Nonlinear Dynamics of a Current Controlled D.C. Drive with PID Controller

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ABSTRACT: This paper describes a closed loop model of a current controlled PMDC motor drive with PID Controller. The output speed of the PMDC motor is compared with a preset reference speed. The differences between these two signals are fed as an error signal to the PID controller of the system. The output of the speed controller is the actuating signal that controls the duty cycle of converter and hence controls the converter output. Through this controlled converter output, required voltage gets injected into the motor to bring it back to its desired speed. As a small change in the input voltage can cause a large change in the motor current and lead to a particular drive control feature.

 Keywords: Permanent Magnet DC motor, DC-DC Step-down Chopper, Closed Loop Control, Proportional-Integral-Derivation controller.

I. RELATED WORK

PM motor drives have been a topic of interest for the last twenty years. Different authors have carried out modeling and simulation of such drives. The three most common speed control methods of a dc motor are field resistance control, armature voltage control, and armature resistance control [6].

Depending on the armature voltage control method simulink model of a current mode buck-type dc chopper-fed permanent-magnet (PM) dc motor drive with proportional controller in its feedback loop is realized & nonlinear dynamics and chaotic behaviour of that system is also analysed [15]. But the importance of PID controllers in process industry cannot be overemphasized because more than half of the industrial controllers in use today utilize PID or modified PID control schemes [8].

The purpose of this paper is to investigate both numerically and through simulink model the characteristics of different electrical parameters [current, speed, different torques working etc] of current mode buck-type dc chopper-fed permanent-magnet (PM) dc motor with PID controller in its feedback loop.

II. INTRODUCTION

Developments of high performance motor drives are very essential for industrial applications. A high performance motor drive system must have good dynamic speed command tracking and load regulating response. DC motors provide excellent control of speed for acceleration and deceleration and chopper fed permanent magnet PMDC motor allows precise voltage control, which is necessary for speed and torque control applications.

DC drives, because of their simplicity, ease of application, reliability and favourable cost have long been a backbone of industrial applications. DC drives are less complex as compared to AC drives system. DC drives are normally less expensive for low horsepower ratings. DC motors have a long tradition of being used as adjustable speed machines and a wide range of options have evolved for this purpose. Cooling blowers and inlet air flanges provide cooling air for a wide speed range at constant torque.

PMDC motors are conveniently portable and well fit to special applications, like industrial equipments and machineries that are not easily run from remote power sources. PMDC motor is considered a SISO (Single Input and Single Output) system having torque/speed characteristics compatible with most mechanical loads. This makes a PMDC motor controllable over a wide range of speeds by proper adjustment of the terminal voltage using various innovative design and control techniques [6].

In case of this project a DC-DC Step-down Chopper is used to adjust the terminal voltage as per requirement [12]. This is done by controlling switch of the chopper through a closed loop control [13]. As in process industry importance of PID (proportional-integral-derivative) controllers cannot be overemphasized because more than half of the industrial controllers in use today utilize PID or modified PID control schemes [9], for this reason we have also used a PID controller in our closed control loop to generate the actuating signal which controls the chopper switch and hence ultimately the motor speed.

III. PROJECT OVERVIEW

The basic principle which has been adopted for the speed control of this modelled DC drive is variation of armature voltage for speed below and up to rated speed with field voltage constant because of using permanent magnet dc motor where field is produced by permanent magnets. There are two control loops, one for controlling current and another for speed [7]. The output speed has been compared with the reference speed and error signal is fed to speed controller [8]. Now whenever a deviation of motor speed from provided reference speed occurs, controller output will vary as well. This controller output signal termed as speed control signal [Vω] will control the voltage fed to the motor by controlling the duty cycle of the convert (step down chopper) [13]. As a result, required amount of input voltage [V$_{in}$] will be injected to the motor which will bring it back to the desired speed[4][5].

