Quantum-Mechanical Substantiation of The Periodic System of Isotopes: Models of Nuclear Orbitals

A.S. Magula*

Department of Applied Physics, University of Ukrain, Ukrain

Review Article

*For Correspondence:

A.S. Magula

Applied Physics, University of Ukrain, Ukrain.

Email: magula@karazin.ua

ABSTRACT

This article is a theoretical section and a continuation of the article: "Periodic system of isotopes", in which the system was checked against 10 types of experimental data, the periodic change of properties at the level of nuclei and the vertical symmetry of subgroups of isotopes were found. Periodic system of isotopes was constructed with the help of a special algorithm, the principle of multilevel periodicity of the atom, from the electrons to the nucleus. As a description of the multilevel periodicity, this paper presents a unified system of quantum numbers, which is used to describe both electron and nucleon, shells (binomial probabilistic interpretation). With the binomial interpretation the problem of a particle in a one-dimensional potential well has been solved; quantum-mechanical calculations for the probability functions of the orbitals and periods of both electrons and nucleons have been performed-characteristic equations have been obtained, the projections of electronic orbitals have been reproduced and the binomial interpretation has been shown to correspond to the family of spherical harmonics. Based on the principle of multilevel periodicity, expressions are derived and planar projections of nucleon orbitals are constructed, and similarity of the forms with electron orbitals is analyzed and revealed. A critical analysis of the modern.

INTRODUCTION

This paper is a theoretical section and continuation of the article: "The periodic system of isotopes". The periodic system of isotopes is constructed using the principle of multilevel periodicity (PMP). The objectives of the article are the following: interpretation of the principle of multilevel periodicity in the context of quantum mechanics, explanation of the principle; demonstration of a general approach for constructing wave functions of both electron and nucleon shells in a single format of reasoning and calculations; presentation of some ideas that can be used in nuclear modeling, planning reactions and experiments using the periodic system of isotopes, as well as an explanation of the fundamental reasons for the emergence of multilevel periodicity.

Due to the periodic system of isotopes, a change in the properties of nuclei for the periods is shown together withtheir vertical symmetry in the subgroups in accordance with the following experimental data:

Today, the Mendeleev's periodic system is considered to be an empirical rule: it cannot be expressed via mathematical expression. A distinctive advantage of PMP is that this principle suggests a single algorithm for determining the size of the orbitals and periods for electrons and nucleons. This allows reproducing the periodic system of elements within the principle. This feature allows describing the electron and nucleon shells in a single format of numbers similar to quantum ones.

The quantum numbers of electrons and the number of moment projections were selected on the basis of the experience for the Mendeleev's periodic system (**figure 1**). It was proposed to use quantum numbers in nuclear physics (color, baryon charge, and strangeness, isotopic spin) to simulate strong interaction, internal structure of the nucleus and nucleons themselves. It should be noted that the quantum numbers for electrons and nucleons are, in fact, isolated systems separated by different laws of strong and electromagnetic interactions. A unified approach to determining the size of periods and orbitals proposed by PMP requires solving a rather complicated issue of combining electron and nucleon shells in a single quantum system. The principle of multilevel periodicity itself can be subsumed under the category and tasks of the Grand Unified Theory [1].

The main task of the Grand Unified Theories is to describe strong, weak, electromagnetic and gravitational interactions in a unified manner. A distinctive feature of unifying theories, among others, is the search for a suitable geometric formalism, in which there is a tendency towards modeling multidimensional simulation. A quantum field theory in the form of a standard model is the only experimentally confirmed theory for high-energy particles. The quantum field theory is used to study the behavior of physical systems with an infinitely large number of degrees of freedom. In this theory, the algebraic models of Hilbert spaces (n-dimensional analogues of a 3D Euclidean space) are used [2]. In string theory, a quantum string is a hypothetical infinitely thin one-dimensional object with the length of 10-35 m. All the elementary particles are formed by string vibrations. The nature of physical interaction is defined by a string vibration pattern. There are bosonic, fermionic, closed-open, super symmetric strings (superstrings), etc. A superstring is a hypothetical multidimensional object, which has ambiguous definition and, at the same time, contains super symmetry — the correspondence of weak and strong interactions.

The nucleus consists of many particles. It is difficult to accurately calculate or predict the behavior of nucleons via simulation of their behavior and interaction one at a time. At the same time, for the methods of statistical physics, the number of nucleons (hundreds) is too small, while the nucleons are structured by shells having different sizes, hierarchies and properties. Such conditions resulted in a sufficiently large number of nuclear models. Today, there are at least 12 models of atomic nucleus. Among them, there are three models describing the existence and parameters of shells inside the nucleus: the nuclear shell, cluster and statistical models. The nuclear shell model Maria Göppert-Mayer, Johannes Hans Daniel Jensen, Eugene Wigner the nuclear statistical model.

The disciplines and tools, using which the nuclear and atomic models are constructed, can be divided into three categories: physical (properties, results of experiments and observations, hypotheses), algebraic (calculations, mathematical models) and geometric (functional and configuration spaces, spatial models, visualization).

