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# Parametric Instability of a Lower Hybrid Wave in Two Ion Species Plasma

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# **Research Article**

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#### ABSTRACT

Three wave parametric instability of a large amplitude lower hybrid wave in two-ion species plasma is studied analytically. The pump decays into a low frequency mode or quasi mode in the ion cyclotron range of frequency and a lower hybrid wave sideband. The dominant channel of decay is the one for which the low frequency mode is an ion cyclotron wave with frequency close to the ion cyclotron frequency of either ion species. For typical D-T plasma of a tokamak, the growth rate for  $\omega$  close to deuterium cyclotron frequency, increases with the wave number of the ion cyclotron wave and decreases as the ratio of deuterium to tritium density increases.

# INTRODUCTION

Parametric instabilities have been an important feature of large amplitude lower hybrid waves in tokamak as well as smaller plasma devices. During the first decades of nuclear fusion research, high power lower hybrid waves in the 500 MHz-1 GHz frequency range were candidates for heating magnetized plasmas to thermonuclear temperatures. More recently, such waves were found to be mostly attractive at higher frequency (1-5 GHz) for driving non-inductive currents in tokamak plasmas, thus opening the possibility of running tokamak in steady state, and leading to the conceptual design of a steady state tokamak reactor. Radio frequency (RF) heating and current drive of tokamak in the lower hybrid range of frequency has been an active field of research for over three decades. In this frequency range, toroidal plasma current is sustained by replenishing the collisional momentum loss of current carrying suprathermal electrons while ions are heated directly through the Landau damping or via the excitation of parametric instabilities <sup>[1-5]</sup>. The prominent channels of three wave parametric decay involve an ion-cyclotron mode or quasi mode and a lower hybrid sideband. In the four wave coupling oscillating two stream instability is the most dominant process. Resonant decay into two lower hybrid waves is also a completing process <sup>[6-11]</sup>.

The current drive experiments on a number of tokamak machines have been successfully performed. Fully non inductive discharges of up to 3.6 MA in JT-60U <sup>[12]</sup>, 3 MA in the Joint European Torus (JET) <sup>[13]</sup> and 0.5 MA during 6 min in Tore Supra <sup>[14]</sup> have been achieved with lower hybrid waves. The projected ITER lower hybrid systems will inject approximately 50 MW of power and they are optimized for the off-axis current drive functions <sup>[15]</sup>. Liu *et al.* have investigated the Lower hybrid wave (LHW) heating in deuterium plasma in the HT-7 tokamak <sup>[16]</sup>. The ion and electron temperatures were increased by 0.4 keV and 0.3 keV respectively for  $P_{LH}$ ~300kW. The ion heating has been observed in the HL-1M tokamak when the plasma density exceeded 3.5 × 10<sup>13</sup> cm<sup>-3</sup> <sup>[17]</sup>. LHW experiments have been performed in HT-6M tokamak by Li *et al.*, they have reported the quasi-steady state H-mode with high plasma density by the injection of lower hybrid heating (LHH) and lower hybrid current drive (LHCD) with a power threshold of 50 kW <sup>[18]</sup>. A good confinement was obtained in ASDEX by combining LHCD and neutral beam injection (NBI) <sup>[19]</sup>. A full wave

current drive with a very high density  $(4.5 \times 10^{19} \text{m}^3)$  was obtained in the high-field Frascati Tokamak Upgrade (FTU)<sup>[20]</sup>. Fisch <sup>[21,22]</sup> has given an elegant review of lower hybrid wave heating and current drive in tokamak. Recently, Ahmad <sup>[23]</sup> has pointed out the parametric excitation of ion-ion hybrid mode by a lower hybrid wave in D-T plasma where growth is faster for higher deuterium to tritium density ratio.

Observations in space indicate that LH waves are among the most important waves in the earth's magnetosphere. In fact, LH wave play a central role in the process of collision less energy and momentum transport in space plasmas. Recently there has been much interest <sup>[24,25]</sup> in magneto sonic and other waves in multi-ion plasmas.