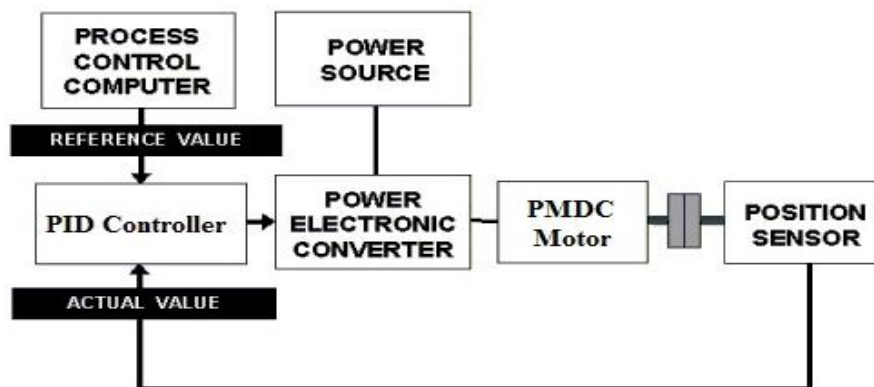A simple block diagram representation of my proposed design is shown below:

Fig.1 Block diagram of proposed system

In case of a practical situation, output speed of motor can be measured by a position sensor [may be a Tacho-generator].

IV. MODELLING OF OUR DC DRIVE SYSTEM

According to our discussion till now the best way to design a dc drive with speed control option is by using a DC-DC step down converter to feed the PMDC motor. Speed control will be achieved by controlling the duty cycle of the power electronics device [GTO] used as switch. For regulation of the duty cycle, the control or error signal can be produced by comparing the motor output speed with the reference speed and this generated error signal is fed to PID [speed] controller. Controller output will vary whenever there is a difference in the reference speed and the actual motor speed. The output of the PID [speed] controller or the actuating signal so generated will control the duty cycle of converter. Now the converter output will give the required V$_{in}$ required to bring motor back to the desired speed [14].

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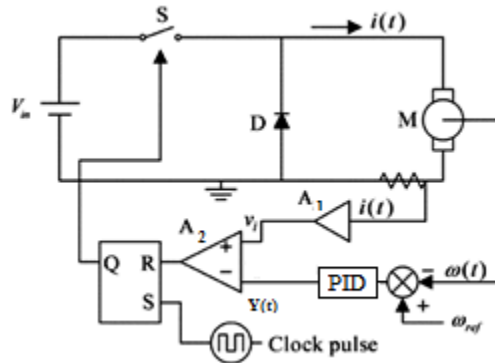Schematic diagram of this kind of drive is shown here:

Fig. 2 A current-mode controlled buck-type dc chopper-fed PM dc motor drive operating in the continuous condition mode

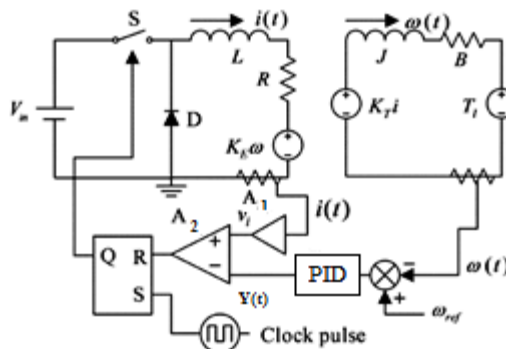The corresponding equivalent circuit is shown in Fig. 3 which will be used throughout the analysis:

Fig. 3 Equivalent circuit Current-controlled DC drive system

Let us assume that operational amplifier A_1, have gains g_i, the current control signal V_i and speed control signal [in our case output of the PID controller] y(t) can be expressed as:

$$y(t) = k_p(\omega_{ref} - \omega) + \frac{k_p}{T_i} \int(\omega_{ref} - \omega) + k_d \frac{d}{dt}(\omega_{ref} - \omega) \qquad (1)$$

$$V_i(t) = g_i \, i(t) \qquad (2)$$

Here,
$i(t)$= armature current of the dc motor, $\omega(t)$ = speed of the dc motor, ω_{ref} = reference speed of the dc motor.

