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On Nano Generalised Pre Closed Sets and Nano Pre Generalised Closed Sets in Nano Topological Spaces

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ABSTRACT: The purpose of this paper is to define and study a new class of sets called Nanogeneralised pre closed sets and Nanopre generalised closed sets in Nano topological spaces.Basic properties of Nanogeneralised pre closed sets and Nanopregeneralised closed sets are analysed.

KEYWORDS: *Nanogeneralisedpreclosed sets, Nanopregeneralised closed sets,Nanopreclosure, Nanopreinterior.*

I. INTRODUCTION

In 1970, Levine[5] introduced the concept of generalised closed sets as a generalization of closed sets in topological spaces.In [6] Maki et al introduced the concepts of gpclosed sets and pgclosed sets in an analogous manner.The notion of Nano topology was introduced byLellisThivagar[3] which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it and also defined Nano closed sets ,Nano-interior and Nano-closure. In this paper, we have introduced a new class of sets onNano topological spaces calledNanogeneralised pre closed sets and Nanopregeneralised closed sets and the relation of these new sets with the existing sets.

II. RELATED WORK

In General Topology, Generalised pre closed sets and Pre generalised closed sets were studied by Maki et al[6].The concept of Nano Topology was introduced by LellisThivagar[3]. He studied about the weak forms of nano open sets such as Nano α- open sets , Nano semi open sets and Nano pre open sets .He investigated the properties of the above sets. Later Bhuvaneswari et al introduced Nano generalised closed sets and studied some of its properties. Also Nano generalised semi closed sets, Nano generalised α- closed sets and Nano generalised regular closed sets were introduced by them. Based on this, a new class of sets called Nano generalised pre closed sets and nano Pre generalised closed sets are introduced here and some of the forms are discussed

III. PRELIMINARIES

Definition:3.1[7]A subset A of a topological space (X,τ) is called a preopen set if $A \subseteq Int(Cl(A))$. The complement of a preopen set of a space X is called preclosed set in X.

Definition:3.2 [2]A pre-closure of a subset A of X is the intersection of all preclosed sets that contains A and it is denoted by pcl(A).

Definition:3.3[2]The union of all preopen subsets of X contained in A is called preinterior of A and it is denoted by pInt(A).

Definition:3.4 [5] A subset A of (X,τ) is called a generalised closed set (briefly g-closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

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Definition:3.5 [6]A subset A of (X,τ) is called a generalisedpre closed set (briefly gp-closed) if$pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

Definition:3.6 [6] A subset A of (X,τ) is called a pre generalised closed set (briefly pg-closed) if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is preopen in X.

Definition:3.7[4]Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as indiscernibility relation. Then U is divided into disjoint equivalence classes . Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be the approximation space.Let $X \subseteq U$.Then,

(i)The lower approximation of X with respect to R is the set of all objects which can be for certain classified as X with respect to R and is denoted by $L_R(X)$.$L_R(X)=U\{R(x):R(x) \subseteq X, x\epsilon U\}$ where R(x) denotes the equivalence class determined by $x\epsilon U$.
(ii)The upper approximation of X with respect to R is the set of all objects which can be possibly classified as X with respect to R andis denoted by $U_R(X)$. $U_R(X) = U\{R(x):R(x)\cap X\neq\Phi, x\epsilon U\}$
(iii)The boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not – X with respect to R and it is denoted by $B_R(X)$. $B_R(X) = U_R(X) - L_R(X)$

Property:3.8[4] If (U,R)is anapproximationspaceand $X,Y\subseteq U$,then
1. $L_R(X) \subseteq X \subseteq U_R(X)$
2. $L_R(\Phi) = U_R(\Phi) = \Phi$
3. $L_R(U) = U_R(U) = U$
4. $U_R(X\cup Y) = U_R(X) \cup U_R(Y)$
5. $U_R(X\cap Y) \subseteq U_R(X) \cap U_R(Y)$
6. $L_R(X\cup Y) \supseteq L_R(X) \cup L_R(Y)$
7. $L_R(X\cap Y) = L_R(X) \cap L_R(Y)$
8. $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$
9. $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$
10. $U_R[U_R(X)] = L_R[U_R(X)]= U_R(X)$
11. $L_R[L_R(X)] = U_R[L_R(X)]= L_R(X)$

Definition:3.9 [4] Let U be the universe , R be an equivalence relation on U and$\tau_R(X) = \{U,\Phi, L_R(X), U_R(X), B_R(X)\}$where $X \subseteq U$. Then by property 2.8, $\tau_R(X)$ satisfies the following axioms:
(i) U and $\Phi\epsilon\tau_R(X)$.
(ii) The union of the elements of any sub-collection of$\tau_R(X)$ is in $\tau_R(X)$.
(iii) The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

Then $\tau_R(X)$ is a topology on U called the Nano topology on U with respect to X. $(U, \tau_R(X))$ is called the Nano topological space. Elements of the Nano topology are known as Nano open sets in U. Elements of $[\tau_R(X)]^c$ are called Nano closed sets with $[\tau_R(X)]^c$ being called dual Nano topology of $\tau_R(X)$.

