

Modeling of Vector Controlled Induction Motor in Different Reference Frames

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ABSTRACT— This paper describes the various mathematical models of both squirrel cage and wound rotor induction motors in different reference frames with different state-space variables. The work suggests the d-q axis unified approach for both types of induction motors by using the state-space analysis and it is a strong tool in the modeling of the symmetrical induction motors. When an electrical motor is represented as a mathematical model with inputs and outputs, it can be analyzed and described in many ways, considering different reference frames and state-space variables. The applications of each model are also discussed. Models are simulated with MATLAB/SIMULINK software for transient response of the squirrel cage induction motor in terms of electromagnetic torque and rotor angular velocity and results are discussed.

KEYWORDS— modeling of induction motor, sensor less control, d-q axes model, and different reference frames.

I. INTRODUCTION

Due to advances in control system, induction motor is used as variable speed drive. When an electrical motor is represented as a mathematical model with inputs and outputs, it can be analyzed and described in many ways, considering different reference frames and state-space variables. In three-phase symmetrical or two-phase unsymmetrical version, the induction motor is employed with vector control strategy. Thus, induction motor can be analyzed as DC motor [1].

The dynamic operation of the induction motor drive system has an important role in the overall performance of the system and there are two fundamental methods for the induction motor control: one is the Direct measurement of the machine parameters, which are compared to the reference signals through closed control loops and other is

the estimation of the machine parameters in the sensor less control schemes, with the following implementation methodologies: slip frequency calculation method, speed estimation using state equation, estimation based on slot space harmonic voltages, flux estimation and flux vector control, direct control of torque and flux, observer-based speed sensor less control with parameter adaptation, neural network based sensor less control, fuzzy-logic based sensor less control.[2]

The development of the precise system model is fundamental to each stage in the design, analysis and control of all electrical machines. The level of accuracy required for these models entirely depends on the design stage under consideration. In some cases, the mathematical description used in machine design requires very fine tolerance levels as stated by Nabae and Murata [3], [4]. However, in the development of suitable models for control purposes, consider certain assumptions that simplify the resulting machine model. Additionally, since modern electric machines are continuously fed from switching power conversion stages, the developed motor models should be valid for arbitrary applied voltage and current waveforms [5]. Generally, the following assumptions are made while implementing the induction motor models

- No magnetic saturation.
- No saliency effects.
- Negligible spatial MMF harmonics.
- The effects of the stator slots may be neglected.
- There is no fringing of the magnetic circuit.
- The magnetic field intensity is constant quantity and directed radially across the air-gap.
- Hysteresis and eddy current effects are negligible.

The goal of this paper is to establish the commonly used d-q models in different reference frames. Matlab/Simulink software is used to simulate the

dynamic models of squirrel cage induction motor. The paper has been organized as follows: section II briefly explains the measurement of motor parameters, section III demonstrate the mathematical model of the induction motor, section IV describes the d-q axis models, and the results of implemented Matlab models are shown in section V.

II. MEASUREMENT OF ROTOR PARAMETERS

A. Stator Resistance

The stator phase resistance is measured by applying a DC voltage and the resulting current with the rotor at standstill. This procedure gives only the DC resistance at a certain temperature, the AC resistance is calculated by considering the wire size, the stator frequency and the operating temperature.

B. No-Load Test

This test is performed by applying a balanced rated voltage on the stator windings at the rated frequency and driven at synchronous speed by DC motor or synchronous motor, preferably a DC motor. The no-load test provides information about exciting current and rotational losses.

C. Locked-Rotor Test

The rotor of the induction motor is locked and a set of low three phase voltages is applied to calculate rated stator currents. The input power per phase is measured along with the input voltage and stator current. The locked rotor test provides the information about leakage impedances and rotor resistance. [7]

III. MODELING OF INDUCTION MOTOR

A. Space vector equations for three-phase induction motor

For the modeling of three-phase induction motor generally two theories are used. First one is the two real axis reference frame theory initially developed by Park for the synchronous machine [8]. Second one is the space complex vector theory elaborated by Kovacs and Racz [9]. Both theories are used to describe the complete equations system of continuous-time linear model of the induction motor with certain assumptions. Usually, the following assumptions are made [10]:

- Geometrical and electrical machine configuration is symmetrical.
- Space harmonics of the stator and rotor magnetic flux are negligible.
- Infinitely permeable iron.
- Stator and rotor windings are sinusoidally distributed in space and replaced by an equivalent concentrated winding.
- Magnetic saturation, anisotropy effect, core loss and skin effect are negligible.
- Windings resistance and reactance do not vary with the temperature.
- Currents and voltages are sinusoidal terms.

- End and fringing effects are neglected

In this approach, all variables are represented by polar vectors indicating the magnetic and angular position for the rotating sinusoidal distribution. Let V_{ph} = Phase Voltage, V_s = Space Vector Stator Voltage, V_r = Space Vector Rotor Voltage, I_s = Space Vector Stator Current, I_r = Space Vector Rotor Current, I_m = Space Vector Magnetisation Current, λ_s = Space Vector Stator Flux, λ_r = Space Vector Rotor Flux, f = Supply Frequency, ω_e = Synchronous Speed, ω_r = Electrical Rotor Angular Velocity, ω_m = Mechanical Rotor Angular Velocity, R_s = Stator Resistance, R_r = Rotor Resistance, L_s = Stator Self-Inductance, L_r = Rotor Self-Inductance, L_m = Mutual Inductance, J = Moment of Inertia, L_{1s} = Stator Leakage Inductance, L_{1r} = Rotor Leakage Inductance, K_s = Stator Coupling Factor, K_r = Rotor Coupling Factor, σ = Leakage co-efficient, τ_s = Stator Time Constant, τ_r = Rotor Time Constant, τ'_s = Transient Stator Time Constant, τ'_r = Transient Rotor Time Constant, p = Derivative Operator.