From Fig. 3 it can be see that both $y(t)$ and $V_i(t)$ are inserted into the comparator A_2 and this comparator outputs the pulse to the reset of an R-S latch. This R-S latch controls the switch S of the dc-dc step down converter and a clock pulses of period T sets the R-S latch. Once the clock pulse sets the latch, switch S is "ON" and the diode is "OFF". From this instance the switch S will be in "ON" until y(t) exceeds V_i(t) and at that point of time the latch will begins to reset. After this reset S is turned off and D is on. Here on S remains open until the arrival of the next clock pulse where it will be closed again. If both set and reset signals occur at the same time, the reset will dominate the set so that S keeps open until the occurrence of another clock pulse. Therefore, the system equation can be divided into two stages as given by [15] -
STATE EQUIATION IS GIVEN BELOW:
A. STAGE 1:
When $V_\omega > V_i$ means switch "S" is in ON state

$$\frac{di}{dt} = \frac{1}{L}(V_{in} - Ri_a - K_E\,\omega) \tag{3}$$

$$\frac{d\omega}{dt} = \frac{1}{J}(K_T\,i_a - B_\omega - T_L) \tag{4}$$

$$\frac{dy(t)}{dt} = -K_p\frac{d\omega}{dt} + \frac{K_p}{T_i}(\omega_{ref} - \omega) + K_D\frac{B}{J} \tag{5}$$

$$\frac{d}{dt}\begin{bmatrix} i_a \\ \omega(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} \frac{-R}{L} & \frac{-K_E}{L} & 0 \\ \frac{K_T}{J} & \frac{-B}{J} & 0 \\ \frac{-K_pK_T}{J} & -K_p(\frac{B}{J}+\frac{1}{T_i}) & 0 \end{bmatrix}\begin{bmatrix} i_a \\ \omega(t) \\ y(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{L} & 0 & 0 \\ 0 & \frac{-1}{J} & 0 \\ 0 & \frac{K_p}{J} & \frac{K_p}{T_i} \end{bmatrix}\begin{bmatrix} V_{in} \\ T_L \\ \omega_{ref} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{K_D B}{J} \end{bmatrix} \tag{6}$$

B. STAGE 2:

When $V_i > V\omega$ means switch "S" is OFF state.

$$\frac{di}{dt} = \frac{1}{L}(-Ri_a - K_E\,\omega) \tag{7}$$

$$\frac{d\omega}{dt} = \frac{1}{J}(K_T\,i_a - B\,\omega - T_L) \tag{8}$$

$$\frac{dy(t)}{dt} = -K_p\frac{d\omega}{dt} + \frac{K_p}{T_i}(\omega_{ref} - \omega) + K_D\frac{B}{J} \tag{9}$$

$$\frac{d}{dt}\begin{bmatrix} i_a \\ \omega(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} \frac{-R}{L} & \frac{-K_E}{L} & 0 \\ \frac{K_T}{J} & \frac{-B}{J} & 0 \\ \frac{-K_pK_T}{J} & -K_p(\frac{B}{J}+\frac{1}{T_i}) & 0 \end{bmatrix}\begin{bmatrix} i_a \\ \omega(t) \\ y(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{-1}{J} & 0 \\ 0 & \frac{K_p}{J} & \frac{K_p}{T_i} \end{bmatrix}\begin{bmatrix} V_{in} \\ T_L \\ \omega_{ref} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{K_D B}{J} \end{bmatrix} \tag{10}$$

Where R is armature resistance, L is armature inductance, Vin is dc supply voltage, K_E is back-EMF constant, K_T is torque constant, B is viscous damping, J is load inertia, & T_L is load torque.

