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More on Generalizations of Weakly Continuity

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Abstract: This paper presents an operator $\tilde{\tilde{g}}s$ -closure and some of it's properties are studied.Also two new classes of mappings via $\tilde{\tilde{g}}s$ -closed sets are introduced and their characterizations are investigated.

Keywords: $\tilde{\tilde{g}}s$ -closed sets, $\tilde{\tilde{g}}s$ -closure, $\tilde{\tilde{g}}s$ -interior, $\tilde{\tilde{g}}s$ -Frontier.
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1 INTRODUCTION

Many continuous mappings by utilizing different kinds of generalized closed sets were introduced and investigated. Recently, R.Balachandran,P.Sundaram and H.Maki in 1991 [1],defined a new class of generalized continuous mappings. IN 1972,Crosseley and Hildebrand [2] introduced the class of irresolute mappings.In this paper, our objective is to introduce an operator $\tilde{\tilde{g}}s$ -closure and investigate its properties. Also two new notions such as the classes of mappings $\tilde{\tilde{g}}s$ -continuity and $\tilde{\tilde{g}}s$ -irresoluteness are introduced and investigated. Some of their characterizations are studied in terms of $\tilde{\tilde{g}}s$ -closure.

2 PRELIMINARIES

Throughout this paper, (X,τ) (or simply X) represents topological spaces on which no separation axioms are assumed unless otherwise mentioned.For a subset A of X,cl(A),int(A)and A^c denote the closure of A,the interior of A and the complement of A respectively. Let us recall the following definitions, which are useful in the sequel.

Definition 2.1 *A subset of* (X,τ) *is called* [(i)]

1. a $\tilde{g}$ -semi closed (briefly $\tilde{g}$ s-closed) set [?] if $sClA \subseteq U$ whenever $A \subseteq U$ and U is #gs-open in (X,τ).
2. $\tilde{\tilde{g}}$ − semi closed set (briefly $\tilde{\tilde{g}}s-closed$)[4] if $scl(A) \subseteq u$ whenever $A \subseteq u$ and u is $\tilde{g}s-$ open set in (X,τ). Complement of $\tilde{\tilde{g}}s-$ closed (resp. $\tilde{g}$ s-closed) set is known as $\tilde{\tilde{g}}s-$ open (resp. $\tilde{g}$ s-open)set in X.

Notations 2.2 *The collection of all* $\tilde{\tilde{g}}s-closed$ *(resp.* $\tilde{\tilde{g}}s-op$*) sets in X is denoted by* $\tilde{\tilde{G}}sC(X)$ *(resp.*

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$\tilde{\tilde{G}}sO(X)$. *For* $x \in X$, $\tilde{\tilde{G}}sO(X,x) = \{u \in \tilde{\tilde{G}}O(X) \mid x \in u\}$.

Definition 2.3 *A function* $f: X \to Y$ *is called a* [(i)]

1. contra-continuous [3] if $f^{-1}(V)$ is closed subset of (X,τ) for every open subset V of (Y,σ).
2. semi continuous function [5] if $f^{-1}(V)$ is semi open set of (X,τ) for every open sub set V of (Y,σ).
3. α -continuous function [6] if $f^{-1}(V)$ is α -open sub set of (X,τ) for every open sub set V of (Y,σ).
4. $\tilde{g}$ -continuous functions [?] if $f^{-1}(V)$ is $\tilde{g}$ -open sub set of (X,τ) for every open sub set V of (Y,σ).
5. sg -continuous functions [7] if $f^{-1}(V)$ is sg -open sub set of (X,τ) for every open sub set V of (Y,σ).

3 $\tilde{\tilde{g}}s$ -CLOSURES

Definition 3.1 For any subset A of a space X, $\tilde{\tilde{g}}s-$closure of A is denoted by $\tilde{\tilde{g}}scl(A)$ and defined by intersection of all $\tilde{\tilde{g}}s-$closed sets containing A. That is, $\tilde{\tilde{g}}scl(A) = \cap\{F : A \subseteq F, F \in \tilde{\tilde{G}}sC(X)\}$.

Definition 3.2 Let x be a point of X and N be a subset of X N is called a $\tilde{\tilde{g}}s-$neighbourhood of x in X if there exists a $\tilde{\tilde{g}}s-$open set U in X such that $x \in U \subseteq N$.