From the Fig (1), the stator voltage equation is

$$V_s = \frac{2}{3} (1.V_{as} + \bar{\alpha}V_{bs} + \bar{\alpha}^2V_{cs})$$

$$V_s = \frac{2}{3} (1.I_{as} + \bar{\alpha}I_{bs} + \bar{\alpha}^2I_{cs}).R_s + p \frac{2}{3} (1.\lambda_{as} + \bar{\alpha}\lambda_{bs} + \bar{\alpha}^2\lambda_{cs})$$

Or in a condensed form

$$V_s = R_s I_s + p \lambda_s \tag{1}$$

Similarly, the rotor equation is

$$V_r = R_r I_r + p \lambda_r \tag{2}$$

And stator and rotor flux linkage-current equations are

$$\lambda_s = L_s I_s + L_m e^{j\theta} I_r \tag{3}$$

$$\lambda_r = L_r I_r + L_m e^{-j\theta} I_s \tag{4}$$

Where θ is relative rotor position angle

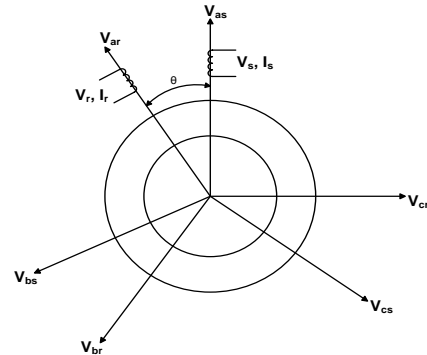


Fig (1) Model of the 3-φ induction motor in Space Vector Notation

B. Three-Phase to Two-Phase Transformation

To develop models of the induction motor, three-phase (a, b, c) to two-phase (d, q) transformation is needed. Fig (2) shows the three-phase to two phase transformation windings. Assuming that each of the three-phase windings has T_1 turns per phase and equal current magnitudes, the two-phase windings will have $3T_1/2$ turns per phase for MMF equality. Let the q-axis is assumed to be lagging the a-axis by θ_c . The relationship between abc and dqo is as follows

$$\begin{bmatrix} V_{qs} \\ V_{ds} \\ V_o \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos \theta_c & \cos \left(\theta_c - \frac{2\pi}{3} \right) & \cos \left(\theta_c + \frac{2\pi}{3} \right) \\ \sin \theta_c & \sin \left(\theta_c - \frac{2\pi}{3} \right) & \sin \left(\theta_c + \frac{2\pi}{3} \right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \times \begin{bmatrix} V_{as} \\ V_{bs} \\ V_{cs} \end{bmatrix}$$

$$V_{qds} = [T_{abc}] V_{abc}$$

$[T_{abc}]$ is also valid for currents, flux-linkages in a machine (5)

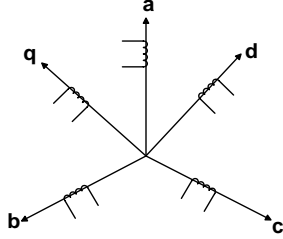


Fig (2) Three-phase to Two-phase Transformation

In stationary reference frame $\theta_c = 0$,
 In rotor reference frame $\theta_c = \theta_r$,
 In synchronous reference frame $\theta_c = \theta_e$.

C. Generalized Model in Arbitrary Reference Frame

The analysis of induction motor drive system is carried out by representing the stator and rotor variables are in a common reference frame. By using the same space vector notations, we can define an arbitrary reference frame, which rotates with the angular velocity ω_c , and according to Fig (3), the following relation is valid

$$x_{qd}^c = x_{qd} \cdot e^{-j\theta_c}$$

Where θ_c is the time variable relative angle between the arbitrary reference frame and the stationary reference frame. The reverse transformation relation is

$$x_{qd} = x_{qd}^c \cdot e^{j\theta_c}$$

In an arbitrary common reference frame, the voltage and flux linkage equations (1)–(4) becomes

$$V_s^c = R_s \cdot I_s^c + j \cdot \omega_c \cdot \lambda_s^c + p \lambda_s^c \quad (6)$$

$$V_r^c = R_r \cdot I_r^c - j \cdot (\omega_c - \omega_r) \lambda_r^c + p \lambda_r^c \quad (7)$$

$$\lambda_s^c = L_s \cdot I_s^c + L_m \cdot I_r^c \quad (8)$$

$$\lambda_r^c = L_r \cdot I_r^c + L_m \cdot I_s^c \quad (9)$$

Equations (6) to (9) represent the mathematical model of the induction motor in a common arbitrary reference frame. The main advantage of the arbitrary reference frame is mutual inductance doesn't depend on the relative rotor position. The magnetisation flux and current in an arbitrary reference frame becomes

$$\lambda_m^c = L_m (I_s^c + I_r^c) \quad (10)$$

$$I_m^c = I_s^c + I_r^c \quad (11)$$

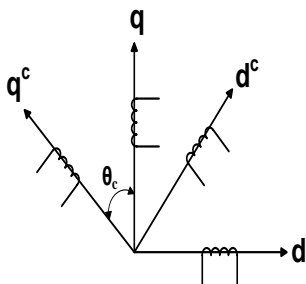


Fig (3) Transformation into an arbitrary reference Frame

D. Instantaneous Electromagnetic Torque

The electromagnetic torque of the induction motor is the ratio of the air-gap power by mechanical rotor speed (ω_m) in rad/sec. The induction motor electromagnetic torque can be written as

$$T_e = \frac{3P}{2} L_m (I_{qs}^c \cdot I_{dr}^c - I_{ds}^c \cdot I_{qr}^c) \quad (12)$$

Where P is the number of poles and T_e is the instantaneous electromagnetic torque. The electromagnetic torque can be expressed in many other forms by substituting the equations (8), (9), (10) & (11). Neglecting mechanical damping, the torque and rotor speed are related as

$$\frac{d\omega_r}{dt} = \frac{P}{2J} (T_e - T_L) \quad (13)$$

where T_L is the load torque.

E. Modeling of induction motor in Different Reference Frames

When the mathematical model of the induction motor is established, several reference frames can be employed depending on the application and the chosen strategy control. There are several main reference frames: stationary reference frame fixed to the stator, rotor reference frame fixed to the rotor shaft, synchronous rotating reference frame revolving with an angular velocity equal to: the air-gap flux, the rotor flux, the stator voltage, the rotor current space vectors. The significance of the index is as follows:

- s –stationary reference frame
- r –rotor reference frame
- e –synchronous rotating reference frame

a. Stationary Reference Frame Model Equations

The choice of the reference frame for the dynamic analysis of the induction motor, especially when the stator circuits are unbalanced or discontinuous and the rotor-applied voltages are balanced or zero, is more convenient to be fixed to the stator frame. This stationary reference frame was first employed by Stanley [11]. Transformation from the arbitrary reference frame to the stationary reference frame fixed to the stator is made by substituting $\omega_c = 0$ and the induction motor equations system can be written as

$$V_{qs}^s = R_s \cdot I_{qs}^s + p \lambda_{qs}^s \quad (14)$$

$$V_{ds}^s = R_s \cdot I_{ds}^s + p \lambda_{ds}^s \quad (15)$$

$$V_{qr}^s = R_r \cdot I_{qr}^s - \omega_r \cdot \lambda_{dr}^s + p \lambda_{qr}^s \quad (16)$$

$$V_{dr}^s = R_r \cdot I_{dr}^s + \omega_r \cdot \lambda_{qr}^s + p \lambda_{dr}^s \quad (17)$$

$$\lambda_s^c = L_s \cdot I_s^c + L_m \cdot I_r^c$$

$$\lambda_r^c = L_r \cdot I_r^c + L_m \cdot I_s^c$$

b. Rotor Reference Frame Model Equations

The possibility of the reference frame for the dynamic analysis of the induction motor, especially when the rotor circuits are unbalanced, is more convenient to be fixed to the rotor frame. This reference frame is initially developed for synchronous machine and then applied to

the induction motor by Brereton [12]. The method of referring the machine variables to a rotor reference frame is most useful for field oriented control systems. Transformation from the arbitrary reference frame to the rotor reference frame is made by substituting $\omega_c = \omega_r$ and the induction motor equations system becomes

$$V_{qs}^r = R_s \cdot I_{qs}^r + \omega_r \cdot \lambda_{ds}^r + p \lambda_{qs}^r \quad (18)$$

$$V_{ds}^r = R_s \cdot I_{ds}^r - \omega_r \cdot \lambda_{qs}^r + p \lambda_{ds}^r \quad (19)$$

$$V_{qr}^r = R_r \cdot I_{qr}^r + p \lambda_{qr}^r \quad (20)$$

$$V_{dr}^r = R_r \cdot I_{dr}^r + p \lambda_{dr}^r \quad (21)$$

$$\lambda_s^c = L_s \cdot I_s^c + L_m \cdot I_r^c$$

$$\lambda_r^c = L_r \cdot I_r^c + L_m \cdot I_s^c$$

c. Synchronous Reference Frame Model Equations

The synchronously rotating reference frame is suitable when incorporating the dynamic characteristics of an induction motor into a digital computer program used to study the dynamic and transient stability of the system. The synchronous reference frame may also be useful in variable frequency applications if we may assume that the stator voltages are a sinusoidal balanced set. It was systematically developed by Kovas [9], Krause [10] and Lorenz [6]. Transformation from the arbitrary reference frame to the synchronous reference frame is made by substituting $\omega_c = \omega_e$ and the induction motor equations system becomes

$$V_{qs}^e = R_s \cdot I_{qs}^e + \omega_e \cdot \lambda_{ds}^e + p \lambda_{qs}^e \quad (22)$$

$$V_{ds}^e = R_s \cdot I_{ds}^e - \omega_e \cdot \lambda_{qs}^e + p \lambda_{ds}^e \quad (23)$$

$$V_{qr}^e = R_r \cdot I_{qr}^e + (\omega_e - \omega_r) \cdot \lambda_{dr}^e + p \lambda_{qr}^e \quad (24)$$

$$V_{dr}^e = R_r \cdot I_{dr}^e - (\omega_e - \omega_r) \cdot \lambda_{qr}^e + p \lambda_{dr}^e \quad (25)$$

$$\lambda_s^c = L_s \cdot I_s^c + L_m \cdot I_r^c$$

$$\lambda_r^c = L_r \cdot I_r^c + L_m \cdot I_s^c$$

IV. D-Q AXES MODELS

The three-phase induction motor can be modeled by using different state-space variables and keeping as inputs the stator voltages and the load torque, and as outputs the electromagnetic torque and the rotor angular velocity. The possible set of currents and flux linkages per second space vectors are defined as

$$[x] = [I_s \ I_r \ I_m \ \lambda_s \ \lambda_r \ \lambda_m]^T \quad (26)$$

The d-q axes are orthogonal and fixed to the stator, q axis coincide with the magnetic axis of the as winding. As there are four voltage equations, it is necessary to consider two of the space vectors as state-variables in order to obtain a solution for the equations system. Let the selected pair of state-space variables are denoted as x_1, x_2 . The set of six state-space variables will be expressed in terms of the two selected state-space variables:

$$[x] = [I_s \ I_r \ I_m \ \lambda_s \ \lambda_r \ \lambda_m]^T = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \\ a_{51} & a_{52} \\ a_{61} & a_{62} \end{bmatrix} \cdot [x_1 \ x_2]^T$$

There are mainly three types of d-q models: current state-space variables models, flux linkage state-space variables models, mixed currents-flux linkage state-space variables models. By using d-q model to obtain a

complete version of the three-phase IM model, viewed as the key for a motion control system. The general form of d-q model equations written in state variables system is given by:

$$p x = A \cdot x + B \cdot u \quad (27)$$

Where x is the selected set of state-variables and represent also the output of the model, u is the input vector (stator voltage), A is the coefficient matrix of x, B is the coefficient matrix of u, and p is the differential operator ($\frac{d}{dt}$). Flowchart for the dynamic simulation of the induction motor in d-q model is shown in Fig (4) and in this paper commonly used state space-variables models are presented below:

$$L_{1s} = L_s - L_m, L_{1r} = L_r - L_m, K_s = \frac{L_m}{L_s}, K_r = \frac{L_m}{L_r},$$

$$\sigma = 1 - K_s \cdot K_r, \tau_s = \frac{L_s}{R_s}, \tau_r = \frac{L_r}{R_r}, \tau'_s = \sigma \cdot \tau_s, \tau'_r = \sigma \cdot \tau_r$$

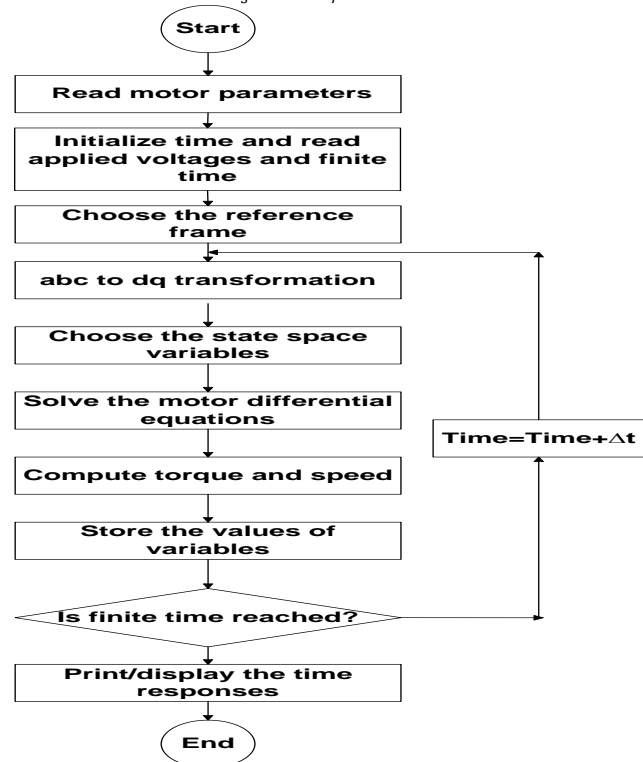


Fig (4) Flowchart for dynamic simulation of the IM

A. $[I_{qs} \ I_{ds} \ I_{qr} \ I_{dr}]$:

In this mathematical d-q model, stator and rotor space vector currents are selected as state-space variables $x = [I_{qs} \ I_{ds} \ I_{qr} \ I_{dr}]$. The stator current space vector is considered generally as the right choice, because it corresponds to directly measurable quantities. This model is derived by substituting equations (8) & (9) in the equations (6) & (7) and expressed in a matrix form as follows:

$$\begin{bmatrix} I_{qs}^c \\ I_{ds}^c \\ I_{qr}^c \\ I_{dr}^c \end{bmatrix} = \begin{bmatrix} \frac{-1}{\tau_s} & -\omega_c & -K_s \cdot \mathfrak{p} & -\omega_c \cdot K_s \\ \omega_c & \frac{-1}{\tau_s} & \omega_c \cdot K_s & -K_s \cdot \mathfrak{p} \\ -K_r \cdot \mathfrak{p} & -(\omega_c - \omega_r) \cdot K_r & \frac{-1}{\tau_r} & (\omega_c - \omega_r) \\ (\omega_c - \omega_r) \cdot K_r & -K_r \cdot \mathfrak{p} & -(\omega_c - \omega_r) & \frac{-1}{\tau_r} \end{bmatrix} \begin{bmatrix} I_{qs}^c \\ I_{ds}^c \\ I_{qr}^c \\ I_{dr}^c \end{bmatrix} + \begin{bmatrix} \frac{1}{L_s} & 0 & 0 & 0 \\ 0 & \frac{1}{L_s} & 0 & 0 \\ 0 & 0 & \frac{1}{L_r} & 0 \\ 0 & 0 & 0 & \frac{1}{L_r} \end{bmatrix} \begin{bmatrix} V_{qs}^c \\ V_{ds}^c \\ V_{qr}^c \\ V_{dr}^c \end{bmatrix}$$

B. $[I_{qs} \ I_{ds} \ I_{qm} \ I_{dm}]$:

In this mathematical d-q model, stator and magnetisation currents space vectors are selected as state-space variables $x = [I_{qs} \ I_{ds} \ I_{qm} \ I_{dm}]$. By selecting the magnetisation current space vector among the state-space variables, it is possible to include the saturation effect in modeling the IM. Also, the stator current space vector is a measurable quantity, and determines a precise and accurate option for implementing controllers [13]. This model is derived by substituting equations (8), (9) & (11) in the equations (6) & (7) and expressed in a matrix form as follows:

$$\begin{bmatrix} I_{qs}^c \\ I_{ds}^c \\ I_{qm}^c \\ I_{dm}^c \end{bmatrix} = \begin{bmatrix} \frac{-R_s}{L_{1s}} & -\omega_c & \frac{-L_m}{L_{1s}} \cdot \mathfrak{p} & -\omega_c \cdot \frac{L_m}{L_{1s}} \\ \omega_c & \frac{-R_s}{L_{1s}} & \omega_c \cdot \frac{L_m}{L_{1s}} & \frac{-L_m}{L_{1s}} \cdot \mathfrak{p} \\ \frac{1}{\tau_r} + (1-K_r) \cdot \mathfrak{p} & (\omega_c - \omega_r) \cdot (1-K_r) & \frac{-1}{\tau_r} & -(\omega_c - \omega_r) \\ -(\omega_c - \omega_r) \cdot (1-K_r) & \frac{1}{\tau_r} + (1-K_r) \cdot \mathfrak{p} & (\omega_c - \omega_r) & \frac{-1}{\tau_r} \end{bmatrix} \begin{bmatrix} I_{qs}^c \\ I_{ds}^c \\ I_{qm}^c \\ I_{dm}^c \end{bmatrix} + \begin{bmatrix} \frac{1}{L_{1s}} & 0 & 0 & 0 \\ 0 & \frac{1}{L_{1s}} & 0 & 0 \\ 0 & 0 & \frac{1}{L_r} & 0 \\ 0 & 0 & 0 & \frac{1}{L_r} \end{bmatrix} \begin{bmatrix} V_{qs}^c \\ V_{ds}^c \\ V_{qr}^c \\ V_{dr}^c \end{bmatrix}$$

C. $[\lambda_{qs} \ \lambda_{ds} \ \lambda_{qr} \ \lambda_{dr}]$:

In this mathematical d-q model, stator and rotor flux linkages space vectors are selected as state-space variables $x = [\lambda_{qs} \ \lambda_{ds} \ \lambda_{qr} \ \lambda_{dr}]$. When flux linkages space vectors are selected as state-space variables, the models are less complex than the current state-space variables models because of it contains information about two current space vectors components. From the equations (8) & (9)

$$I_s^c = \frac{L_r \cdot \lambda_s^c - L_m \cdot \lambda_r^c}{L_s \cdot L_r - L_m^2} \tag{30}$$

$$I_r^c = \frac{L_s \cdot \lambda_r^c - L_m \cdot \lambda_s^c}{L_s \cdot L_r - L_m^2} \tag{31}$$

This model is derived by substituting equations (30) & (31) in the equations (6) & (7) and expressed in a matrix form as follows:

$$\begin{bmatrix} \lambda_{qs}^c \\ \lambda_{ds}^c \\ \lambda_{qr}^c \\ \lambda_{dr}^c \end{bmatrix} = \begin{bmatrix} \frac{-1}{\tau_s} & -\omega_c & \frac{K_r}{\tau_s} & 0 \\ \omega_c & \frac{-1}{\tau_s} & 0 & \frac{K_r}{\tau_s} \\ \frac{K_s}{\tau_r} & 0 & \frac{-1}{\tau_r} & -(\omega_c - \omega_r) \\ 0 & \frac{K_s}{\tau_r} & (\omega_c - \omega_r) & \frac{-1}{\tau_r} \end{bmatrix} \begin{bmatrix} \lambda_{qs}^c \\ \lambda_{ds}^c \\ \lambda_{qr}^c \\ \lambda_{dr}^c \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_{qs}^c \\ V_{ds}^c \\ V_{qr}^c \\ V_{dr}^c \end{bmatrix}$$