Two ion species plasma in tokamak has the possibility of extra channels of parametric decay. In this paper, we study the parametric decay of a large amplitude lower hybrid wave into an ion cyclotron wave and lower hybrid sideband wave in two ion species plasma such as those considered for nuclear fusion. The channels of decay are: i) resonant decay into an ion cyclotron mode near the ion cyclotron frequency of the either ion species, ii) nonlinear cyclotron damping on either ion species. The lower hybrid pump wave  $(\omega_0, \vec{k}_0)$ , imparts an oscillatory velocity  $\vec{v}_0$  to electrons. The latter beats with the density perturbation due to the low frequency mode  $(\omega, \vec{k})$  to produce a nonlinear density perturbation, driving the sideband lower hybrid wave  $(\omega_1, \vec{k}_1)$ , where  $\omega_{1=}\omega - \omega_0$ ,  $\vec{k}_1 = \vec{k} - \vec{k}_0$ . The sideband couples with the pump to produce a ponder motive force on electrons that drive the low frequency mode.

In Sec. 2, we obtain the linear response of electrons to the pump and sideband waves and the low frequency mode. In Section 3, we study the nonlinear coupling and obtain the nonlinear dispersion relation and the growth rate. The results have been discussed in Section 4.

## LINEAR RESPONSE

Consider a two-ion species plasma (e.g., tokamak) of equilibrium electron density  $n_0^0$ , in a static magnetic field  $B_s \hat{z}$ . The ion species are characterized by mass  $m_1, m_2$ , density  $n_{01}^0, n_{02}^0$  and charge  $Z_1 e, Z_2 e$  such that  $n_{01}^0 Z_1 + n_{02}^0 Z_2 = n_0^0$ . A large amplitude lower hybrid pump wave propagates through the plasma in the x-z plane with electrostatic potential

$$\phi_0 = \mathbf{A}_0 \exp\left[-i\left(\omega_0 t - \vec{k}_0 \cdot \vec{r}\right)\right],\tag{1}$$

$$\omega_0 = \omega_{LH} \left( 1 + \frac{\omega_p^2}{\omega_{p1}^2 + \omega_{p2}^2} \frac{k_{0z}^2}{k_0^2} \right)^{1/2},$$
(2)

$$\omega_{LH} = \left(\frac{\omega_{p1}^2 + \omega_{p2}^2}{1 + \omega_p^2 / \omega_c^2}\right)^{1/2},$$
(3)

where  $\vec{k}_0 = k_{0x}\hat{x} + k_{0z}\hat{z}$ ,  $k_{0x} >> k_{0z}, \omega_c >> \omega_{c1}, \omega_{c2} >> \omega_{ci}, \omega_0 >> k_{0z} v_{thi}, \omega_0 >> k_{0z} v_{th}, \omega_> > k_{0z} v_{th}, k_{01}v_{th}/_c <<1$ , *m* is the mass of electrons,  $v_{th} = (2T_e/m)^{1/2}$  and  $v_{thi} = (2T_i/m_j)^{1/2}$ , j = 1, 2.  $\omega_p = (4\pi n_0^0 e^2 / m)^{1/2}$ ,  $\omega_{p} = (4\pi n_0^0 Z^2 e^2 / m_j)^{1/2}$ , j = 1, 2 and  $\omega_c = (eB_e/m_c)^{1/2}$ , -e and *m* are the electron charge and mass.

The pump wave imparts oscillatory velocity to electrons  $\vec{v}_0$ , which is governed by the equation of motion

$$\left(\frac{\partial \vec{\mathbf{v}}_0}{\partial t} + \vec{\mathbf{v}}_0 \cdot \nabla \vec{\mathbf{v}}_0\right) = \frac{e}{m} \nabla \phi_0 - \vec{\mathbf{v}}_0 \times \vec{\omega}_c,$$

where we have ignored the pressure term. Linearizing this equation and replacing  $\partial/\partial t$  by -i $\omega_{n}$ ,

we obtain

$$\vec{\mathbf{v}}_{0\perp} = -\frac{ie\phi_0}{m\omega_c^2} \left( \vec{k}_0 \times \vec{\omega}_c + i\omega_0 \vec{k}_0 \right), \tag{4}$$

$$\mathbf{V}_{0z} = -\frac{e\kappa_{0z}\varphi_0}{m\omega_0},\tag{5}$$

Using the velocity perturbation in the linearized equation of continuity

$$\frac{\partial n_0}{\partial t} + \nabla \left( n_0^0 \, \vec{\mathbf{v}} \right) = 0 \,.$$

we obtain the density perturbation

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$$n_0 = -\frac{n_0^0 e\phi_0}{m} \left( \frac{k_{0z}^2}{\omega_0^2} - \frac{k_{0\perp}^2}{\omega_c^2} \right).$$
(6)