Definition 3.3 A point $x \in X$ is said to be a $\tilde{\tilde{g}}s-$interior point of A if there exists a $\tilde{\tilde{g}}s-$open set U containing x such that $U \subseteq A$. The set of all $\tilde{\tilde{g}}s-$interior points of A is known as the $\tilde{\tilde{g}}s-$interior of A and denoted by $\tilde{\tilde{g}}sint(A)$.

Definition 3.4 For a subset A of a space X, $\tilde{\tilde{g}}sFr(A) = \tilde{\tilde{g}}scl(A) - \tilde{\tilde{g}}sint(A)$ is known as $\tilde{\tilde{g}}s-$frontier of A.

Theorem 3.5 For any subsets A and B of a space X, the following statements hold.

1. $\tilde{\tilde{g}}scl(\phi) = \phi$
2. $A \subseteq \tilde{\tilde{g}}scl(A) \subseteq scl(A)$
3. If $A \subseteq B$, then $\tilde{\tilde{g}}scl(A) \subseteq scl(B)$
4. $x \in \tilde{\tilde{g}}scl(A)$ if and only if for any $\tilde{\tilde{g}}s-$open set u containing x such that $A \cap u \neq \phi$.

Proof.

2. From the definition of $\tilde{\tilde{g}}s-$closure, $A \subseteq \tilde{\tilde{g}}scl(A)$. Suppose that $x \notin scl(A)$. Then, there exist a semi closed set F such that $A \subseteq F$ and $x \notin F$. Since every semi-closed set is $\tilde{\tilde{g}}s-$closed, $x \notin \tilde{\tilde{g}}scl(A)$. Thus, $\tilde{\tilde{g}}scl(A) \subseteq scl(A)$.

3. Suppose that $A \subseteq B$ and $x \notin \tilde{\tilde{g}}scl(B)$. Then, there exists a $\tilde{\tilde{g}}s-$closed set F such that $B \subseteq F$ and $x \notin F$. That is, there exists a $\tilde{\tilde{g}}s-$closed set F such that $A \subseteq F$ and $x \notin F$. Again by the def. of $\tilde{g}s-$closure, $x \notin \tilde{\tilde{g}}scl(A)$. Therefore, $\tilde{\tilde{g}}scl(A) \subseteq \tilde{\tilde{g}}scl(B)$.

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4. Necessity Suppose that $x \in \tilde{\tilde{g}}scl(A)$ and U is any $\tilde{\tilde{g}}s-$open set containing x such that $A \cap U = \phi$. Then, $X - U$ is a $\tilde{\tilde{g}}s-$closed set in X containing A. Therefore, $\tilde{\tilde{g}}scl(A) \subseteq X - U$. Since $x \notin X - U$, $x \notin \tilde{\tilde{g}}scl(A)$, a contradiction.

Sufficiency Suppose that every $\tilde{\tilde{g}}s-$closed set is containing x intersects A. If $x \notin \tilde{\tilde{g}}scl(A)$, then there exists a $\tilde{\tilde{g}}s-$closed set F such that $A \subseteq F$, and $x \notin F$. Then, $X - F$ is a $\tilde{\tilde{g}}s-$open set in X containing x such that $A \cap (X - F) = \phi$, a contradiction. Therefore, $x \in \tilde{\tilde{g}}scl(A)$.

Remark 3.6 In (2), converse is not true in general from the following example.

Example 3.7 Let $X = \{a, b, c\}$ and $\tau = \{\phi, \{a, b\}, X\}$ $\tilde{\tilde{g}}scl(\{a\}) = \{a, c\}$ and $scl(\{a\}) = X$. Therefore, $\tilde{\tilde{g}}scl(\{a\}) \neq scl(\{a\})$.

Lemma 3.8 If A is $\tilde{\tilde{g}}s-$closed set in X, then $A = \tilde{\tilde{g}}scl(A)$.

Proof. Suppose that A is $\tilde{\tilde{g}}s-$closed set in X. By def. of $\tilde{\tilde{g}}s-$closure, $\tilde{\tilde{g}}scl(A) = \cap\{F : A \subseteq F, F \in \tilde{\tilde{G}}s \subset (X)\}$. Then, $A \subseteq \tilde{\tilde{g}}scl(A)$ and $\tilde{\tilde{g}}scl(A) \subseteq F$ for every $\tilde{\tilde{g}}s-$closed set containing A. Since A is $\tilde{\tilde{g}}s-$closed containing A, $\tilde{\tilde{g}}scl(A) \subseteq A$. Thus, $A = \tilde{\tilde{g}}scl(A)$.