It should be noted that in the recognized and widespread models of the atomic and nuclear structure, physical and algebraic formalism is of predominant importance, while geometric formalism is of subordinate importance and is often underdeveloped or adjusted to the results of calculations and experiments. The existence of periodicity for the elements and isotopes suggests the existence of a fundamental cause for atom segmentation (division into periods and subgroups). The basic cause of quantization, the discreteness of which is reflected in quantum numbers, is also not quite clear. In natural phenomena, these causes should be found in the form of either a general structural pattern of matter in space and time or in the form of unknown binding interaction, unknown particle or something else. The search for particles and properties of interactions has been carried out rather successfully for a long time, but an empirical nature of the periodic system has not yet been "deflated". Knowledge on the structure and nature of time is extremely poor. In this regard, the search of a general physical system for describing the shells of electrons and nucleons should be carried out by means of quantum mechanics and in terms of a wave function for comparison with generally accepted results and visualization achievements. Moreover, not only physical, algebraic or geometric formalism should be used, but also a correspondence to this cause should be found in the patters existing in nature. Visualization is of great practical and academic importance, especially for finding correspondences in nature, as well as for the development of applied use. These are the criteria and main directions described in this article.

The examples for underdevelopment of geometric formalism and visualization methods in quantum mechanics and physics as a whole are the following:
When obtaining the wave function graphs for electron orbitals, the complex solutions are reduced to the real form. This can be done using artificial algebraic manipulations with coordinates, such as squaring, reduction to the sum or difference of squares: z^2, x^2-y^2, etc. Otherwise, many wave functions simply could not be constructed.

A classical spherical coordinate system is constructed anisotropically. There is a specified Z axis and XOY plane. A horizontal angle is measured in this plane, and a vertical angle is measured either from the XOY plane or from the Z axis. However, there are other X, Y axes and other planes. While using this coordinate system, it is necessary to remember that 3D space is isotropic and all the directions are equal. This is one of the main properties of space.

The priority of one axis or plane can lead to errors and one-sided results when simulating spherically organized systems, where all the directions are independent. The fourth degree of freedom for a particle, introduced by the German physicist Wolfgang Pauli (later called the spin), made it possible to explain a number of phenomena: magnetism, splitting of spectral lines of a fine structure in a magnetic field. However, today there is no single approach to the geometry and visualization of 4D space, as well as to multidimensional spaces in general. These spaces are set parametrically in most cases.

The complex numbers determine the additional dimension in the "indefinite" direction of the imaginary axis. No method for representation of this axis together with the three real axes has been developed yet.

In almost all the sources, 4D Minkowski space **(figure 1)** is presented only in the form of two light cones of a special relativity theory, where one of the spatial coordinate axes is aligned with the time axis. This is actually a 3D image of one cone for simulated 4D space. In this space, a 3D object being under study is represented and described by parallel transfer, also in 3D space. The temporal dimension is not shown and it does not go into or out of the spatial dimension, but is understood. The theory is used to simulate 4 spatial dimensions only algebraically. It is impossible to get information on the structure of 4D space from this theory.
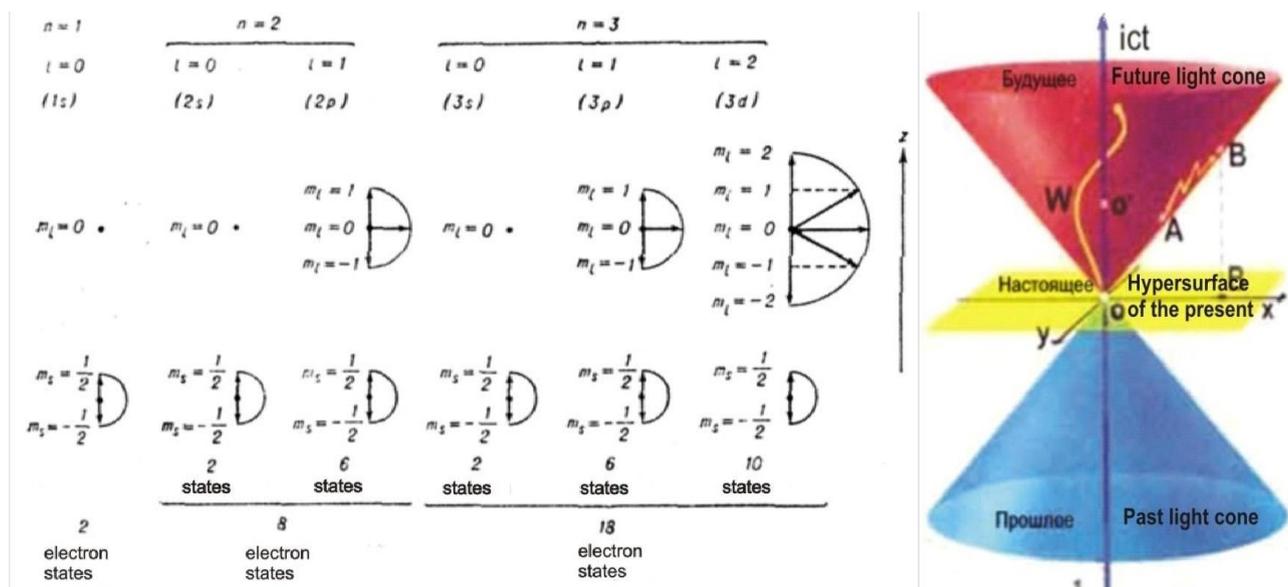

Figure 1. (a) Quantum numbers, electron angular; (b): Schematic view of 4DMinkowski space.

Different aspects of a more general picture for the structure of matter can be better described by different theories. There are several atomic models that can be historically highlighted: "Raisin bun" by J.J. Thomson Nuclear model by E. Rutherford Wave model by L. De Broglie Quantum mechanical model by E. Schrödinger Orbital model by G.