The pump wave decays into an electrostatic ion cyclotron mode / quasimode of potential  $\varphi$  and a lower hybrid sideband wave of potential s,

$$\phi = \operatorname{A} \exp\left[-i\left(\omega t - \vec{k}.\vec{r}\right)\right],\tag{7}$$

$$\phi_s = A_s \exp\left[-i\left(\omega_s t - k_s . \vec{r}\right)\right],$$
where
(8)

 $\omega_{s} = \omega - \omega_{0}, \ \vec{k}_{s} = \vec{k} - \vec{k}_{0}.$ 

The linear response of electrons to the sideband at  $(\omega_s, \vec{k}_s)$  is same as given by Equations (4–5), with  $\omega_0, \vec{k}_0$  replaced by  $\omega_s, \vec{k}_s$ .

$$\vec{\mathbf{v}}_{s\perp} = -\frac{ie\phi_s}{m\,\omega_c^2} \Big(\vec{k}_s \times \vec{\omega}_c + i\omega_s \vec{k}_s\Big),\tag{9}$$

$$\vec{\mathbf{v}}_{sz} = -\frac{ek_{sz}\phi_s}{m\,\omega_s},\tag{10}$$

The linear density perturbations of electrons and ions at  $\omega, \vec{k}$  can be written in terms of electron and ion susceptibilities  $X_{e}$ ,  $X_{i1}$ ,  $X_{i2}$ ,

$$n^{L} = \frac{k^{2}}{4\pi e} \chi_{e} \phi'$$

$$n_{i1} = -\frac{k^{2}}{4\pi Z_{1} e} \chi_{i1} \phi,$$

$$n_{i2} = -\frac{k^{2}}{4\pi Z_{2} e} \chi_{i2} \phi,$$
(11)
where

wnere

$$\chi_e \approx 1 + \frac{\omega_p^2}{k^2 v_h^2},\tag{12}$$

$$\chi_{ij} = \frac{2\omega_{pj}^2}{k^2 v_{ihj}^2} \left[ 1 + \frac{\omega}{k v_{ihj}} \sum_n Z \left( \frac{\omega - n\omega_{cj}}{k_z v_{ihj}} \right) I_n \left( b_j \right) e^{-b_j} \right], j=1,2$$
(13)

where  $v_{thj} = (2T_j / m_j)$ , are the ions thermal speeds,  $\omega_{cj} = Z_j eBs / m_j c$  are the ion-cyclotron frequencies.  $I_n(b_1)$  and  $I_n(b_2)$  are the modified Bessel functions of order *n* and arguments  $b_1$  and  $b_2$ , and  $b_1 = k_{\perp}^2 \mathbf{v}_{th1}^2 / 2\omega_{c1}^2$ ,  $b_2 = k_{\perp}^2 \mathbf{v}_{th2}^2 / 2\omega_{c2}^2$ .

# NONLINEAR RESPONSE AND GROWTH RATE

The sideband couples with the pump to produce a low frequency ponder motive force  $\vec{F}_p = -m\vec{v}\cdot\nabla\vec{v}$  on the electrons.  $\vec{F}_p$ has two components, perpendicular and parallel to the magnetic field. The response of electrons to  $ar{F}_{p\perp}$  is strongly suppressed by the magnetic field and is usually weak. In the parallel direction, the electrons can effectively respond to  $\vec{F}_{pz}$ , hence, the low frequency nonlinearity arises mainly through  $\vec{F}_{pz} = -m\vec{v}.\nabla v_z$ .