Remark 3.9 Converse of the above lemma is not always possible from the following example.

Example 3.10 Let $X = \{a, b, c, d\}$ and $\tau = \{\phi, c, \{a, d\}, \{a, c, d\}, X\}$. Here $\tilde{\tilde{g}}scl(\{a\}) = \{a\}$, but $\{a\}$ is not a $\tilde{\tilde{g}}s-$closed set in X.

Lemma 3.11 If A is $\tilde{\tilde{g}}s-$open, then $A = \tilde{\tilde{g}}sint(A)$.

Proof. By the def. always $\tilde{\tilde{g}}sint(A) \subseteq (A)$. Sup. that there exists $x \in X$ such that $x \notin \tilde{\tilde{g}}sint(A)$ and $x \in A$. By the def of $\tilde{\tilde{g}}s-$interior of A, every $\tilde{\tilde{g}}s-$open set containing x intersects (X - A). Since A $\tilde{\tilde{g}}s-$open set containing x, $A \cap (X - A) \neq \phi$, a contradiction our assumption is wrong and hence $x \notin A$. Therefore, $A = \tilde{\tilde{g}}sint(A)$.

Remark 3.12 The following example illustrates that the converse of the above theorem is not true in general.

Example 3.13 Let $X = \{a, b, c, d\}$ and $\tau = \{\phi, \{c\}, \{a, d\}, \{a, c, d\}, X\}$. Then $\tilde{\tilde{g}}sint(\{b, c, d\}) = \{b, c, d\}$, but $\{b, c, d\}$ is not a $\tilde{\tilde{g}}s-$open set in x.

Theorem 3.14 For any subset A of a space X,

i. $X - \tilde{\tilde{g}}sint(A) = \tilde{\tilde{g}}scl(X - A)$

ii. $X - \tilde{\tilde{g}}scl(A) = \tilde{\tilde{g}}sint(X - A)$

Proof. Suppose that $x \in X - \tilde{\tilde{g}}sint(A)$. Then $x \notin \tilde{\tilde{g}}sint(A)$. By the def., of $\tilde{\tilde{g}}s-$interior, every $\tilde{\tilde{g}}s-$open set containing x meets $(X - A)$. By theorem (1)-(4), $x \in \tilde{\tilde{g}}scl(X - A)$. On the other hand if $x \in \tilde{\tilde{g}}scl(X - A)$, then every $\tilde{\tilde{g}}s-$open set containing x intersects (X - A). Therefore, there is no $\tilde{\tilde{g}}s-$open set containing x and

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contained in A. By the def., of $\overset{\approx}{g}s-$interior, $x \notin \overset{\approx}{g}sint(A)$. Then $x \in X-\overset{\approx}{g}sint(A)$. Therefore, $X-\overset{\approx}{g}sint(A)=\overset{\approx}{g}scl(X-A)$.

2. Replace A by $(X-A)$ in (1) we get $X-\overset{\approx}{g}sint(X-A)=\overset{\approx}{g}scl(A)$. That is, $\overset{\approx}{g}sint(X-A)=X-\overset{\approx}{g}scl(A)$.

Theorem 3.15 For a subset A of a space X, the following are equivalent.

i. $\overset{\approx}{G}sO(X)$ is closed under union.

ii. A is $\overset{\approx}{g}s-$closed if and only if $A=\overset{\approx}{g}cl(A)$

iii. A is $\overset{\approx}{g}s-$open if and only if $A=\overset{\approx}{g}sint(A)$

Proof. (1) $\Longrightarrow$ (2).

Necessity: Suppose that A is $\overset{\approx}{g}s-$closed in X. By hypothesis and by the def. of $\overset{\approx}{g}s-$closure, it is known that $\overset{\approx}{g}scl(A)$ is $\overset{\approx}{g}s-$closed in X and $\overset{\approx}{g}scl(A)=A$.

Sufficiency: Suppose that $\overset{\approx}{g}scl(A)=A$. For each $x \in X-A$, $x \in X \setminus A$, $x \notin \overset{\approx}{g}scl(A)$. By theorem (1)-(4), there exists a set $U_x \in \overset{\approx}{G}sO(X,x)$ such that $U_x \cap A=\phi$. Then, $x \in U_x \subseteq X-A$ and hence $X-A=\bigcup_{x\in(X-A)} U_x$. By hypothesis, $X-A$ is $\overset{\approx}{g}s-$open in X. Therefore, $X-A$ is $\overset{\approx}{g}s-$open. Thus, A is $\overset{\approx}{g}s-$closed.