D. $[\lambda_{qs} \ \lambda_{ds} \ \lambda_{qm} \ \lambda_{dm}]$:

In this mathematical d-q model, stator and air-gap flux linkages space vectors are selected as state-space variables $x = [\lambda_{qs} \ \lambda_{ds} \ \lambda_{qm} \ \lambda_{dm}]$. This choice presents the advantage of an easier saturation effect modeling, but the disadvantage of an increased computational burden. This easier saturation effect modeling imposes many practical solutions for vector-control schemes. This model is derived by substituting equations (30), (31) & (10) in the equations (6) & (7) and expressed in a matrix form as follows:

$$\begin{bmatrix} \lambda_{qs}^c \\ \lambda_{ds}^c \\ \lambda_{qm}^c \\ \lambda_{dm}^c \end{bmatrix} = \begin{bmatrix} \frac{-1}{\tau_s \cdot (1-K_s)} & -\omega_c & \frac{1}{\tau_s \cdot (1-K_s)} & 0 \\ \omega_c & \frac{-1}{\tau_s \cdot (1-K_s)} & 0 & \frac{1}{\tau_s \cdot (1-K_s)} \\ \frac{K_s}{\tau_r} + \frac{K_s \cdot (1-K_r)}{\sigma} \cdot \mathfrak{p} & \frac{(\omega_c - \omega_r) \cdot K_s \cdot (1-K_r)}{\sigma} & \frac{-1}{\tau_r} & -(\omega_c - \omega_r) \\ -(\omega_c - \omega_r) \cdot K_s \cdot (1-K_r) & \frac{K_s}{\tau_r} + \frac{K_s \cdot (1-K_r)}{\sigma} \cdot \mathfrak{p} & (\omega_c - \omega_r) & \frac{-1}{\tau_r} \end{bmatrix} \begin{bmatrix} \lambda_{qs}^c \\ \lambda_{ds}^c \\ \lambda_{qm}^c \\ \lambda_{dm}^c \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{K_r \cdot (1-K_s)}{\sigma} & 0 \\ 0 & 0 & 0 & \frac{K_r \cdot (1-K_s)}{\sigma} \end{bmatrix} \begin{bmatrix} V_{qs}^c \\ V_{ds}^c \\ V_{qr}^c \\ V_{dr}^c \end{bmatrix}$$

E. $[\lambda_{qs} \ \lambda_{ds} \ I_{qs} \ I_{ds}]$:

In this mathematical d-q model, stator flux linkage and stator current space vectors are selected as state-space variables $x = [\lambda_{qs} \ \lambda_{ds} \ I_{qs} \ I_{ds}]$. This mathematical model is selected when stator flux oriented strategy is implemented and derived by substituting equations (9) & (31) in the equations (6) & (7) and expressed in a matrix form as follows:

$$\begin{bmatrix} \lambda_{qs}^c \\ \lambda_{ds}^c \\ I_{qs}^c \\ I_{ds}^c \end{bmatrix} = \begin{bmatrix} 0 & -\omega_c & -R_s & 0 \\ \omega_c & 0 & 0 & -R_s \\ \frac{1}{L_s \tau_r} + \frac{1}{L_s \sigma} \cdot \mathfrak{p} & \frac{(\omega_c - \omega_r)}{L_s \sigma} & \frac{-1}{\tau_r} & -(\omega_c - \omega_r) \\ \frac{-(\omega_c - \omega_r)}{L_s \sigma} & \frac{1}{L_s \tau_r} + \frac{1}{L_s \sigma} \cdot \mathfrak{p} & (\omega_c - \omega_r) & \frac{-1}{\tau_r} \end{bmatrix} \begin{bmatrix} \lambda_{qs}^c \\ \lambda_{ds}^c \\ I_{qs}^c \\ I_{ds}^c \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{\sigma - 1}{L_m \sigma} & 0 \\ 0 & 0 & 0 & \frac{\sigma - 1}{L_m \sigma} \end{bmatrix} \begin{bmatrix} V_{qs}^c \\ V_{ds}^c \\ V_{qr}^c \\ V_{dr}^c \end{bmatrix}$$

F. $[\lambda_{qr} \ \lambda_{dr} \ I_{qr} \ I_{dr}]$:

In this mathematical d-q model, rotor flux linkage and rotor current space vectors are selected as state-space variables $x = [\lambda_{qr} \ \lambda_{dr} \ I_{qr} \ I_{dr}]$. This model is optimum solution for rotor flux orientation control strategy in a drive system. However, as it is impossible to measure the rotor currents if the machine is equipped with cage rotor, there are limitations in using this model for vector control strategies. This model is derived by

substituting equations (8) & (30) in the equations (6) & (7) and expressed in a matrix form as follows:

$$\begin{bmatrix} I_{qr}^c \\ I_{dr}^c \\ \lambda_{qr}^c \\ \lambda_{dr}^c \end{bmatrix} = \begin{bmatrix} \frac{-1}{\tau_s} & -\omega_c & \frac{1}{L_r \tau_s} + \frac{1}{L_r \sigma} \cdot p & \frac{\omega_c}{L_r \sigma} \\ \omega_c & \frac{-1}{\tau_s} & \frac{-\omega_c}{L_r \sigma} & \frac{1}{L_r \tau_s} + \frac{1}{L_r \sigma} \cdot p \\ -R_r & 0 & 0 & -(\omega_c - \omega_r) \\ 0 & -R_r & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} I_{qr}^c \\ I_{dr}^c \\ \lambda_{qr}^c \\ \lambda_{dr}^c \end{bmatrix} + \begin{bmatrix} \frac{\sigma - 1}{L_m \sigma} & 0 & 0 & 0 \\ 0 & \frac{\sigma - 1}{L_m \sigma} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} V_{qs}^c \\ V_{ds}^c \\ V_{qr}^c \\ V_{dr}^c \end{bmatrix}$$