The parallel pondermotive force on electrons at  $\omega, \vec{k}$  can be written as

$$F_{pz} = -\frac{m}{2} (\vec{v}_0 \cdot \nabla v_{1z} + \vec{v}_1 \cdot \nabla v_{0z}),$$
  
i.e.  $F_{pz} = iek_z \phi_p = -\frac{m}{2} (\vec{v}_{0\perp} \cdot \nabla_\perp v_{1z} + \vec{v}_{1\perp} \cdot \nabla_\perp v_{0z}),$  (14)

The pondermotive potential  $\varphi_{P}$  turns out to be

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$$\phi_{p} = \frac{e\phi_{0}\phi_{s}}{2m\omega_{c}^{2}}\frac{\vec{k}_{\perp}.\vec{k}_{0\perp}\times\omega_{c}}{ik_{z}\omega_{0}\omega_{s}}\left(\omega_{0}k_{z}-\omega k_{0z}\right).$$
(15)

The electron density fluctuations in response to  $\varphi_{P}$  and the self-consistent potential  $\varphi$  at  $\omega, \vec{k}$  can be written as

$$n = \frac{\kappa^{-} \chi_{e}}{4\pi e} \left( \phi + \phi_{p} \right), \tag{16}$$

The density perturbations of the two-ion species can be written as

$$n_{i1} = -\frac{k^2}{4\pi Z_1 e} \chi_{i1} \phi,$$

$$n_{i2} = -\frac{k^2}{4\pi Z_2 e} \chi_{i2} \phi.$$
(17)

Using Eqs. (16) and (17) in the Poisson's equation

$$\nabla^2 \varphi = 4\pi e(n - Z_1 n_{11} - Z_2 n_{12}),$$
  
we obtain,

$$\varepsilon \varphi = -\chi_e \varphi_P, \tag{18}$$

where

$$\varepsilon = 1 + \chi_e + \chi_{i1} + \chi_{i2}$$

The density perturbation at  $\omega, \vec{k}$  couples with the oscillatory velocity of electrons  $\vec{v}_0$ , to produce a nonlinear density perturbation at the sideband  $\omega_1, \vec{k}_1$ . Solving the equation of continuity for the nonlinear density perturbation at the sideband,

$$\frac{\partial n_s^{NL}}{\partial t} + \frac{1}{2} \nabla \left( n \, \vec{\mathbf{v}}_0^* \right) = 0 \,,$$
  
we obtain

$$n_s^{NL} = n \frac{\vec{k}_s \cdot \vec{v}_0^*}{2\omega_s}$$
(19)

The linear density perturbation of electrons due to the self consistent potential  $\phi_{\rm s}$  is

$$n_s^L = \frac{k_s^2}{4\pi e} \chi_s \phi_s, \tag{20}$$

The ion density perturbations can be written as

$$n_{sj} = \frac{k_{sj}^2}{4\pi Z_j e} \chi_{sj} \phi_s , \qquad (21)$$

where

$$\chi_{s} = \frac{\omega_{p}^{2}}{\omega_{c}^{2}} \frac{k_{\perp s}^{2}}{k_{s}^{2}} - \frac{\omega_{p}^{2}}{\omega_{s}^{2}} \frac{k_{s}^{2}}{k_{s}^{2}},$$
$$\chi_{j} = -\frac{\omega_{p}^{2}}{\omega_{s}^{2}}.$$

Here we have ignored the nonlinearity arising through ions as it is suppressed by their large mass.

Using these electron and ion density perturbations in the Poisson's equation

$$\nabla^2 \varphi_{s} = 4\pi e(n_s - Z_1 n_{s1} - Z_2 n_{s2}),$$

we obtain

$$\mathcal{E}_{s}\phi_{s} = \frac{4\pi e}{k_{s}^{2}}n_{s}^{NL} = -\frac{k^{2}}{k_{s}^{2}}\left(1 + \chi_{i1} + \chi_{i2}\right)\frac{\vec{k}_{s}.\vec{v}_{0}^{*}}{2\omega_{s}}\phi$$
(22)

Equations (18) and (22) yield the nonlinear dispersion relation

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ε ε=µ,

where

$$\mu = \chi_e \left( 1 + \chi_{i1} + \chi_{i2} \right) \frac{\left| \vec{k} \cdot \vec{v}_0 \right|^2}{4\omega_s^2} \frac{k^2}{k_s^2} \left( 1 - \frac{\omega}{\omega_0} \frac{k_{0z}}{k_z} \right), \tag{24}$$

We solve Eq. (23) in the case of resonant decay.