By defining the state vector $X(t)$ and the following matrixes A,E1,E2,E3 as:

$$A = \begin{bmatrix} \frac{-R}{L} & \frac{-K_E}{L} & 0 \\ \frac{K_T}{J} & \frac{-B}{J} & 0 \\ \frac{-K_pK_T}{J} & -K_p(\frac{B}{J}+\frac{1}{T_i}) & 0 \end{bmatrix}; \; X(t) = \begin{bmatrix} i_a \\ \omega(t) \\ y(t) \end{bmatrix}; \; E1 = \begin{bmatrix} \frac{1}{L} & 0 & 0 \\ 0 & \frac{-1}{J} & 0 \\ 0 & \frac{K_p}{J} & \frac{K_p}{T_i} \end{bmatrix}; \; E_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{-1}{J} & 0 \\ 0 & \frac{K_p}{J} & \frac{K_p}{T_i} \end{bmatrix}; \; E_3 = \begin{bmatrix} 0 \\ 0 \\ \frac{K_d B}{J} \end{bmatrix}$$

The system equation given by can be rewritten as

$$\dot{X}(t) = A\,X(t) + E_K \qquad ; (k = 1,2) \tag{11}$$

Since k is topological dependent hence time dependent, the system equation given by is in fact a time-varying state equation. Thus this closed-loop drive system is a second-order non-autonomous dynamical system

Given the desired initial conditions $X(t_o)$, the analytical solution of the system equation given by can be expressed as:

$$X(t) = \phi(t - t_0)X(t_o) + \int\phi(t - \tau)E_K\,d\tau$$

$$= -A^{-1}E_K + \phi(t- t_o)(X(t_o) + A^{-1}E_K) \qquad (k = 1,2) \tag{12}$$

Where $\phi(t) = e^{At}$ is so called the state transition matrix. by defining the following parameters

$$\alpha = \frac{1}{2}\left(\frac{R}{L} + \frac{B}{J}\right), \qquad \Delta = \frac{1}{4}(\frac{R}{L} - \frac{B}{J})^2 - \frac{K_E K_T}{LJ}$$

For different values of Δ the eigen values λ_1, λ_2 of the matrix A can be expressed as this:

$$\Delta = 0 \quad \lambda_1 = 0, \; \lambda_{2,3} = -\alpha$$
$$\Delta > 0 \quad \lambda_1 = 0, \; \lambda_{2,3} = -\alpha \pm \sqrt{\Delta}$$
$$\Delta < 0 \quad \lambda_1 = 0, \; \lambda_{2,3} = -\alpha \pm \sqrt{\Delta} \tag{13}$$

Hence the corresponding state transition matrix can be obtained as for given below:

$$\Delta = 0, \quad \phi(t) = e^{-at}[I - t(2\alpha I - A)]$$
$$\Delta > 0, \quad \phi(t) = [e^{\lambda 1 t}(\lambda 2 I - A) - e^{\lambda 2 t}(\lambda 1 I - A)]/(\lambda 2 - \lambda 1)$$
$$\Delta < 0, \quad \phi(t) = e^{-at}\left[I \cos(\beta t) + \frac{1}{\beta}(\alpha I + A) \sin(\beta t)\right]$$

Where **I** is the identity matrix and $\beta = \sqrt{-\Delta}$, for the system to be locally stable, the roots of the characteristic equation should be in the left half plane.

V. MATLAB SIMULATION RESULTS

As discussed up to this point, our last objective is to model and simulate [1] a buck chopper fed PMDC motor with a PID controller oriented feedback loop through MATLAB software [2][3]. This objective is fulfilled with following parameter values:

TABLE I
PARAMETER VALUES OF MODELLED DC DRIVE

Various machine parameters	Parameter Values
Viscous damping [B]	0.000564 Nm/rad/s
Load inertia [J]	0.000971 Nms^2
Back-EMF constant [K_E]	0.1356 V/rad/s
Torque constant [K_T]	0.1324Nm/A
Armature Resistance [R_a]	3.5 Ω
Armature Inductance [L_a]	36 mH
Input voltage [V_{in}]	100 V
Reference speed [ω_{ref}]	100 r.p.m
Load torque [T_{load}]	Case 1. 0.39Nm Case 2. 0.27Nm