White Ring-shaped model (1963) by K. Snelson. The orbital and quantum-mechanical models are of the greatest practical importance in chemistry. These models most closely reproduce the periodic law of elements. In this regard, it is necessary to demonstrate how, using the principle of multilevel periodicity, by means of quantum mechanics, it is possible to construct the forms of nuclear orbitals, and within the framework of one system of quantum numbers, in a new way, to reproduce the forms of electron orbitals, proposing a theoretical unification of systems of electron shells and nuclei. And also, most importantly, to offer an explanation for the fundamental reason for the periodicity of chemical elements and isotopes.

A physical sense for the principle of multilevel periodicity should be considered in more detail. In this aspect, "one of the most excellent properties of quantum mechanics should be demonstrated – how much can be deduced from so little". The space for the filled shell of the orbital or period can be represented as a configuration space for N regions equal to the number of particles contained in this shell. In accordance with the principle of multilevel periodicity, N is equal to:

$$N = 2*(Cn1k1 + Cn2k2) \quad (1)$$

where Cn1k1 and Cn2k2 corresponding to the shell are the binomial coefficients of the Pascal's triangle (figure 2):
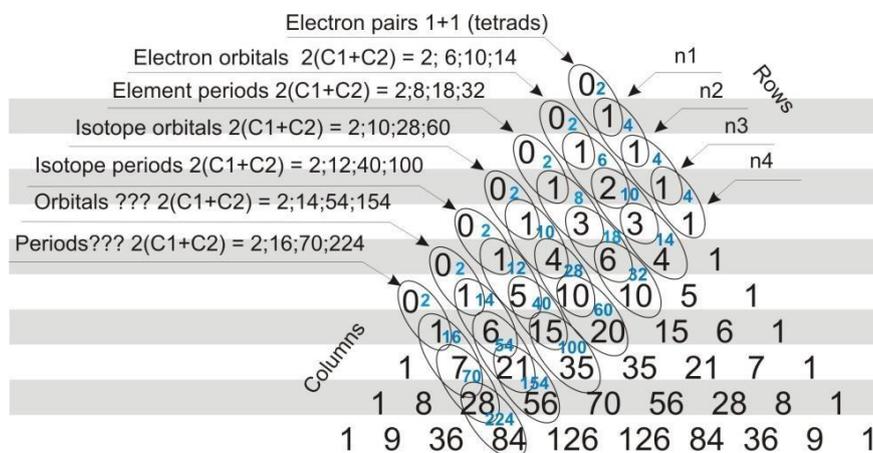

Figure 2. The Pascal's triangle; The principle of multilevel periodicity.

Binomial coefficients are calculated by several explicit formulas, for non-negative integers d and f there is the expression (2). In literature, as a rule, denotations n and k are used (2, on the left) for the formulas of binomial coefficients. To avoid confusion with the main quantum number n and the wave number k, the coefficients in (2) are denoted as n = d and k = f (2, on the right).

$$C\_n^k = (n¦k) = n!/k!(n-k)!; \quad C\_d^f = (d¦f) = d!/f!(d-f)! \quad (2)$$

where d = 0, 1, 2, 3, 4 … and f = 0 … d are the numerical coefficients. The number of particles contained in the shell of the period and orbital for electrons and nucleons is as follows:

$$N = 2*(Cd1f1 + Cd2f2) \quad (3)$$

Since the principle of multilevel periodicity covers both electronic and nucleon shells, the experimental data should be analyzed for both types of particles. The changes in the coefficients d and f should be compared to each other depending on the shell type, and then the coefficients should be compared to the changes in the quantum numbers: the principal quantum number n and the orbital quantum number l.

Binomial coefficients should be analyzed in terms of two disciplines: geometry and combinatorics. Geometric approach is important for understanding spatial quantization and anisotropy of the wave vector k. Combinatorial approach is important for understanding the probabilistic picture within this interpretation of quantum numbers. The interpretation will be called binomial. From the point of view of geometry, the sum Cd1f1 + Cd2f2 (3) quantitatively shows the specific volume for the configuration space of the filled shell for N/2 pairs of particles, where the volume Vshell/2 (Cd1f1 + Cd2f2) corresponds to each particle at the point moment of time.

From the point of view of combinatorics, the binomial coefficients Cd1f1 and Cd2f2 (as well as multinomial ones) are the number of d combinations by f or the number of ordered partitions for the d-element set X1-Xd into Cdf unique subsets consisting of f elements.

It is important to pay attention to the term "unique". The Pauli fundamental principle also has the uniqueness property, whereby electrons (fermions as a whole) in the atom can have only a different set of quantum numbers, which is unique for each particle.

In formula (2), the numbers d and f are changed similar to the quantum numbers n and l (d= 0, 1, 2, 3, 4 … and f=0 … d; the principal quantum number n = 1, 2, 3, 4 and the orbital quantum number l=0 … n) with the only difference that there is a state d = 0 for d, and when the energy levels are numbered by n, the count is from 1; "0" is present for l (the state d = 0 is considered below, following the calculations).

In the classical format, one period of chemical elements corresponds to each n, one orbital corresponds to each l. In the binomial format, a subshell, which is included in the composition of two adjacent periods and orbitals of both electrons and nucleons, corresponds to each d and each f **(Table 1)**. There are important differences in the correspondence of numbers to the shells: in the binomial interpretation, two pairs of numbers d1, f1 and d2, f2 correspond to each shell, as the number of particles for each shell consists of twice the sum (particle pairs) of two binomial coefficients.

One pair of quantum numbers n and l corresponds to each orbital in the classical format. It follows that from the point of view of the binomial interpretation, each shell, orbital or period, consists of two subshells. At a first glance, this complication may look like regression. However, using this format, being the binomial analog of quantum numbers d and f, the forms of the orbitals for both the electrons of the atom and the nucleons of the nucleus can be constructed and their descriptions can be combined within one theoretical model.