In case  $\omega, \vec{k}$  and  $\omega_s, \vec{k}_s$ , in the absence of the pump, satisfy the linear dispersion relations corresponding to ion cyclotron and lower hybrid waves respectively, the decay process is termed as resonant decay.

In the vicinity of  $\omega \approx \omega_{c1}$ , when  $\omega_{-1} >> k_z v_{th1}$ , the linear dispersion relation for the low frequency mode (=0) takes the form

$$1 + \frac{\omega_p^2}{k^2 v_{th}^2} + \frac{2\omega_{p1}^2}{k^2 v_{th1}^2} \left[ 1 - I_0(b_1)e^{-b_1} - \frac{\omega I_1(b_1)e^{-b_1}}{\omega - \omega_{c1}} \right] + \frac{2\omega_{p2}^2}{k^2 v_{th2}^2} \left[ 1 - I_0(b_2)e^{-b_2} - \frac{\omega I_1(b_2)e^{-b_2}}{\omega - \omega_{c2}} \right] = 0,$$

giving

$$\omega = \omega_r \approx \omega_{c1} \left( 1 + \frac{I_1(b_1)e^{-b_1}}{D} \right),$$
where
$$(25)$$

$$D = \left(1 - I_0(b_1)e^{-b_1}\right) + \left\{1 - I_0(b_2)e^{-b_2} - \frac{\omega_{c1}}{\omega_{c1} - \omega_{c2}}I_1(b_2)e^{-b_2}\right\} \frac{\omega_{p2}^2 \mathbf{v}_{th1}^2}{\omega_{p1}^2 \mathbf{v}_{th2}^2} + \frac{\omega_{p2}^2}{\mathbf{v}_{th1}^2} \frac{\mathbf{v}_{th1}^2}{\omega_{p1}^2} + \frac{k^2 \mathbf{v}_{th1}^2}{2\omega_{p1}^2} \cdot \frac{k^2 \mathbf{v}_{th1}^2}{2\omega_{$$

The linear dispersion relation for the sideband ( $\epsilon_s$ =0) gives

$$\omega_s = \omega_s \approx -\omega_H \left( 1 + \frac{k_s^2}{k_s^2} \frac{\omega_p^2}{\omega_{p1}^2 + \omega_{p2}^2} \right)^{1/2}.$$
(26)

In the presence of nonlinear coupling we write

 $\omega = \omega_r + i\gamma$ ,

$$\omega_{s=}\omega_{sr}+i\gamma$$
,

and expand  $\varepsilon$  and  $\varepsilon$ s around  $\omega_r$  and  $\omega_{sr}$  as  $\varepsilon(\omega) = \varepsilon(\omega_r) + i\gamma \frac{\partial \varepsilon}{\partial \varepsilon} = i\gamma \frac{\partial \varepsilon}{\partial \varepsilon}$ 

$$\varepsilon(\omega) = \varepsilon(\omega_r) + i\gamma \frac{\partial}{\partial \omega} = i\gamma \frac{\partial}{\partial \omega}$$
or

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$$\varepsilon(\omega) \approx \frac{2\omega_{p_1}^2}{k^2 v_{h_1}^2} \frac{D^2 i\gamma}{\omega_{c_1} I_1(b_1) e^{-b_1}}.$$
(27)

$$\varepsilon_{s}(\omega_{s}) = \varepsilon_{s}(\omega_{s}) + i\gamma \frac{\partial \varepsilon_{s}}{\partial \omega_{s}} = i\gamma \frac{\partial \varepsilon_{s}}{\partial \omega_{s}}$$
  
or

$$\varepsilon_s(\omega_s) \approx \frac{2}{\omega_s} \left( 1 + \frac{\omega_p^2}{\omega_c^2} \right) i\gamma \,. \tag{28}$$

where we have suppressed the subscript r on  $\omega_r$  and  $\omega_s$  for the sake of brevity. Then the nonlinear dispersion relation gives

$$\gamma^{2} = -\frac{\mu}{\frac{\partial \varepsilon}{\partial \omega} \frac{\partial \varepsilon_{s}}{\partial \omega_{s}}} \approx \mu \frac{\omega_{c1} |\omega_{s}| I_{1}(b_{1}) e^{-b_{1}}}{4 \left(1 + \frac{\omega_{p}^{2}}{\omega_{c}^{2}}\right)} \frac{k^{2} v_{h1}^{2}}{\omega_{p1}^{2} D^{2}},$$
(29)

