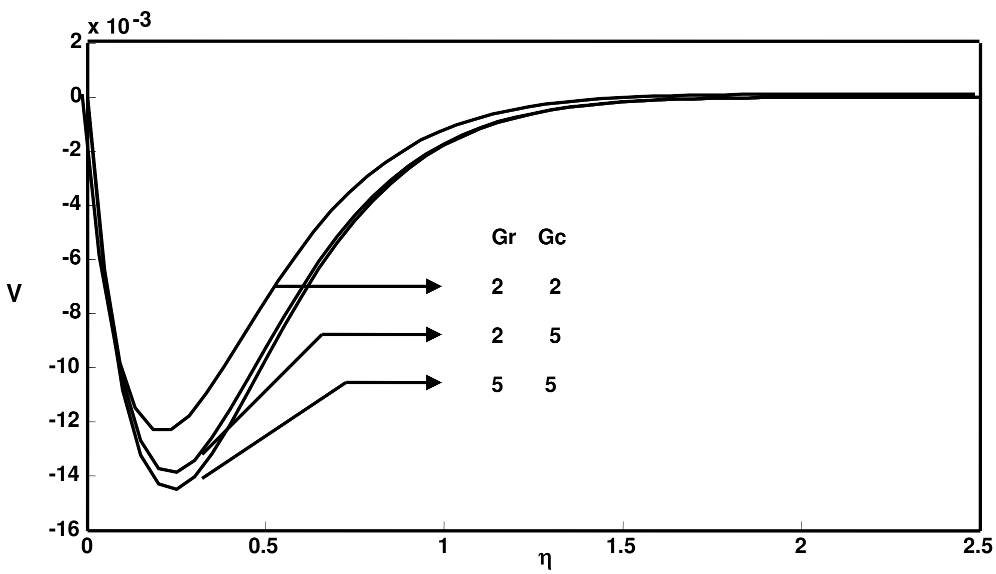

Figure 7 Secondary Velocity Profiles for different Gr and Gc

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 2, Issue 10, October 2013

IV. CONCLUSION

The closed form solution of rotation effects on unsteady hydromagnetic flow past a uniformly accelerated infinite vertical plate in the presence of variable temperature and mass diffusion has been studied. The effect of different parameters like thermal Grashof number, mass Grashof number and t are studied graphically. It is observed that the velocity increases with increasing values of Gr, Gc and t. But the trend is just reversed with respect to the rotation parameter or magnetic field parameter M.

REFERENCES

[1] Basanth Kumar Jha, Ravindra Prasad, "Free convection and mass transfer effects on the flow past an accelerated vertical plate with heat sources", Mechanics Research Communications Vol.17, pp.143-148, 1990.

[2] Gupta AS, Pop I, Soundalgekar VM, "Free convection effects on the flow past an accelerated vertical plate in an incompressible dissipative fluid", Rev. Roum. Sci. Techn.-Mec. Apl. Vol.24, pp.561-568, 1979.

[3] Kafousias NG, Raptis A, "Mass transfer and free convection effects on the flow past an accelerated vertical infinite plate with variable suction or injection", Rev. Roum. Sci. Techn.-Mec. Apl. Vol.26, pp.11-22, 1981.

[4] Muthucumarswamy R, Tina Lal, Ranganayakulu D, "Effects of rotation on MHD flow past an accelerated isothermal vertical plate with heat and mass transfer", Theoretical Applied Mechanics, Vol.37, pp. 189-202, 2010.

[5] Raptis A, Singh A K, "MHD free convection flow past an accelerated vertical plate", Letters in Heat and Mass Transfer. Vol.8, pp.137-143, 1981.

[6] Raptis A, Singh AK,. "Rotation effects on MHD free convection flow past an accelerated vertical plate", Mechanics Research Communications, Vol.12, pp.31-40, 1985.

[7] Raptis A, Tzivanidis G J, Peridikis C P, "Hydromagnetic free convection flow past an accelerated vertical infinite plate with variable suction and heat flux", Letters in heat and mass transfer, Vol.8, pp.137-143, 1981.

[8] Singh AK, Singh J, "Mass transfer effects on the flow past an accelerated vertical plate with constant heat flux", Astrophysics and Space Science, Vol.97, pp.57-61, 1983.

[9] Singh AK, "Hydromagnetic free-convection flow past an impulsively started vertical plate in a rotating fluid", International Communications in Heat and Mass transfer, Vol.11, pp.399-406, 1984.

[10] Soundalgekar V.M, "Effects of mass transfer on flow past a uniformly accelerated vertical plate", Letters in heat and mass transfer, Vol.9, pp.65-72, 1982.

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 2, Issue 10, October 2013

Dr R Muthucumaraswamy, received his B.Sc. degree in Mathematics, from Gurunanank College, University of Madras in 1985, M.Sc. degree in Applied Mathematics from Madras Institute of Technology, Anna University in 1987, M.Phil. degree in Mathematics from Pachaiyappas' College, University of Madras in 1991 and Ph.D. degree in Mathematics from Anna University in 2001. His area of specialization in Theoretical and Computational Fluid Dynamics. He published 180 papers in National/International journals and in conferences. He completed two funded projects from Defence Research and Developmental Organization in 2009 and 2012. He received best teacher award in the year 2001. Currently, working as a Professor and Head, Department of Applied Mathematics, Sri Venkateswara College of Engineering, Sriperumbudur, India.

Ms Tina Lal, received his B.Sc. degree in Mathematics from Ethiraj College for women, University of Madras in the year 1999, M.Sc. degree in Mathematics from Stella Maris College, University of Madras in 2001 and M.Phil. degree in Mathematics, from Anna University 2002. She is She served in Sri Venkateswara College of Engineering from 2002 to 2012. Currently, she is working in Central Polytechnic, Chennai. Her area of specialization is theoretical fluid dynamics, in particular heat and mass transfer studies in rotating fluid. She published three research papers in reputed international journals.

Dr D. Ranganayakulu, received his B.Sc. degree in Mathematics, from University of Madras in 1979, M.Sc. degree in Mathematics from Sri Venkateswara University, Tirupathi, in 1981, M.Phil. degree in Mathematics from Presidency College in 2000 and Ph.D. degree in Mathematics from Madras Christian College in 2004. His area of specialization is Theoretical Computer Science, Graph Theory and Fluid Dynamics. He published 15 papers in National/International journals and in conferences. Currently, he works as an Associate Professor in the Department of Mathematics, SIVET College, Gowrivakkam, Chennai, India.