It should be assumed that the coefficients d and f have geometric sense for the degrees of freedom. This assumption is made on the basis of the following two features:

While observing the increase in the number of particles N from shell to shell, it can be shown that in the configuration space, d and f can be presented by directions. If it is assumed that the particles are fixed in the shell at the point moment of time, a straight line can be drawn between the pair particles (for example, electrons with opposite spin) through the center of an atom; under the influence of Coulomb repulsive forces, the charged particles tend to be evenly distributed in the configuration space of the shell (period or orbital). Therefore, these lines will be distributed equidistantly;

This assumption is also true on the basis of the probability function for the binomial distribution (4), where d and f are the exponents, and therefore, in geometric sense, d and f are the degrees of freedom.

This assumption should be verified using the quantum-mechanical calculations. As applied to the wave function, d and f can be previously defined as the number of degrees of freedom for the configuration space.

The integer random variable X has the binomial distribution, if the probability of its possible values is calculated by the Bernoulli equation:

$$P\_k = P(X=k) = \sum_{(f=0)}^{d} \llbracket C\_d^f\, p^f\, q^{((d-f))} = \llbracket (p+q) \rrbracket^d, (f=0,1,2,..,d) \rrbracket \quad (4)$$

where p is the probability of event with the value 1 "success" and q with the value 0 "failure"; p+q=1; C_d^f is the binomial coefficient; X1, .., Xn is the random variable from the finite sequence of independent random variables having the same Bernoulli distribution. The Bernoulli distribution is a special case of binomial distribution, and binomial distribution, in turn, is the most common form of discrete distribution.

In accordance with the principle of multilevel periodicity, the shells either of electrons or nucleons are located immediately in two configuration spaces Rd1 and Rd2. f1 and f2 are the number of allowed degrees of freedom for the

pairs of particles in the shell of a particular type **(figure 1, table 1)**.

For example, from the point of view of this interpretation of quantum numbers, the p-orbital of electrons is located in two configuration spaces, in Rd1 and Rd2, where d1 = 1 and d1 = 2. Configuration space of the shell is a superposition of two spaces. This will be shown with examples and calculations.

As is known from the experiments, a photon (Jung experiment, 1803), electron (experiments of Davisson-Germer, 1927, Jensen, 1961) and other particles (fullerene-60, Zeilinger experiment, 1999) can behave like a wave or "corpuscle" depending on the presence or absence of additional external interaction (observer, measurement). Both cases should be analyzed as applied to the binomial interpretation of quantum numbers.

In, the diagram of a well-known double-slit experiment is shown. In the experiment, it is shown that the electrons diffracting through two slits behave like waves, and the interference pattern is shown on the screen. It is interesting to note that the intensity of lines on the screen is distributed in accordance with one of the rows of the Pascal's triangle or their sum and is proportional to the binomial coefficients. "Which line" is determined on the basis of the ratio of the distance between the slits and from the slits to the screen.

The magnetic quantum number (m = ± l) takes the following values: m=0, ±1, ±2, ±3. m shows the sign of the wave function for the electron orbital and its orientation in the coordinate system. The absolute value of m is changed similar to l and f: $|m|=f=l$ (l is the orbital quantum number). From the point of view of geometry, m (m = ±l ±f) corresponds to the half-axis of the direction of showing its sign.

In 1896, the Dutch physicist Peter Zeeman placed a device that is similar to a hydrogen lamp, but filled with hot sodium vapor, in a strong magnetic field. It was found that in the magnetic field, the number of lines in the emission spectra is increased. The spectra become complex, but it is shown that each p-line splits in a magnetic field into 3 new lines, each d-line splits into 5 lines, each f-line splits into 7 lines, and s-lines remain unchanged. The number of spectral lines for the electrons of different orbitals is equal to the sum of the binomial coefficients $C_{d1}^{f1} + C_{d2}^{f2}$, — the second diagonal row of the Pascal's triangle **(figure 1)**, the second column at f= 1 **(Table 1)**. For the electron orbitals, the number of spectral lines in the magnetic field is also equal to d1+d2, as $d = C_d^f$ for this type of shells [4].
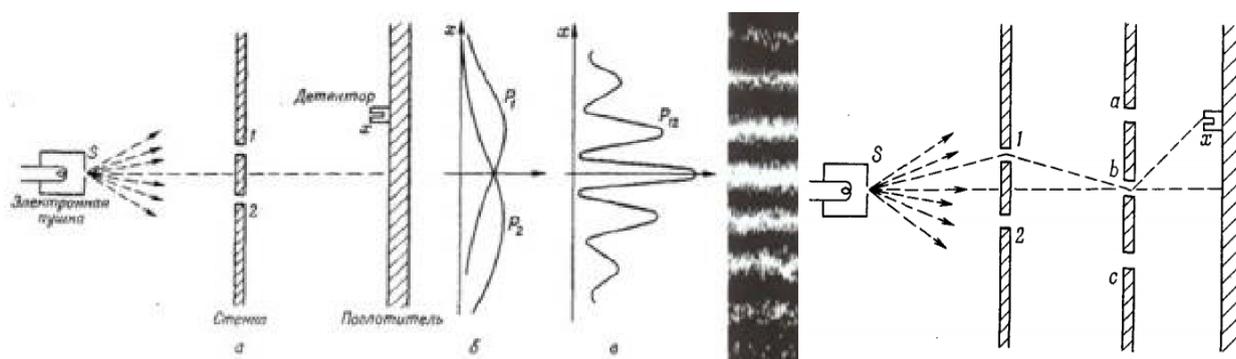

Figure 3. The interference experiment with electrons and the interference experiment with two walls.