Time Plot of speed & armature current with different load torques:

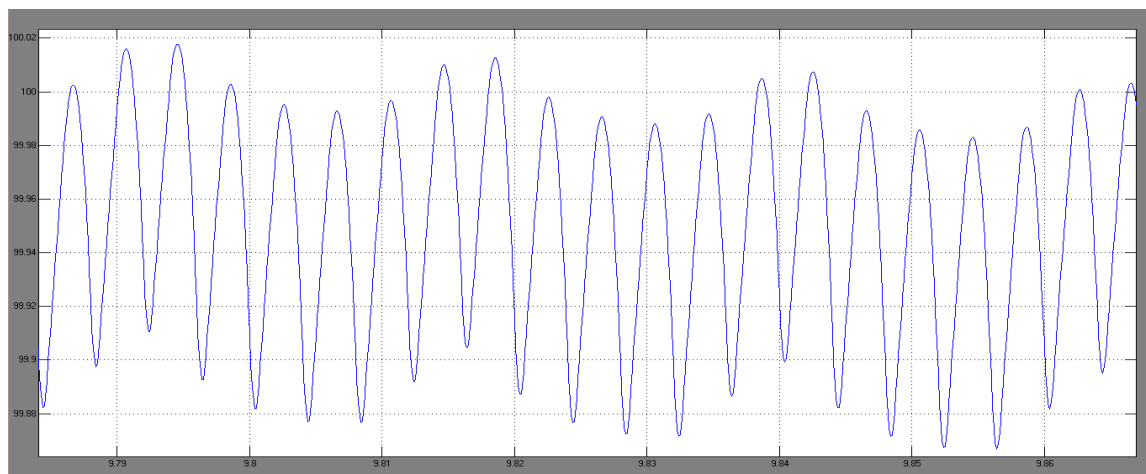*a. Case 1. For $T_l = 0.39Nm$ and $w_{ref} = 100$*

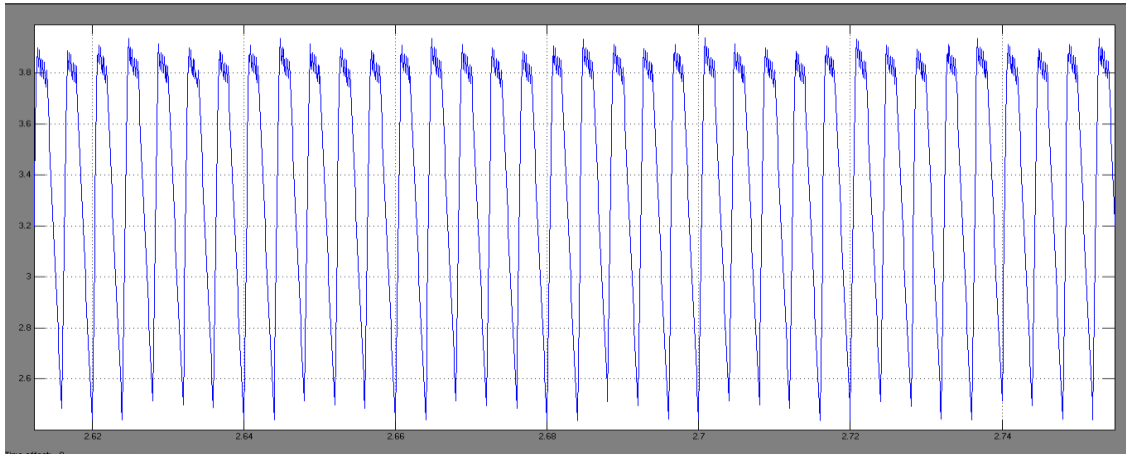Fig.4 Time plot of motor speed.