(2) $\Longrightarrow$ (3).

Necessity: Suppose that A is $\overset{\approx}{g}s-$open in X. Then $(X-A)$ is $\overset{\approx}{g}s-$open in X. By hypothesis $\overset{\approx}{g}s-cl(X-A)=X-A$. By theorem (2), $X-\overset{\approx}{g}sint(A)=X-A$. Therefore, $A=\overset{\approx}{g}sint(A)$.

Sufficiency: Suppose that $A=\overset{\approx}{g}sint(A)$. By theorem (2), $X-A=X-\overset{\approx}{g}sint(A)=\overset{\approx}{g}scl(X-A)$. By hypothesis, $(X-A)$ is $\overset{\approx}{g}s-$closed and hence A is $\overset{\approx}{g}s-$open in X.

(3) $\Longrightarrow$ (1).

Sup, that $\{G_\alpha : \alpha \in J\}$ be any family of $\overset{\approx}{g}s-$open sets in X. Let $G=\bigcup_{\alpha\in J} G_\alpha$. If $x \in G$, then $x \in G_{\alpha_x}$ for some $\alpha_x \in J$. Since G_α is open in X. there exist a $\overset{\approx}{g}s-$open set U_x such that $x \in U_x \subseteq G_{\alpha_x} \subseteq G$ for every $x \in G$. Therefore $x \in \overset{\approx}{g}sint(G)$ and hence $G=\overset{\approx}{g}sint(G)$. By hypothesis, G is $\overset{\approx}{g}s-$open in X.

Definition 3.16 A mapping $f: X \to Y$ is said to be $\overset{\approx}{g}s-$continuous if the inverse image of every closed set in Y, is $\overset{\approx}{g}s-$closed in X.

Example 3.17 $X=Y=\{a,b,c\}$, $\tau=\{\phi,\{a,b\},X\}$, $\sigma=\{\phi,\{a\},Y\}$. Define $f:(X,\tau)\to(y,\sigma)$ by $f(a)=b$, $f(b)=c$ and $f(c)=c$. Then f is $\overset{\approx}{g}s-$continuous from X into Y.

Theorem 3.18 In a topological space (X,τ), the following statements hold:

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i. Every continuous function is $\tilde{\tilde{g}}s-$continuous.

ii. Every semi continuous function is $\tilde{\tilde{g}}s-$continuous.

iii. Every α -continuous function is $\tilde{\tilde{g}}s-$continuous.

iv. Every $\tilde{g}-$continuous function is $\tilde{\tilde{g}}s-$continuous.

Proof. By [4] proposition 3.2 (1) holds, proposition 3.4, (2) holds, proposition 3.6, (3) holds, proposition 3.8, (4) holds.

Remark 3.19 Converse of the above theorem not holds always from the following examples.

Example 3.20 $X = Y = \{a,b,c\}$, $\tau = \{\phi,\{a,b\},X\}$, $\sigma = \{\phi,\{a\},Y\}$. Define $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=b$, $f(b)=a$ and $f(c)=c$. Then f is $\tilde{\tilde{g}}s-$continuous but not continuous, semi-continuous, or $\alpha-$continuous.

Example 3.21 $X = Y = \{a,b,c\}$, $\tau = \{\phi,\{a\},\{a,b\},X\}$ and $\sigma = \{\phi,\{a,b\},Y\}$. Define $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=b$, $f(b)=c$, and $f(c)=a$. Then f is $\tilde{\tilde{g}}s-$continuous but not $\tilde{g}s-$continuous.

We state the necessary conditions under which the converse of theorem 3.8 hold.

Theorem 3.22 [(i)]

i. In $T_{\tilde{\tilde{g}}}-$space every $\tilde{\tilde{g}}s-$continuous function is continuous.

ii. In $T^{\#}_{\tilde{\tilde{g}}}-$space, every $\tilde{\tilde{g}}s-$continuous function is $\alpha-$continuous.

iii. In ${}_{\tilde{\tilde{g}}}T_s$ space every $\tilde{\tilde{g}}s-$continuous function is semi-continuous.

iv. In ${}^{\#}T_{\tilde{\tilde{g}}}-$space every $\tilde{\tilde{g}}s-$continuous functions is $\tilde{g}-$continuous.

Proof. By [4] def. 6.1, 6.7, 6.16 and 6.22, it holds.