Remark:3.10 [4] If $\tau_R(X)$ is the Nano topology on U with respect to X , then the set $B = \{ U, L_R(X), B_R(X)\}$ is the basis for$\tau_R(X)$.

Definition:3.11 [4] If $(U, \tau_R(X))$ is a Nano topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$,then
(i)The Nano interior of the set A is defined as the union of all Nano open subsets contained in A and is denoted by NInt(A). NInt(A) is the largest Nano open subset of A.
(ii)The Nano closure of the set A is defined as the intersection of all Nano closed sets containing A and is denoted by NCl(A). NCl(A) is the smallest Nano closed set containing A.

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Definition:3.12 [4] Let (U, τ_R(X)) be a Nano topological space and A ⊆ U. Then A is said to be
(i) Nano semi open if A⊆NCl[NInt(A)]
(ii) Nano semi closed if NInt [NCl (A)] ⊆A
(iii)Nanopre openif A⊆NInt [NCl (A)]
(iv)Nano**α** open if A⊆NInt[NCl(NInt(A))].

NSO(U,X), NSF(U,X), NPO(U,X) and τ_R^{α} (X) respectively denote the families of all Nano semi open ,Nano semi closed,Nano pre-open and Nano**α** open subsets of U.

Definition: 3.13 [1] A subset A of (U, τ_R(X)) is called Nanogeneralised closed set (briefly Ng-closed) if Ncl(A) ⊆ V whenever A⊆ V and V is Nano open in (U, τ_R(X)).

IV.FORMS OF NANOGENERALISEDPRE CLOSED SETS AND NANOPRE GENERALISEDCLOSED SETS

Throughout this paper, (U, τ_R(X)) is a Nano topological space with respect to X where X ⊆ U, R is an equivalence relation on U.Then U/R denotes the family of equivalence classes of U by R.

In this section ,we define and study the forms of Nanogeneralised pre closed sets and Nanopre generalised closed sets.

Definition:4.1 If (U, τ_R(X)) is a Nano topological space with respect to X where X ⊆ U and if A ⊆U,then
(i)The Nano pre closure of A is defined as the intersection of all Nano pre closed sets containing A and it is denoted by Npcl(A). Npcl(A) is the smallest Nano pre closed set containing A.
(ii)The Nano pre interior of A is defined as the union of all Nanopreopen subsets of A contained in A and it is denoted by NpInt(A). NpInt(A) is the largest Nanopre open subset of A .

Definition:4.2 A subset A of (U, τ_R(X)) is called Nanogeneralisedpreclosed set (briefly Ngp-closed) if Npcl(A) ⊆ V whenever A⊆ V and V is Nano open in (U, τ_R(X)).

Definition:4.3A subset A of (U, τ_R(X)) is called Nano pregeneralised closed set (briefly Npg-closed) if Npcl(A) ⊆ V whenever A⊆ V and V is Nanopre open in (U, τ_R(X)).