Fig 5 Time plot of motor armature current.

b. Case 2. For $T_l = 0.27Nm$ and $W_{ref} = 100r.p.m$

Fig. 6 Time plot of motor speed.
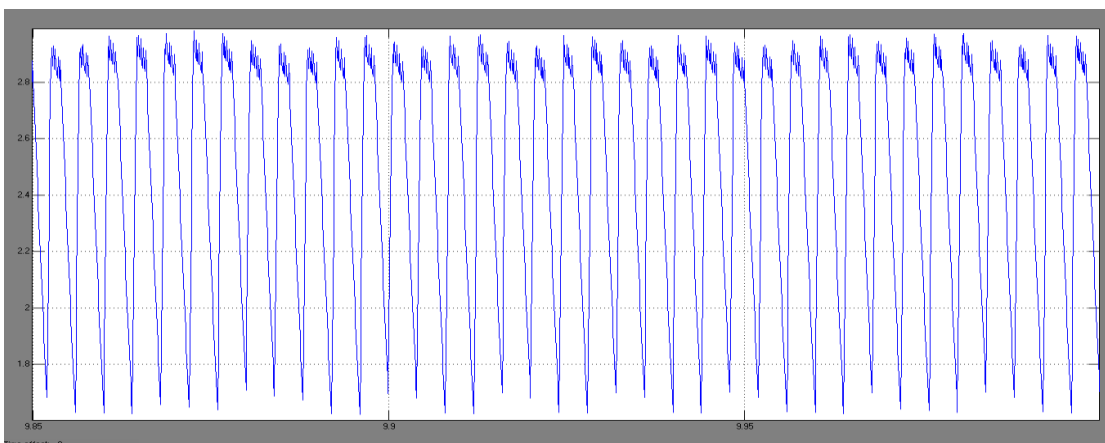

Fig. 7 Time plot of motor armature current.
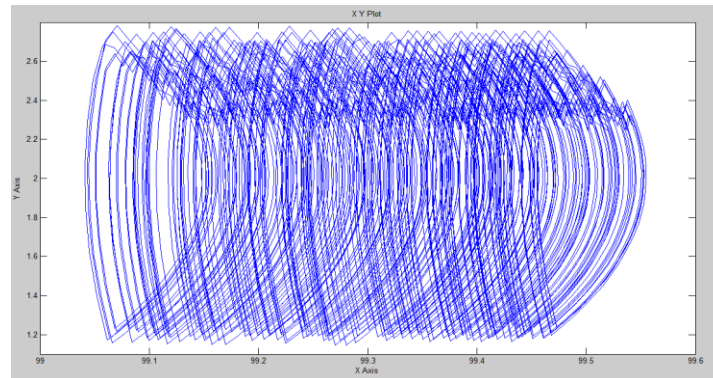
c. Phase Portrait

Fig.8 Phase portrait trajectory of motor armature current versus motor speed in chaos.

From these results it can be seen that for different values of load torque [T_l]our designed DC drive operating with a PMDC motor that reaches to its steady state operating condition and maintaining the speed at given reference speed. This is happening because at steady state, load torque becoming equal to steady state torque of the motor. From Fig.8 it can also be said that some chaotic behaviour is present in our system [10][11].

VI. CONCLUSION

After designing the whole dc drive using MATLAB, the model has been simulated with a pre set reference speed of 100rpm and with variable load torque. It is seen that although the load torque is changed motor reached at steady state operation with the given reference speed which is 100rmp. This means that at steady state the load torque is getting equal to steady state torque. From this it can be concluded that the speed of the PMDC motor can be controlled preciously as required with our proposed model.

At last observing from phase portrait of armature current versus speed it can be said that some chaotic behaviour is present in this designed system. From simple understanding of the system it can be assumed that our switching circuit for the chopper switch control may have brought these chaotic behaviour to this system.

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BIOGRAPHY

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