Theorem 3.23 In a topological space (X,τ), every $\tilde{\tilde{g}}s-$continuous functions is sg-continuous.

Proof. By [4] proposition 3.14, every $\tilde{\tilde{g}}s-$closed set is sg-closed set. Therefore, it holds.

Remark 3.24 Converse is not true in general from the following example.

Example 3.25 Let $X = Y = \{a,b,c,d\}$ and $\tau = \{\phi,\{c\},\{a,d\},\{a,c,d\},X\}$ and $\sigma = \{\phi,\{a,b,c\},Y\}$. Define $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=d$, $f(b)=a$, $f(c)=b$, $f(d)=c$. Then f is sg-continuous but not $\tilde{\tilde{g}}s-$continuous.

Theorem 3.26 Assume that $\tilde{\tilde{G}}sO(X)$ is closed under any union. Then the following are equivalent for a function $f:X\to Y$.

i. f is $\tilde{\tilde{g}}s-$continuous;

ii. For every open set V of Y, $f^{-1}(V)\in\tilde{\tilde{G}}sO(Y)$;

iii. For each $x\in X$ and each open set V of Y containing $f(x)$, there exits $u\in\tilde{\tilde{G}}sO(X,x)$ such that $f(u)\subseteq v$;

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iv. $f(\tilde{\tilde{g}}scl(A)\subseteq cl(f(A))$ for any subset A of X;

v. $\tilde{\tilde{g}}scl(f^{-1}(B))\subseteq f^{-1}(cl(B))$ for any subset B of y;

vi. $f^{-1}(int(B))\subseteq \tilde{\tilde{g}}sint(f^{-1}(B))$ for any subset B of Y.

Proof. (1) $\Rightarrow$ (2) It follows from $f^{-1}(Y-V) = X - f^{-1}(V)$ for any subset V of Y and def., of $\tilde{\tilde{g}}s-$open set.

(2) $\Rightarrow$ (3) Let $x\in X$ and V be any open set in Y containing $f(x)$. Since f is $\tilde{\tilde{g}}s-$continuous, $f^{-1}(V)\in \tilde{\tilde{G}}sO(Y)$ and $x\in f^{-1}(V)$. If we take $U = f^{-1}(V)$, then $U\in \tilde{\tilde{G}}sO(X,x)$ and $f(U)\subseteq V$.

(3) $\Rightarrow$ (4) Let A be any subset of X and $x\in \tilde{\tilde{g}}scl(A)$ and V be any open set in Y containing f(x). By hypothesis, there exists $U\in \tilde{\tilde{G}}sO(X,x)$ such that $f(U)\subseteq V$. Since $x\in \tilde{\tilde{g}}scl(A)$, by theorem 3.5 (4), $U\cap A\neq\phi$ and hence $\phi\neq f(U\cap A)\subseteq f(U)\cap f(A)\subseteq V\cap f(A)$. Therefore, $f(x)\in cl(f(A))$. Thus, $f(\tilde{\tilde{g}}scl(A)\subseteq cl(f(A))$.

(4) $\Rightarrow$ (5) Let B be any subset of Y. By hypothesis, $f(\tilde{\tilde{g}}scl(f^{-1}(B)))\subseteq cl(f(f^{-1}(B)))\subseteq cl(B)$ and hence $\tilde{\tilde{g}}scl(f^{-1}(B))\subseteq f^{-1}(cl(B))$.

(5) $\Rightarrow$ (6) Let B be any subset in Y. By hypothesis, $\tilde{\tilde{g}}scl(f^{-1}(Y\setminus B))\subseteq f^{-1}(cl(Y-B))$. Then, $X-\tilde{\tilde{g}}sint(f^{-1}(B))\subseteq X - f^{-1}(int(B))$. Therefore, $f^{-1}(int(B))\subseteq \tilde{\tilde{g}}sint(f^{-1}(B))$.

(6) $\Rightarrow$ (1) Let F be any closed set in Y. By hypothesis, $f^{-1}(Y-F) = f^{-1}(int(Y-F))\subseteq \tilde{\tilde{g}}sint(f^{-1}(Y-F)) = X-\tilde{\tilde{g}}scl(f^{-1}(F))$.

Then, $\tilde{\tilde{g}}scl(f^{-1}(F))\subseteq f^{-1}(F)$. Therefore, $\tilde{\tilde{g}}scl(f^{-1}(F) = F^{-1}(F)$. By theorem (3), $f^{-1}(F)$ is $\tilde{\tilde{g}}s-$closed in X and hence f is $\tilde{\tilde{g}}s-$continuous.