G. $[\lambda_{qm} \ \lambda_{dm} \ I_{qs} \ I_{ds}]$:

In this mathematical d-q model, air-gap flux linkage and stator current space vectors are selected as state-space variables $x = [\lambda_{qm} \ \lambda_{dm} \ I_{qs} \ I_{ds}]$. It preserves information regarding both stator and rotor parameters. This choice also presents the advantage of an easier saturation effect modeling. This model is suitable choice for the air-gap flux orientation control strategy and derived by substituting equations (8), (9), (10) & (31) in the equations (6) & (7) and expressed in a matrix form as follows:

$$\begin{bmatrix} I_{qs}^c \\ I_{ds}^c \\ \lambda_{qm}^c \\ \lambda_{dm}^c \end{bmatrix} = \begin{bmatrix} \frac{-R_s}{L_{1s}} & & -\omega_c & \frac{-1}{L_{1s}} \cdot p & \frac{-\omega_c}{L_{1s}} \\ \omega_c & & \frac{-R_s}{L_{1s}} & \frac{\omega_c}{L_{1s}} & \frac{-1}{L_{1s}} \cdot p \\ \frac{L_m}{\tau_r} + L_m \cdot (1 - K_r) \cdot p & (\omega_c - \omega_r) \cdot L_m \cdot (1 - K_r) & \frac{-1}{\tau_r} & & -(\omega_c - \omega_r) \\ -(\omega_c - \omega_r) \cdot L_m \cdot (1 - K_r) & \frac{L_m}{\tau_r} + L_m \cdot (1 - K_r) \cdot p & (\omega_c - \omega_r) & \frac{-1}{\tau_r} & \end{bmatrix} \cdot \begin{bmatrix} I_{qs}^c \\ I_{ds}^c \\ \lambda_{qm}^c \\ \lambda_{dm}^c \end{bmatrix} + \begin{bmatrix} \frac{1}{L_{1s}} & 0 & 0 & 0 \\ 0 & \frac{1}{L_{1s}} & 0 & 0 \\ 0 & 0 & K_r & 0 \\ 0 & 0 & 0 & K_r \end{bmatrix} \cdot \begin{bmatrix} V_{qs}^c \\ V_{ds}^c \\ V_{qr}^c \\ V_{dr}^c \end{bmatrix}$$

IV. SIMULATION RESULTS

The simulation scheme for the modeling of squirrel cage induction motor in different frames is shown in Fig (5) and flowchart for the dynamic simulation of the squirrel cage induction motor in d-q model is shown in Fig (4). For squirrel cage induction motor, rotor voltage is equal to zero i.e. $V_{qr} = V_{dr} = 0$ and the following induction motor parameters are chosen for the simulation studies:

$R_s = 0.78\Omega, R_r = 0.15\Omega, L_s = 0.0434H, L_r = 0.0407H$
 $L_m = 0.041 H, f = 50 Hz, J = 0.095 kg - m^2, P = 4$.
 At $t=0$, the motor is connected to a 400 V, 50 Hz three-phase supply. The load torque T_L is applied at $t=1$ sec. Figures (6) to (26) shows the results of computer simulation using the MATLAB/SIMULINK models with $T_L = 40$ N-m.

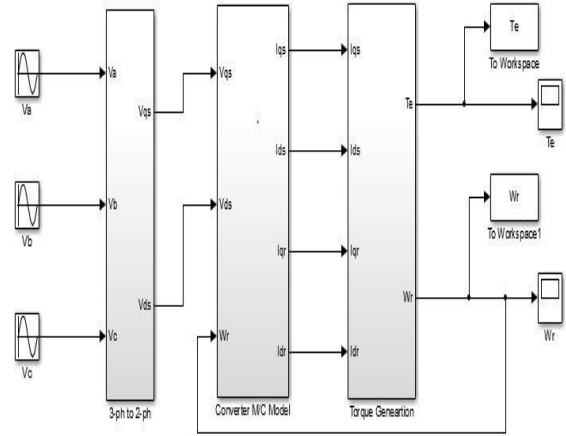


Fig (5) Simulation scheme of squirrel cage induction motor model

$[I_{qs} \ I_{ds} \ I_{qr} \ I_{dr}]$:

Stationary Reference Frame:

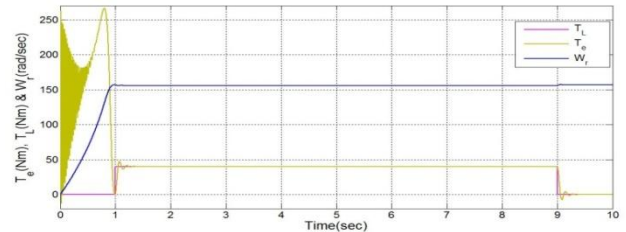


Fig (6) Electromagnetic Torque, Load Torque, Rotor Angular speed

Rotor Reference Frame:

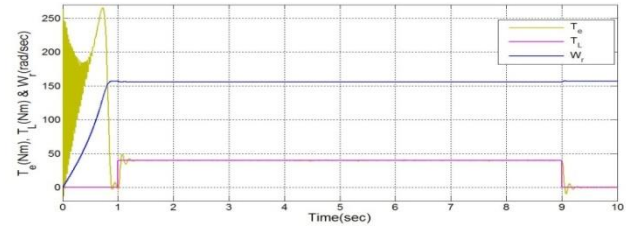


Fig (7) Electromagnetic Torque, Load Torque, Rotor Angular speed

Synchronous Reference Frame:

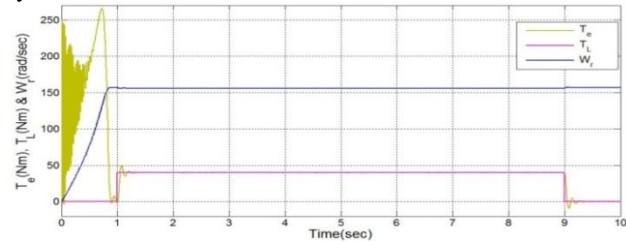


Fig (8) Electromagnetic Torque, Load Torque, Rotor Angular speed

$[I_{qs} \ I_{ds} \ I_{qm} \ I_{dm}]$:

Stationary Reference Frame:

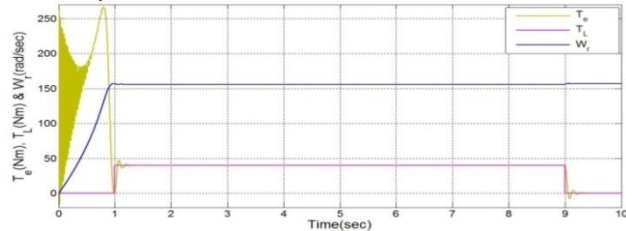


Fig (9) Electromagnetic Torque, Load Torque, Rotor Angular speed

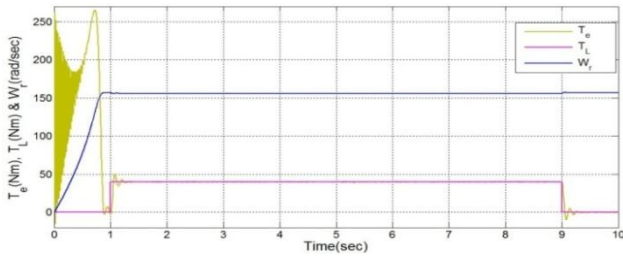


Fig (10) Electromagnetic Torque, Load Torque, Rotor Angular speed Synchronous Reference Frame:

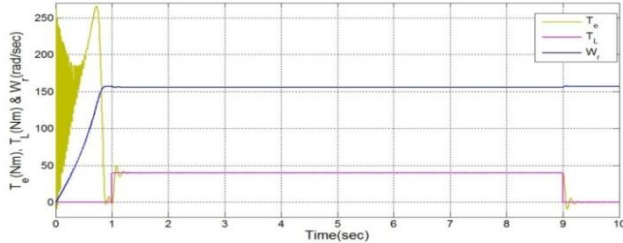


Fig (11) Electromagnetic Torque, Load Torque, Rotor Angular speed

$[\lambda_{qs} \ \lambda_{ds} \ \lambda_{qr} \ \lambda_{dr}]$:

Stationary Reference Frame:

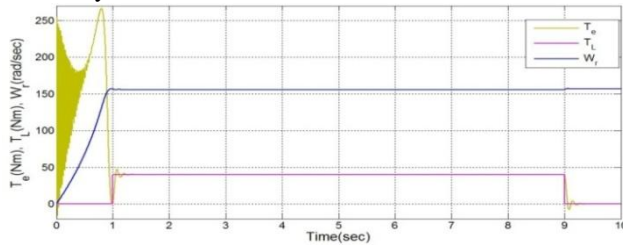


Fig (12) Electromagnetic Torque, Load Torque, Rotor Angular speed Rotor Reference Frame:

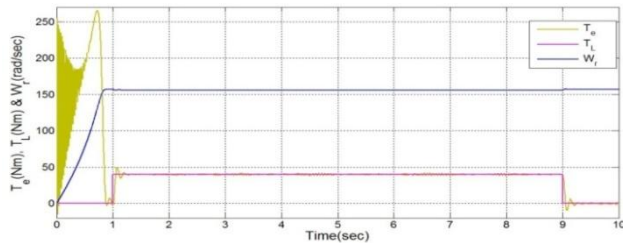


Fig (13) Electromagnetic Torque, Load Torque, Rotor Angular speed Synchronous Reference Frame:

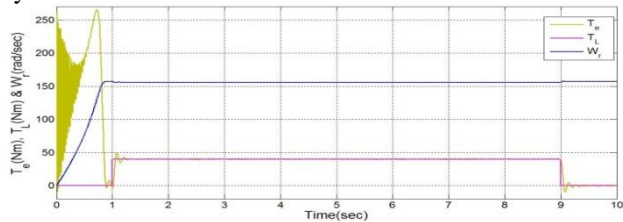


Fig (14) Electromagnetic Torque, Load Torque, Rotor Angular speed

$[\lambda_{qs} \ \lambda_{ds} \ \lambda_{qm} \ \lambda_{dm}]$:

Stationary Reference Frame:

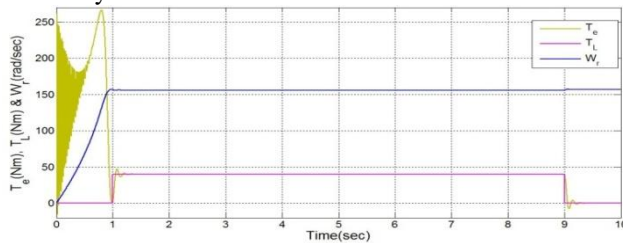


Fig (15) Electromagnetic Torque, Load Torque, Rotor Angular speed

Rotor Reference Frame:

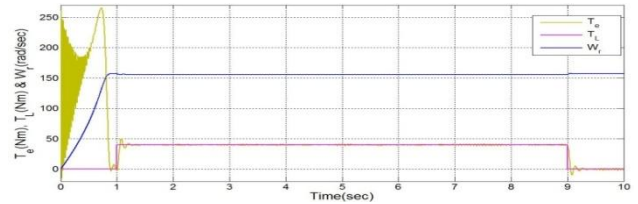


Fig (16) Electromagnetic Torque, Load Torque, Rotor Angular speed Synchronous Reference Frame:

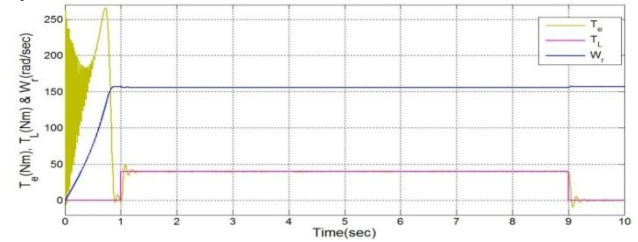


Fig (17) Electromagnetic Torque, Load Torque, Rotor Angular speed

$[\lambda_{qs} \ \lambda_{ds} \ I_{qs} \ I_{ds}]$:

Stationary Reference Frame:

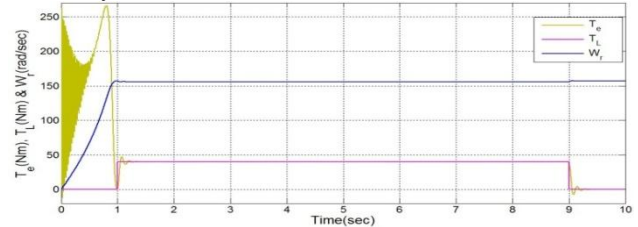


Fig (18) Electromagnetic Torque, Load Torque, Rotor Angular speed Rotor Reference Frame:

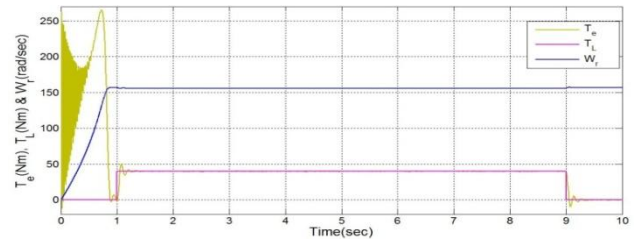


Fig (19) Electromagnetic Torque, Load Torque, Rotor Angular speed Synchronous Reference Frame:

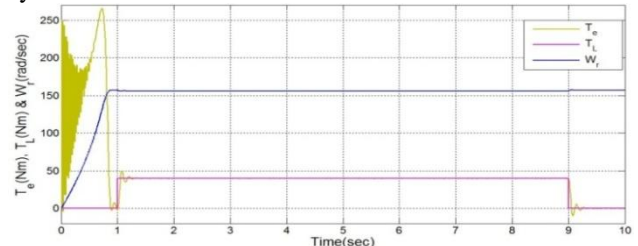


Fig (20) Electromagnetic Torque, Load Torque, Rotor Angular speed

$[\lambda_{qr} \ \lambda_{dr} \ I_{qr} \ I_{dr}]$:

Stationary Reference Frame:

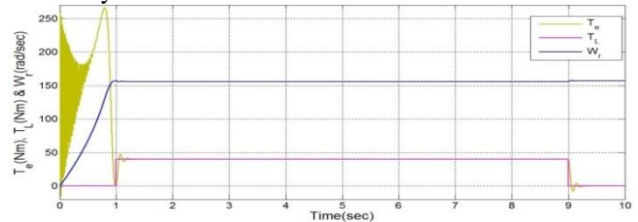


Fig (21) Electromagnetic Torque, Load Torque, Rotor Angular speed Rotor Reference Frame:

REFERENCES

- [1] R. Saidur, S. Mekhilef, M. B. Ali, A. Safari, H. A. Mohammed, "Applications of variable speed drive (VSD) in electrical motors energy savings," *Renewable and Sustainable Energy Reviews*, vol. 16, no. 1, pp. 543-550, January 2012.
- [2] M. Ibrahim, Alsofyani, N. R. N. Idris, "A review on sensorless techniques for sustainable reliability and efficient variable frequency drives of induction motors," *Renewable and Sustainable Energy Reviews*, vol. 24, pp. 111-121, August 2013.
- [3] A. Nabaie, O. Kenichi, U. Hiroshi, R. Kurosawa, "An approach to flux control of induction motors operated with variable-frequency power supply," *IEEE Trans. Ind. Appl.*, vol. IA-16, no. 3, pp. 342-350, 1980.
- [4] T. Murata, T. Tsuchiya, I. Takeda, "Vector control for induction machine on the application of optimal control theory," *IEEE Trans. Ind. Appl.*, vol. 37, no. 4, pp. 282-290, 1990.
- [5] D. W. Novotny and T. A. Lipo, "Vector control and dynamics of AC drives," Oxford University Press, New York.
- [6] R. D. Lorenz, T. A. Lipo, D. W. Novotny, "Motion control with induction motors," *Proceedings of the IEEE*, vol. 82, no. 8, 1215-1240, 1994.
- [7] R. Krishnan, "Electric motor drives modeling, analysis and control," 1st ed., 2001 Prentice-Hall International, New Jersey.
- [8] R. H. Park, "Two-reaction theory of synchronous machines generalised method of analysis-part 1," *AIEE Trans.*, vol. 48, pp. 716-727, 1929.
- [9] P. K. Kovacs, "Transient phenomena in electrical machines," Elsevier Science Publishers, Amsterdam.
- [10] P. C. Krause, O. Wasynczyk, "Analysis Of Electrical Machinery," IEEE Press, New York.
- [11] H. C. Stanley, "An analysis of the induction motors," *AIEE Trans.*, vol. 57, pp. 751-755, 1938.
- [12] B. K. Bose, "Power electronics and drives," Prentice-Hall, Englewood Cliffs, New Jersey.
- [13] W. Leonhard, "Control of electric drives," Springer Verlag, New York.

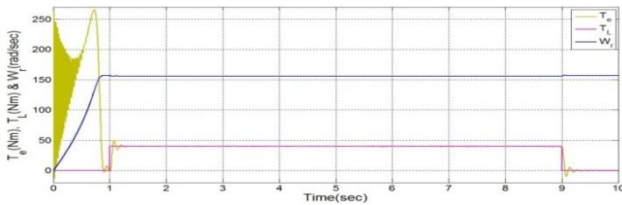


Fig (22) Electromagnetic Torque, Load Torque, Rotor Angular speed Synchronous Reference Frame:

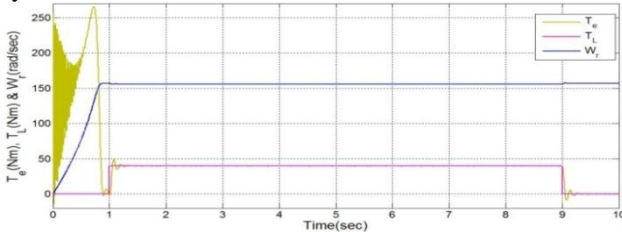


Fig (23) Electromagnetic Torque, Load Torque, Rotor Angular speed

$[\lambda_{qm} \ \lambda_{dm} \ I_{qs} \ I_{ds}]$:

Stationary Reference Frame:

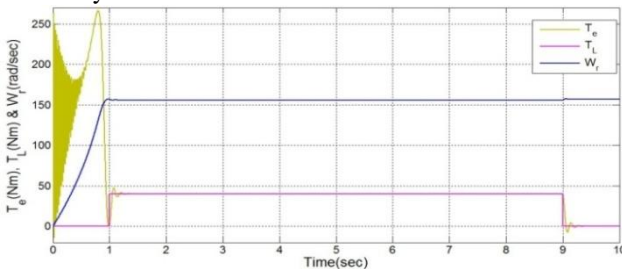


Fig (24) Electromagnetic Torque, Load Torque, Rotor Angular speed Rotor Reference Frame:

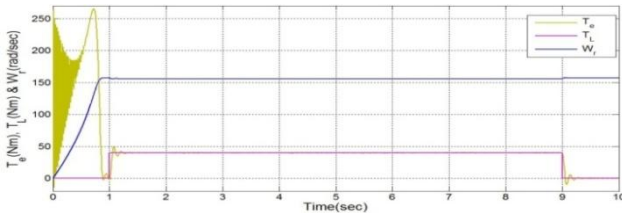


Fig (25) Electromagnetic Torque, Load Torque, Rotor Angular speed Synchronous Reference Frame:

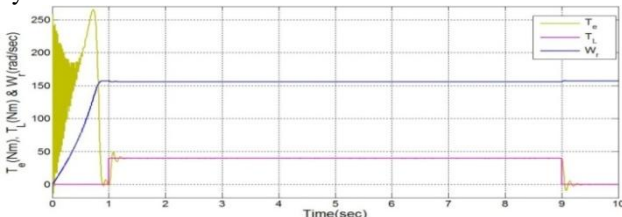


Fig (26) Electromagnetic Torque, Load Torque, Rotor Angular speed

V. CONCLUSIONS

In this paper, various mathematical d-q models of both squirrel cage and wound rotor induction motors in different reference frames are presented. This paper presents the d-q axes unified approach for both types of induction motors. The applications of each model are also discussed. Models either only with magnetisation current space vector or air-gap flux space vector, or both includes saturation effect in modeling of induction motor. MATLAB/SIMULINK software was used to implement the dynamic response of squirrel cage induction motor d-q models in different reference frames and these models are analyzed in terms of torque and rotor angular velocity.