The probabilistic aspect of this interpretation should be considered. In **figure 3**, the interference experiment on the electron passage through the two walls demonstrating the composition laws of the probability amplitude from the Feynman lectures on physics is shown: "Here are the two walls: one with two slits 1 and 2, and the other one with three slits a, b and c. There is a detector behind the second wall at the "x" point, and the amplitude of the probability that the particle reaches "x" should be found. One way of solution is to calculate the superposition or interference of the waves passing through the slits; but it can be done differently saying that there are six possible ways. The electron may pass through slit 1, then through slit "a" and then to "x", or it may pass through slit 1, then through slit "b" and then to "x", etc. In accordance with the probability theory, the amplitudes of mutually exclusive paths are summed up so that the amplitude of transition from s to x should be written as the sum of six separate amplitudes. On the other hand, in case of successive events, each of them can be written as a product of three amplitudes. For example, one of them is the amplitude of transition from s to 1 multiplied by the amplitude of transition from 1 to "a" and by the amplitude of transition from "a" to "x". Using our abbreviation, the full amplitude of transition from s to x can be written in the following form".

$$\langle x\,|\,s\rangle = \langle x\,|\,a\rangle\langle a\,|\,1\rangle\langle 1\,|\,s\rangle + \langle x\,|\,b\rangle\langle b\,|\,1\rangle\langle 1\,|\,s\rangle + \langle x\,|\,c\rangle\langle c\,|\,1\rangle\langle 1\,|\,s\rangle + \langle x\,|\,a\rangle\langle a\,|\,2\rangle\langle 2\,|\,s\rangle + \langle x\,|\,b\rangle\langle b\,|\,2\rangle\langle 2\,|\,s\rangle + \langle x\,|\,c\rangle\langle c\,|\,2\rangle\langle 2\,|\,s\rangle \quad (5)$$

The expression for the composition of probability amplitudes is similar to the expansion into a polynomial or sum of several polynomials, where the product is the composition of a chain of independent events, and the sum is the variety of chains for such events. The probability of two independent events is equal to the probabilities of these events.

The probability multiplication theorem can be generalized in case there is an arbitrary number of events. In general form, it is formulated as follows: the probability of the product of several events is equal to the product of probabilities for these events, and the probability of each subsequent event is calculated, if all the previous ones took place.

Therefore, in accordance with the Bernoulli's equation for the probability (4), as well as with the assumption that d and f have the meaning of the degrees of freedom, by the probability multiplication theorem, a particle is symmetrically distributed in space in several directions at the same time. The electron distribution with respect to several directions at the same time does not contradict the experimental data obtained as a result of electron passage through the two slits at the same time 2, when the electron behaves like a wave.

Using this explanation, the values for the parameters p^f and q^{d-f} in the Bernoulli's equation should be considered as applied to the geometric and combinatorial understanding of the binomial interpretation and the correspondence to the electron orbitals. From the point of view of geometry, in corpuscular representation of a particle for the filled shell, p^f is the total probability of a particle being close or symmetrical to the directions f in the configuration space R^d. Since the configuration space of the shell consists of the superposition of two spaces $d = d_1+d_2$, the total number of the directions f within the shell is $f= f_1+f_2$, and of the total number of the particles is 2 $(C_n^{1}k_1 + C_n^{2}k_2)$. q^{d-f} is the probability of the particle being close or symmetrical to the directions d-f not covered by the orbital or period shell. For example, in case of the pe-orbital: in accordance with the principle of multilevel periodicity, the sum of binomial coefficients corresponds to the pe-orbital: $C_1^0+C_1^2 = 1+2 = 3$. In other words, 3 pairs of electrons in the configuration space of $d_p= d_1+d_2= 1+2= 3$ directions, where each of the orbital branches is distributed in space along each of three directions equally distributed in space, are the axes OX, OY, OZ **(Table 1)**. At $f_1 = 0$, $d_1 = 1$. That is, for the first pair of electrons, the particles of the pe-orbital are freely distributed, being not adjacent and not symmetrical to any direction (f_1

= 0), and can take any direction during bond formation (d1=1). For the filled pe-orbital, each pair is adjacent or symmetrical to one (f = 1) of three directions (dp= d1+d2=1 + 2= 3) for the configuration space. In general, the result corresponds to the modern understanding of e spatial distribution for the filled pe-orbital. For the de-orbitals, there will be such directions: dd=d2+d3=2+3=5, f1=1 and f2=2 **(Table 1)**. That is, the first two pairs of electrons are distributed close or symmetrically to one direction each (f1 = 1); the next three pairs of electrons are each distributed with respect to 3 directions out of 5 (f2= 2); for the f-orbital, the configuration space has 7 specified directions: df=d3+d4=3+4=7, and 14 electrons are distributed as follows: 6 e is symmetrical to 2 directions at the same time, f1=2; 8 e is symmetrical to 3 directions at the same time, f2=3. Such a distribution is possible when the electron behaves like a wave. If this understanding of the probabilities pf and qd-f (4) is true, then the form of pe-orbitals can be represented using the coefficients d and f, just as it was done using the quantum numbers n, l, m. As the principle of multilevel periodicity covers the orbitals and periods of electrons and nucleons, it should be assumed that all the shell data can also be represented using the coefficients d and f. This understanding and assumption should be verified using calculations and constructions.