Notations 3.27 Let $f:(X,\tau)\to(Y,\sigma)$ be any function. $NGC(f)=\{x\in X : f$ is not $\tilde{\tilde{g}}s-$continuous$\}$ at $x\in X$.

Theorem 3.28 Assume that $\tilde{\tilde{G}}sO(X)$ is closed under any union. Then $NGC(f)$ is identical with the union of the $\tilde{\tilde{g}}s-$frontiers of the inverse images of $\tilde{\tilde{g}}s-$open sets containing $f(x)$.

Proof. Suppose that f is not $\tilde{\tilde{g}}s-$continuous at $x\in X$. Then, there exists an open set V in Y containing $f(x)$ such that $f(U)$ is not a subset of V for every $U\in \tilde{\tilde{G}}sO(X,x)$. Hence $U\cap(X-f^{-1}(V))\neq\phi$ for every $U\in GO(X,x)$. It follows that $x\in \tilde{\tilde{g}}scl(X-f^{-1}(V))$ and hence $x\in X-\tilde{\tilde{g}}sint(f^{-1}(V))$. Then $x\notin \tilde{\tilde{g}}sint(f^{-1}(V))$. Since $x\in f^{-1}(V)\subseteq \tilde{\tilde{g}}scl(f^{-1}(V))$, $x\in \tilde{\tilde{g}}sFr(f^{-1}(V))$. On the other hand, let f be $\tilde{\tilde{g}}s-$continuous at $x\in X$ and V be any open set in Y containing $f(x)$. Then $x\in f^{-1}(V)$ is a $\tilde{\tilde{g}}s-$ open set in X. Thus $x\in \tilde{\tilde{g}}sint(f^{-1}(V))$ and hence $x\notin \tilde{\tilde{g}}sFr(f^{-1}(V))$ for any open set V containing $f(x)$.

Definition 3.29 Let A be a subset of X and $f:X\to A$ be a $\tilde{\tilde{g}}s-$continuous retraction if f is $\tilde{\tilde{g}}s-$continuous

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and the $f|A$ is the identity mapping on A.

Theorem 3.30 Assume that $\tilde{\tilde{g}}sO(X)$ is closed under any union. Let A be a subset of X and $f: X \to A$ be a $\tilde{\tilde{g}}s-$continuous retraction. If X is Hausdorff, then A is a $\tilde{\tilde{g}}s-$closed in X.

Proof. Suppose that A is not $\tilde{\tilde{g}}s-$closed. $A \neq \tilde{\tilde{g}}scl(A)$. Therefore, there exists $x \in X$ such that $x \in \tilde{\tilde{g}}scl(A)$ and $x \notin A$. Then $f(x) \neq x$ because f is $\tilde{\tilde{g}}s-$continuous retraction. Since x is Hausdorff, there exists disjoint open set u and v in x such that $x \in u$ and $f(x) \in V$. Let W be any $\tilde{\tilde{g}}s-$neighbourhood of x. By ..., theorem 4.6, $W \cap U$ is a $\tilde{\tilde{g}}s-$neighbourhood of x. Since $x \in \tilde{\tilde{g}}scl(A)$, by theorem (1)-(4), $(W \cap U) \cap A \neq \phi$. Choose $y \in W \cap U \cap A$. Since $y \in A$, $f(y) = y \in U$ and hence $f(y) \notin V$. Therefore $f(W) \not\subseteq V$ because $y \in W$, a contradiction to $\tilde{\tilde{g}}s-$continuous retraction. Thus A is $\tilde{\tilde{g}}s-$closed in X.

Remark 3.31 Composition of two $\tilde{\tilde{g}}s-$continuous functions is not a $\tilde{\tilde{g}}s-$ continuous in general from the following example.

Example 3.32 $x = y = z = \{a,b,c\}$, $\tau = \{\phi, \{a,b\}, X\}$, $\sigma = \{\phi, \{a\}, y\}$ and $\eta = \{\phi, \{a\}, \{a,b\}, z\}$. Define $f:(X,\tau) \to (Y,\sigma)$ by $f(a) = c$, $f(b) = a$, $f(c) = b$ and define $g:(Y,\sigma) \to (Z,\eta)$ identity function. Then f and g are $\tilde{\tilde{g}}s-$continuous function but $gof:(X,\tau) \to (Y,\sigma)$ defined by $(gof)(x) = g(f(x))$ for all $x \in X$ is not a $\tilde{\tilde{g}}s-$continuous.