Example:4.4
Let U = {a,b,c,d} with U/R = { {a}, {c}, {b,d}} and X = {a,b}.
Then τ_R(X) ={ U,Φ, {a}, {a,b,d}, {b,d}} which are open sets.
The Nano closed sets = { U,Φ, {b,c,d}, {c}, {a,c}} .
The Nano pre closed sets = {Φ,U, {b} , {c}, {d}, {a,c}, {b,c}, {c,d}, {a,b,c}, {b,c,d}, {a,c,d}} .
The Nanogeneralised closed sets are { Φ,U,{c}, {a,c}, {b,c} , {c,d} , {a,b,c} , {b,c,d} ,{a,c,d}}
The Nanogeneralised preclosed sets are
{U,Φ ,{b},{c},{d},{a,c},{b,c}, {c,d} ,{a,b,c} ,{a,c,d},{b,c,.d}}
The Nanopregeneralised closed sets are{Φ,U,{a},{b},{c},{d},{a,b},{a,c},{a,d},{b,c},{c,d},{a,b,c},{a,c,d},{b,c,d}}
Theorem:4.5 Every Nano closed set is a Nanopre closed set .
Proof:Let A be a Nano closed set.Then we have Ncl(A) = A. We have to prove that NCl(NInt(A)) ⊆A which implies that A is a Nanopreclosed set. NCl (NInt(A)) = N Int(A) ⊆A. Hence A is aNanopre closed set.
Remark:4.6 The converse of the above theorem is not true.In the example 4.4 , the sets {a,b,c}, {a,c,d} are Nanopreclosed sets but not Nano closed sets.
Theorem: 4.7 Every Nano closed set is a Nanogeneralised pre closed set.
Proof: Let A be a Nano closed set of U and A ⊆ V, V is Nano open in U. Since A isNano closed, Ncl(A) = A ⊆ V. That isNcl(A)⊆ V. Also, Npcl(A) ⊆Ncl(A) ⊆ V, where V is Nano open in U.Therefore, A is a Nanogeneralised pre closed set.

Theorem:4.8 Every Nano closed set is a Nano pre generalised closed set.

Proof: Let A be a Nano closed set of U and A⊆V ,V is Nanopre open in U. Since A is Nanoclosed ,Ncl(A) = A ⊆ V That is Ncl(A)⊆ V. Also, Npcl(A) ⊆Ncl(A) ⊆ V where V is Nanopre open in U.Therefore, A is a Nano pre generalised closed set.

Remark:4.9 The converse of the theorems (4.7 and 4.8) need not be true which is shown in the example 4.4
Remark :4.10 Every Ng- closed set is Nano pre closed set which is shown in the following example.

Example:4.11 Let U = {a,b,c,d} with U/R = { {a}, {c}, {b,d}} and X = {a,b}.Then $\tau_R(X)$ = { U,Φ, {a}, {a,b,d}, {b,d}} which are open sets.
The Nanogeneralised closed sets are { Φ,U,{c}, {a,c}, {b,c} , {c,d} , {a,b,c} , {b,c,d} ,{a,c,d}}
The Nano pre closed sets are {Φ,U, {b} , {c}, {d}, {a,c}, {b,c}, {c,d}, {a,b,c}, {b,c,d}, {a,c,d}}

Theorem:4.12 Every Nanogeneralised closed set is Nanogeneralised pre closed set.
Proof:Let A be a Nanogeneralised closed set. Then Ncl(A) ⊆ V whenever A ⊆ V and V is Nano open in U. But Npcl(A) ⊆Ncl(A) which implies that Npcl(A)⊆V, A⊆V, V is Nano open in U.Hence A is Nanogeneralised pre closed set.

Remark:4.13 The converse of the above theorem is not true in general. In the example (4.4), let A = {b},V= {a,b,d} whenever A ⊆ V, V is Nano open. Now Npcl(A) = {b}⊆ V. Hence A= {b} is Nanogeneralised pre closed set. But Ncl(A) = {b,c ,d}⊈V. Hence the subset A={b} is not Nanogeneralised closed set. Hence every Nanogeneralisedpre closed set need not be a Nanogeneralised closed set.

Theorem:4.14. Every Nanogeneralised closed set is Nano pre generalised closed set.
Proof: Let A be a Nanogeneralised closed set. Then Ncl(A) ⊆ V whenever A ⊆ V and V is Nano open in U. But every Nano open set is Nanopre open which implies V is Nanopre open.AlsoNpcl(A) ⊆Ncl(A)⊆V, A⊆V, V is Nanopreopen in U.Hence A is Nanopre generalised closed set.
Remark:4.15. The converse of the above theorem is not true in general. In the example (4.4), let A = {a,b}, V= {a,b,c} whenever A ⊆ V, V is Nanopreopen. Now Npcl(A) = {a,b,c}⊆ V. Hence A= {a,b} is Nanopre generalised closed set. But Ncl(A) = U⊈V. Hence the subset A={a,b} is not Nanogeneralised closed set. Hence every Nanopre generalised closed set need not be a Nanogeneralised closed set.