Using the classical problem of a particle in one-dimensional or three-dimensional potential well, the understanding of how the energy in the orbital is quantized in the direction from the nucleus is obtained depending on the quantum number n[4]. Using the binomial interpretation of the probability function, it is also possible to solve this problem and get the understanding of how the energy of the orbital or period shell is quantized both for electrons and nucleons. The solution to this problem should be considered using the binomial interpretation.

Within the rectangular potential well, the Schrödinger's steady-state equation is reduced to the following differential equation:

$\Psi''(x) + k2\ \Psi(x) = 0$

where k= $\sqrt{(2mE/\hbar^2)}$ – is the wave number or spatial frequency.

The general solution of the differential equation is found from the assumption that the probability function of finding a particle can be as follows: $\Psi(x)=e^{kx}$. After finding the roots of the characteristic equation, the general solution is obtained: $\Psi(x)=A\sin(kx)+B\cos(kx)$. Application of the boundary conditions $\psi(0)=0$ results in B=0, thus: $\Psi(x)=A\sin(kx)$.

In the binomial case, instead of assuming that $\Psi(x)=e^{kx}$, it should be assumed that location of the particle in the shell follows the binomial distribution. The binomial distribution tends to be normal, and the normal distribution is an exponential function. Taking into account the frequency of application in various problems, the binomial distribution can be considered to be the most common type of discrete distribution. The binomial probability function should be used as follows:

$\Psi(x) = p^f q^{(d-f)}$

Since $\Psi(x)$ takes the values 1 ("success") and 0 ("failure") with the probabilities p and q=1-p respectively, p+q=1 (4), the particle's motion is cyclic, and the functions sin(kx) and cos(kx) are in antiphase, they can be applied to the description of states 1 ("success") and 0 ("failure"). Applying the condition p+q=1 on the basis of the main trigonometric identity (sin2(kx)+cos2(kx)=1), it can be assumed that:

p = sin2(kx); q = cos2(kx).

When applying the condition, which follows from the Bernoulli's equation, that the total probability is $(p+q)d = 1d$, the equation for the probability function can be found simply by applying the basic trigonometric identity $(\sin 2(kx) + \cos 2(kx) = 1)$ being a circle equation $(x2+y2 = 12$, where: $x = \cos(kx)$; $y = \sin(kx))$:

$\Psi d(x) = (\sin 2(kx) + \cos 2(kx))d = 1d$

It is also the condition for normalization. If it is considered that k is not equal to 1 (unit), then taking into account the integration coefficients, in 1D case at d=1, the expression takes the following form:

$\Psi d(x) = A2\sin 2(kx) + B2\cos 2(kx) = 1$

The expression corresponds to the normalization condition for the wave function. The coefficients A and B are also squared, as the trigonometric functions have the degree 2.

The integration constants should be found from the following conditions:

For the "success" state: $\int_{(-r)}^{r} A^2 [\![\sin]\!]^2 (kx)dx=1$; $A= \sqrt{(2\&2/r)}$

For the "failure" state: $\int_{(-r)}^{r} B^2 [\![\cos]\!]^2 (kx)=0$; $B = 0$

Thus, the probability of detecting a particle ("success") in the binomial interpretation is $\Psi f(x) = pf$:

$\Psi f(x) = \sqrt{(2\&2/r)}\sin 2(kx)$

where k is the spatial frequency [rad/m] or wave number.

Solution for a particle in 1D potential well with infinitely high walls, obtained using the binomial interpretation for 1D case, is somewhat different from the classical one in the form, but not in the essence. In "binomial" case, the expression for the radial probability function is obtained not by solving the differential equation with complex roots, but directly from the Bernoulli's probability equation taking into consideration the probability multiplication theorem. In classical case, energy quantization is shown by the function $\sin(kx)$; for the binomial interpretation, quantization can be shown by the exponential function $\sin 2(kx)$, which matches the points of the boundary conditions more asymptotically. In other words, the probability change can be described more accurately. No additional module function application (abs) is required to provide positive probability at negative kx angles. In addition, the normalization condition for binomial interpretation is the Newton binomial itself $(p+q)d=1d$ for $p+q=1$. In classical case, the normalization condition is as follows:

$A^2 \int_{V} [\![\Psi\Psi^*]\!] dV=1$

where Ψ is the wave function, $\Psi *$ is its complex conjugation, V is the volume being studied.

It is believed that the wave function itself has no physical meaning, but physical meaning is attributed to the square of its module $[\![|\Psi(x_1,x_2,...,x_n,t)|]\!]^2$. For the binomial interpretation in the Bernoulli's equation, physical meaning of the probability function (4) follows from the probability multiplication theorem for independent or sequential events (and/and). The function Ψ itself is the mathematical description of wave behavior in space, where the probability and normalization condition depend on the degree of freedom for the system d of $(p+q)d=1d$. In the multidimensional case, the Bernoulli's equation or Newton binomial $(p+q)d = pd + C\_d^f p^f q^{(d-f)}+ ...+qd$ is the expansion into the sum of summands, where, in accordance with the addition theorem on probability, each one is the description of probabilities for incompatible events (in accordance with the scheme or/or). As applied to quantum objects, the multidimensional case of the Bernoulli's equation reproduces the principle of quantum superposition.

The principle involves an idea that if the system is in the state described by the wave functions Ψ1 and Ψ2, it can also be in the state described by the wave function Ψ = C1Ψ1 + C2 Ψ2 for any complex Cn.

Obviously, it can be said about the superposition (addition) of any number of quantum states, i.e. about the existence of quantum state for the system, which is described by the wave function:

$$\Psi\_\Sigma = C\_1\,\Psi\_1 + C\_1\,\Psi\_1 + \ldots + C\_n\,\Psi\_n$$

Therefore, due to the Bernoulli's equation for describing the probability of quantum events, two expressions from the quantum theory are reproduced: the normalization condition and the principle of quantum superposition.