Theorems on Compositions:

Theorem 3.33 If $f:(X,\tau) \to (Y,\sigma)$ is $\tilde{\tilde{g}}s-$continuous and $g:(y,\sigma) \to (z,\eta)$ is continuous, then $(gof):(X,\tau) \to (z,\eta)$ is $\tilde{\tilde{g}}s-$continuous.

Proof. Suppose that V be any open set in Z. Since g is continuous, $g^{-1}(V)$ is open in Y. Since f is $\tilde{\tilde{g}}s-$continuous, $f^{-1}(g^{-1}(V)$ is $\tilde{\tilde{g}}s-$open in X. That $(gof)^{-1}(V)$ is $\tilde{\tilde{g}}s-$open in X. Therefore, gof is $\tilde{\tilde{g}}s-$continuous.

Theorem 3.34 Suppose that (y,σ) is $T_{\tilde{\tilde{g}}}$ space. If $f(X,\tau) \to (Y,\sigma)$ is $\tilde{\tilde{g}}s-$continuous and $g:(Y,\sigma) \to (Z,\eta)$ is $\tilde{\tilde{g}}s-$continuous then $gof:(X,\tau) \to (Z,\eta)$ is $\tilde{\tilde{g}}s-$continuous.

Proof. Suppose that V be any open set in Z. Since g is $\tilde{\tilde{g}}s-$continuous. $g^{-1}(V)$ is $\tilde{\tilde{g}}s-$open in y. Since Y is $T_{\tilde{\tilde{g}}}$-space, $g^{-1}(V)$ is open in y. Since f is $\tilde{\tilde{g}}s-$continuous, $f^{-1}(g^{-1}(V))$ is $\tilde{\tilde{g}}s-$open in X. $(gof)^{-1}(V)$ is $\tilde{\tilde{g}}s-$open in X. Therefore composition of two $\tilde{\tilde{g}}s-$continuous funiton is $\tilde{\tilde{g}}s-$continuous.

Theorem 3.35 If $f:(X,\tau) \to (Y,\sigma)$ and $g:(Y,\sigma) \to (Z,\eta)$ are continuous, then $gof:(X,\tau) \to (Z,\eta)$ is $\tilde{\tilde{g}}s-$continuous.

Proof. Since composition of two continuous functions is continuous, and since every continuous function is $\tilde{\tilde{g}}s-$continuous gof is $\tilde{\tilde{g}}s-$ continuous.

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Theorem 3.36 Composition of two contra continuous function is always $\tilde{\tilde{g}}s-$continuous function.

Proof. Since every open set is $\tilde{\tilde{g}}s-$open, it follows.

Definition 3.37 A mapping $f:X \to Y$ is said to be $\tilde{\tilde{g}}s-$irresolute if the inverse image of each $\tilde{\tilde{g}}s-$open set in Y is $\tilde{\tilde{g}}s-$open in X.

Example 3.38 Let $X = Y = \{a,b,c\}$, $\tau = \{\phi,\{a\},\{b\},\{a,b\},\{a,c\},X\}$ and $\sigma = \{\phi,\{a\},Y\}$. Define $f:(X,\tau)\to(Y,\sigma)$ as an identity function. Then f is $\tilde{\tilde{g}}s-$irresolute function.

Theorem 3.39 A function $f:X \to Y$ is $\tilde{\tilde{g}}s-$irresolute if and only if $f^{-1}(V)$ is $\tilde{\tilde{g}}s-$closed in X for every $\tilde{\tilde{g}}s-$closed set V in Y.

Proof. **Necessity** Suppose that V is any $\tilde{\tilde{g}}s-$closed set in Y. Then $(Y-V)$ is $\tilde{\tilde{g}}s-$open in X and hence $X - f^{-1}(V)$ is $\tilde{\tilde{g}}s-$open in X. That is $f^{-1}(V)$ is $\tilde{\tilde{g}}s-$closed in X.

Sufficiency: Let V be $\tilde{\tilde{g}}s-$open in Y. Then $Y-V$ is $\tilde{\tilde{g}}s-$closed in Y. By hypothesis, $f^{-1}(Y-V)$ is $\tilde{\tilde{g}}s-$closed in X. That is $X - f^{-1}(V)$ is $\tilde{\tilde{g}}s-$closed in X and hence $f^{-1}(V)$ is $\tilde{\tilde{g}}s-$open in X. Therefore, f is $\tilde{\tilde{g}}s-$irresolute.