Equation (29) simplifies to

$$\gamma^{2} \approx \frac{\omega_{p}^{2} \mathbf{v}_{h1}^{2}}{\omega_{p1}^{2} \mathbf{v}_{h}^{2}} \frac{\left[1 - \left(\omega_{p1}^{2} + \omega_{p2}^{2}\right)/\omega^{2}\right]}{4D^{2} \left(1 + \omega_{p}^{2}/\omega_{c}^{2}\right)} \frac{k^{2} \left|\vec{\mathbf{v}}_{0}\right|^{2}}{4\omega_{s}^{2}} Sin^{2} \delta \frac{k^{2}}{k_{s}^{2}} \omega_{c1} \omega_{0} I_{1}(b_{1}) e^{-b_{1}}.$$
(30)

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(23)

where  $\delta$  is the angle between  $\vec{k}_{\perp}$  and  $\vec{k}_{0\perp}$ .

In order to have numerical appreciation of the growth rate, we have carried computations of growth rate for the following parameters:  $\omega_o/\omega_{LH}=2$ ,  $|V_0|/V_{th1}=2$ ,  $\omega_p^2/\omega_c^2=1/4$ ,  $\delta=/2$ ,  $m_{\chi'}/m_2=367$ ,  $m_{\chi'}/m_2=5508$  (D-T plasma),  $_o/k_{oz}=2$ ,  $T_e/T_1=1.5$ ,  $T_2=T_1$ ,  $Z_1=Z_2=1$ . We have plotted in **Figure 1**, the variation of normalized growth rate,  $\gamma/_{c1}$  as a function of  $k_{\perp}\rho_1$  (where  $\rho_{=}V_{th1}/\omega_{c1}$ ) for  $n_{o\chi'}/m_2=0.5,1,2$ 



**Figure 1.** Variation in normalized growth rate,  $\gamma/\omega_{c1}$ , as a function of  $k_1v_{th1}/\omega_{c1}$  for different values of  $n_{02}/n_{02}$  for the following parameters:  $\omega_0/\omega_{LH}=2$ ,  $|V_0|/V_{th1}=2$ ,  $\omega_p^2/\omega_c^2=0.25$ ,  $\delta=\pi/2$ ,  $m_1/m=3672$ ,  $m_2/m=5508$ , (D-T plasma),  $\omega_0/k_{02}=2$ ,  $T_e/T_1=1.5$ ,  $T_2=T_1$ ,  $Z_1=1$ ,  $Z_2=1$ .

## DISCUSSION

At high power, lower hybrid waves are prone to parametric decay because of their large electrostatic component in the direction perpendicular to the equilibrium magnetic field. This was particularly the case in experiments where the wave frequency was chosen for ion heating through the lower hybrid resonance. A high fraction of the injected power was found to decay nonlinearly into lower frequency daughter waves, and this phenomenon often prevented the power to reach the plasma core and heat the plasma efficiently. At the higher frequencies required for current drive, the question of parametric decay seems to be less severe, but it remains open, especially at high density, and it is the subject of extensive theoretical and experimental investigations. Lower hybrid wave is parametrically unstable when the oscillatory velocity of electrons is of the order or greater than the sound velocity. For tokamak parameters, the powers required are of the order of  $\geq 10$  MW. The decay into an ion cyclotron wave and a lower hybrid wave possesses large growth rate. The coupling between high and low frequency modes is provided primarily by the parallel ponder motive force on electrons involving  $\vec{E} \times \vec{B}$  drift.

For a typical D-T plasma of a tokamak, the growth rate for ion cyclotron wave with frequency  $\omega$  close to deuterium cyclotron frequency increases with the wave number of the ion cyclotron wave. However, the growth rate decreases as the ratio of deuterium to tritium density increases.

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