Then, in accordance with the principles of multilevel periodicity and quantum superposition, the superposition ofthe following functions corresponds to each shell of electrons or nucleons:

$$\Psi\_d^{\wedge}f = 〖2(C〗\_{d1}^{\wedge}f1\,\Psi\_{df1} + C\_{d2}^{\wedge}f2\,\Psi\_{df2})$$

where Ψdf1,2 = pfqd-f, C_d^f is the binomial coefficient, and the numbers d and f correspond to the shell. In the geometric sense, the Bernoulli's equation corresponds to the system with d degrees of freedom or to the d- dimensional configuration space (d from "dimension") for the given f. One of the shell types (orbital or period) for both electrons and nucleons corresponds to f **(Table 1)**.

The probability of finding the particle at any point in space depends on both the radial and angular parts of the wave function:

$$\Psi\_{nlm} = 〖R\_{nl}\,(r)\,Y〗\_{lm}\,(\Theta, \varphi)$$

As it is shown in the example of a particle in one-dimensional potential well, an idea of energy quantization in the radial direction, i.e. from the nucleus, can be gained using the binomial interpretation. It is interesting to note that the binomial interpretation of quantum numbers also allows obtaining the angular components of the wave functions.

For the three-dimensional case, 10 coefficients are obtained, part of the coefficients Cijk is dropped and there remain only 3 coefficients which are independent and correspond to the x, y, z coordinates in the Laplace equation[4]. For the spherical function corresponds to the trinomial extension (x+y+z)n, which, in the combinatorial sense, is similar to the Bernoulli's equation (4) for (p+q)n and differs in the number of terms and variables. The non-use of coordinates at this stage of calculation allows obtaining general solution for the probability in the one-dimensional case for different systems and combinations of coordinates. Application of the binomial expansion for the spherical functions where the variables are:

1. angles, because two angles are enough to set the direction in 3D space;
2. Cartesian coordinates in the projection plane of the wave function Ψ.

Using the Schrödinger equation for the steady-state conditions, the calculations should be made for the angular component of the spatial frequency k in the general form, for the one-dimensional case and all the shell types of the electrons and nucleons. The one-dimensional case is analyzed to facilitate the understanding on the basis of the experience that most of the solutions for the differential equation will be complex, and for them, in the classical case, when representing atomic orbitals in 3D space, the imaginary part is reduced to the real part by artificial algebraic

manipulations, and also for a number of reasons outlined below. k can be calculated for two cases of probability functions, when:

1. p = sin(kα); q = cos(kα) – for the condition p2 + q2 = 12, to determine the spatial arrangement of d and f, since such a condition corresponds to the equation of the unit circle in the rectangular coordinate system, which meansthat the solution will be spatially more accurate.

2. p = sin2 (kα); q = cos2 (kα) – for the condition p + q = 1, for a more accurate determination of the spatial probability distribution on the basis of the previous problem.

The first case should be considered. In accordance with the Bernoulli's equation and the principle of multilevel periodicity, the general expression of the probability function for the orbitals and periods of electrons and nucleons in the 1st case is as follows:

Ψ_d^f= 〖2(C〗 _d1^f1 Ψ_df1+C_d2^f2 Ψ_df2)= 〖2(C〗 _d1^f1 p^f1 q^(d1-f1)+C_d2^f2 p^f2 q^(d2-f2)

The probability function for the orbitals and periods of electrons and nucleons consists of two summands and each summand is included in the expression for two shells, the Schrödinger equation should be solved and k should be found for each of the summands, in order to then substitute into expressions for different shells:

Ψ_d^f=C_d^f p^f q^(d-f)= C_d^f sin^f (kα) 〖cos〗 ^(d-f) (kα)

By finding the derivatives Ψ_d^f, differentiation is performed with respect to the angle α:

〖Ψ_d^f〗 ^'= C_d^f 〖k(sin〗 ^(f-1) (kα) 〖cos〗 ^(d-f+1) (kα)-(d-f)sin^(f+1) (kα) 〖cos〗 ^(d-f-1) (kα))

〖Ψ_d^f〗 ^''= C_d^f 〖k^2 _((f(f-1)) sin〗 ^(f-2) (kx) 〖cos〗 ^(d-f+2) (kα)-_(_(f(d-f+1))) sin^f (kx) 〖cos〗 ^(d-f)
  (kα)-

| D1   E1 |
〖-_((d-f)(f+1)) sin〗 ^f (kα) 〖cos〗 ^(d-f) (kx)+_((d-f)(d-f-1)) sin^(f+2) (kα) 〖cos〗 ^(d-f-2) (kα))   F1
                                        G1

The variable should be replaced for Ψ_d^f (28) and 〖Ψ_d^f〗 ^'', where:

sin^f (kα)=c ;   √(f&c)=sin⁗(kα); D1 = f(f-1);   E1 = f(d-f+1);

〖cos〗 ^(d-f) (kα)=e;   √(d-f&e)=cos⁗(kα);   F1 = (d-f)(f+1); G1 = (d-f)(d-f-1).