Theorem 3.40 Every $\tilde{\tilde{g}}s-$irresolute function is $\tilde{\tilde{g}}s-$continuous.

Proof. If follows from the fact that every open set is $\tilde{\tilde{g}}s-$open.

Remark 3.41 Converse not holds in general from the following example.

Example 3.42 $X = Y = \{a,b,c\}$, $\tau = \{\phi,\{a\},X\} = \sigma$. Define $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=b$, $f(b)=c$ and $f(c)=c$. Then f is $\tilde{\tilde{g}}s-$continuous but not $\tilde{\tilde{g}}s-$irresolute because $f^{-1}(\{a,c\}) = \{b,c\}$ is not a $\tilde{\tilde{g}}s-$open in X whereas $\{a,c\}$ is $\tilde{\tilde{g}}s-$open in Y.

Theorem 3.43 Composition of two $\tilde{\tilde{g}}s-$irresolute function is $\tilde{\tilde{g}}s-$irresolute.

Proof. Let $f:(X,\tau)\to(Y,\sigma)$ and $g:(Y,\sigma)\to(Z,\eta)$ be any two $\tilde{\tilde{g}}s-$irresolute function. Define $gof:(X,\tau)\to(Z,\eta)$ by $(gof)(x) = g(f(x))$ for all $x \in X$. Suppose that G be any $\tilde{\tilde{g}}s-$open in Z. Since g is $\tilde{\tilde{g}}s-$irresolute, $g^{-1}(G)$ is $\tilde{\tilde{g}}s-$open in Y. Since f is $\tilde{\tilde{g}}s-$irresolute, $f^{-1}(g^{-1}(G))$ is $\tilde{\tilde{g}}s-$open in X. Therefore $(gof)^{-1}(G)$ is $\tilde{\tilde{g}}s-$open in X. Thus, gof is $\tilde{\tilde{g}}s-$irresolute.

Theorem 3.44 If $f:(X,\tau)\to(Y,\sigma)$ is $\tilde{\tilde{g}}s-$ irresolute and $g:(Y,\sigma)\to(Z,\eta)$ is $\tilde{\tilde{g}}s-$continuous, then $gof:(X,\tau)\to(Z,\eta)$ is $\tilde{\tilde{g}}s-$continuous function.

Proof. Suppose that G is any open set in Z. Since g is $\tilde{\tilde{g}}s-$continuous function, $g^{-1}(V)$ is $\tilde{\tilde{g}}s-$open in Y. Since f is $\tilde{\tilde{g}}s-$irresolute, $f^{-1}(g^{-1}(V))$ is $\tilde{\tilde{g}}s-$open in X and hence $(gof)^{-1}(V)$ is $\tilde{\tilde{g}}s-$open in X. Thus, gof is $\tilde{\tilde{g}}s-$continuous.

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Theorem 3.45 Assume that $\tilde{\tilde{G}}sO(X)$ is closed under any union. A function $f:(X,\tau)\to(Y,\sigma)$ is $\tilde{\tilde{g}}s-$irresolute if and only if for each point x of X and each $\tilde{\tilde{g}}s-$open set V of Y containing $f(x)$, there exists a $\tilde{\tilde{g}}s-$open set is in X containing x such that $f(u)\subseteq V$.

Proof. **Necessity** Suppose that f is $\tilde{\tilde{g}}s-$irresolute and $x\in X$, V is any $\tilde{\tilde{g}}s-$open set in Y containing $f(x)$. Then $f^{-1}(V)$ is $\tilde{\tilde{g}}s-$open set in X. If $U = f^{-1}(V)$, then u is $\tilde{\tilde{g}}s-$open set in X containing x such that $f(U) = f(f^{-1}(V))\subseteq V$.

Sufficiency Suppose that G is any $\tilde{\tilde{g}}s-$open set in Y and $x\in f^{-1}(G)$. Then $f(x)\in G$. By hypothesis, there exists a $\tilde{\tilde{g}}s-$open set U_x such that $x\in U_x$ and $f(U_x)\subseteq V$. Then $x\in U_x\subseteq f^{-1}(V)$ for every $x\in f^{-1}(V)$. Then, $f^{-1}(V)=\cup\{U_x : x\in f^{-1}(V)\}$. By our assumption, $f^{-1}(V)$ is $\tilde{\tilde{g}}s-$open in X and hence f is $\tilde{\tilde{g}}s-$irresolute.

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