Theorem:4.16 Every Nanopre closed set is Nanogeneralisedpreclosed set and Nano pre generalised closed set .This is shown in the example 4.4
Theorem:4.17 The union of two Nanogeneralised pre closed sets in (U, $\tau_R(X)$) is also a Nanogeneralised pre closed set in (U, $\tau_R(X)$).
Proof: Let A and B be two Nanogeneralised pre closed sets in (U, $\tau_R(X)$). Let V be a Nano open set in U such that A⊆V and B⊆V. Then we have A∪B⊆V. A and B are Nanogeneralised pre closed sets in (U, $\tau_R(X)$), Npcl(A) ⊆V and Npcl(B)⊆V. Now Npcl(A∪B) = Npcl(A) ∪Npcl(B) ⊆V. Thus we have Npcl(A∪B) ⊆V whenever A∪B⊆V, V is Nano open set in (U, $\tau_R(X)$) .This implies A∪B is a Nanogeneralised pre closed set in (U, $\tau_R(X)$).

Remark:4.18 The Intersection of two Nanogeneralised pre closed sets in (U, $\tau_R(X)$) is also a Nanogeneralised pre closed set in (U, $\tau_R(X)$) as seen from the following example.

Example :4.19Let U = {a,b,c,d} with U/R = { {a}, {c}, {b,d}} and X = {a,b}.Then $\tau_R(X)$ = { U,Φ, {a}, {a,b,d}, {b,d}}. The Nanogeneralised pre closed sets are {U,Φ ,{b},{c},{d},{a,c},{b,c}, {c,d} ,{a,b,c} ,{a,c,d},{b,c,.d}}. Here {a,c} ∩ {c,d} ={c} which is again a Nanogeneralised pre closed set.

Theorem:4.20 The union of two Nanopregeneralised closed sets in (U, $\tau_R(X)$) is also a Nanopre generalised closed set in (U, $\tau_R(X)$).

Proof: Let A and B be twoNanopre generalisedclosed sets in

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(U, τ_R(X)). Let V be a Nanopre open set in U such that A⊆V andB⊆V.Then we have A∪B⊆V.As Aand B are Nanopregeneralised closed sets in (U, τ_R(X)), Npcl(A) ⊆V and Npcl(B)⊆V. Now Npcl(A∪B) = Npcl(A)∪Npcl(B) ⊆V. Thus we have Npcl(A∪B) ⊆ V whenever A∪B ⊆ V, V is Nanopre open set in (U, τ_R(X)) .This implies A∪B is a Nanopre generalised closed set in (U, τ_R(X)).

Remark:4.21 The Intersection of two Nano pre generalised pre closed sets in (U, τ_R(X)) is also a Nano pre generalised closed set in (U, τ_R(X)) as seen from the following example.

Example :4.22 Let U = {a,b,c,d} with U/R = { {a}, {c}, {b,d}} and X = {a,b}.Then τ_R(X) = { U,Φ, {a}, {a,b,d}, {b,d}}. The Nano pre generalised closed sets are {Φ , U, {a}, {b},{c}, {d},{a,b} ,{a,c},{a,d} , {b,c} ,{c,d} , {a,b,c} ,{a,c,d} ,{b,c,d}}.Here {a,b,c} ∩ {a,c,d} ={a,c} which is again a Nanopre generalisd closed set.

Theorem :4.23 If A is Nanogeneralised pre closed sets in(U, τ_R(X)) ,then it is Nano pre generalised closed set.
Proof: Let A⊆V and V is Nano open in τ_R(X). Then NpCl(A) ⊆V as V is Nanogeneralised pre closed sets .Since every Nano open set is Nanopre open , NpCl(A) ⊆V where V is Nano preopen set.This implies A isNano pre generalised closed set.The converse of the above theorem is not true as seen in the example 4.4.

Theorem:4.24Let A be a Nanogeneralised pre closed subset of
(U, τ_R(X)).If A⊆B⊆Npcl(A),then B is also a Nanogeneralised pre closed subset of (U, τ_R(X)).

Proof: Let V be a Nano open set of a Nanogeneralisedpre closed subset of τ_R(X) such that B ⊆ V. As A ⊆B , we have A ⊆ V. As A is a Nanogeneralised pre closed set, Npcl(A) ⊆ V. Given B⊆Npcl(A), we have Npcl(B) ⊆Npcl(A). As Npcl(B) ⊆Npcl(A) and Npcl(A) ⊆ V, we have Npcl(B) ⊆ V whenever B⊆V and V is Nano open. Hence B is also a Nanogeneralised pre closed subset of τ_R(X) .

V.CONCLUSION

In this paper ,some of the properties of Nano Generalised pre closed sets and Nano pre Generalised closed sets are discussed . This shall be extended in the future Research with some applications.

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