As a result of Ψ_d^f (28) and 〖Ψ_d^f〗 ^'' the Schrödinger equation (11), takinginto account the change of variables, the following is obtained:
C_d^f k^2 (D_1 e^2/c^2 - E_1 ce-F_1 ce+G_1 c^2/e^2)+k^2 ce=0

(C_d^f D_1 e^4- 〖 〖(C〗 _d^f E〗 _1+C_d^f F_1- 〖1)c^3 e^3+C_d^f G〗 _1 c^4)/(c^2 e^2)=0

As a result of simplification, the general form of the characteristic equation for finding kα from the steady-state Schrödinger equation in the binomial interpretation of the quantum numbers d and f is obtained:

C_d^f D_1 e^4- 〖 〖(C〗 _d^f E〗 _1+C_d^f F_1- 〖1)c^3 e^3+C_d^f G〗 _1 c^4=0

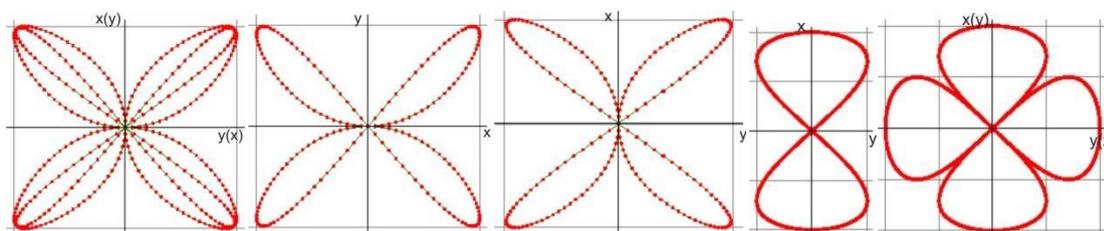C_d^f D_1 〖cos〗 ^(4(d-f)) (kα)- 〖 〖(C〗 _d^f E〗 _1-C_d^f F_1+ 〖1)sin^3f (kα) 〖cos〗 ^(3(d-f)) (kα)+C_d^f G〗 _1 sin^4f (kα)=0

The coefficients C_d^f, D1, E1, F1, G1 are determined using d and f from (2). Thus, for the binomial interpretation of quantum numbers, in the given characteristic equation for the probability of finding a particle for all the shell types, the angular component of the spatial frequency or wave number k is completely determined by the coefficients d and f. Since the principle of multilevel periodicity (PMP) covers the shells of electrons and nucleons, in this case it is possible to construct nucleon orbitals. Since this is being done for the first time, it is necessary to check the correctness of the chosen approach on electron shells[5].

Table 2. Coefficients, probability functions and characteristic equations for finding the angular component of the spatial frequency – kα, for the binomial interpretation of quantum numbers d and f.

Shell		d	f	d-f	Trigonometric form of the angular function Ψ:	CD_1	CE_1	CF_1	$CE1+CF_{1-1}$	CG_1	Characteristic equation
					e orbitals						
s_e		1	1	1	0	$\sin(k_2\alpha)$	0	1	0	0	0	
d_e	p_e	1	1	0	1	$\cos(k_2\alpha)$	0	2	1	2	0	$-2c^3e^3=0$
		2	2	1	1	$2\sin(k_6\alpha)\cos(k_6\alpha)$	0	4	4	7	0	$-7c^3e^3=0$
	f_e	3	3	2	1	$3\sin^2(k_{10}\alpha)\cos(k_{10}\alpha)$	6	6	9	14	0	$6e^4-14c^3e^3=0$
		4	4	3	1	$4\sin^3(k_{14}\alpha)\cos(k_{14}\alpha)$	24	8	16	23	0	$24e^4-23c^3e^3=0$
						e periods						
2e	8e	1	2	0	2	$\cos^2(k_2\alpha)$	0	3	2	4	2	$-4c^3e^3+2c^4=0$
18e		3	3	1	2	$3\sin(k_8\alpha)\cos^2(k_8\alpha)$	0	9	12	20	6	$-20c^3e^3+6c^4=0$
	32e	6	4	2	2	$6\sin^2(k_{18}\alpha)\cos^2(k_{18}\alpha)$	12	18	36	53	12	$12e^4-53c^3e^3+12c^4=0$
		10	5	3	2	$10\sin^3(k_{32}\alpha)\cos^2(k_{32}\alpha)$	60	30	80	109	20	$60e^4-109c^3e^3+20c^4=0$
						Nucleon orbitals (np)						
sя	sя	1	3	0	3	$\cos^3(k_2\alpha)$	0	4	3	6	6	$-6c^3e^3+6c^4=0$

2	10											
dя		4	4	1	3	$4\sin(k_{10}\alpha)\cos^3(k_{10}\alpha)$	0	16	24	39	24	$-39c^3e^3+24c^4=0$
28	fя	10	5	2	3	$10\sin^2(k_{28}\alpha)\cos^3(k_{28}\alpha)$	20	40	90	129	60	$20e^4-129c^3e^3+60c^4=0$
	60	20	6	3	3	$20\sin^3(k_{60}\alpha)\cos^3(k_{60}\alpha)$	120	80	240	319	120	$120e^4-319c^3e^3+120c^4=0$
						Nucleon periods (np)						
2 np	12 np	1	4	0	4	$\cos^4(k_2\alpha)$	0	5	4	8	12	$-8c^3e^3+12c^4=0$
40 np		5	5	1	4	$5\sin(k_{12}\alpha)\cos^4(k_{12}\alpha)$	0	25	40	64	60	$-64c^3e^3+60c^4=0$
	100 np	15	6	2	4	$15\sin^2(k_{40}\alpha)\cos^4(k_{40}\alpha)$	30	75	180	254	180	$30e^4-254c^3e^3+180c^4=0$
		35	7	3	4	$35\sin^3(k_{100}\alpha)\cos^4(k_{100}\alpha)$	210	175	560	734	420	$210e^4-734c^3e^3+420c^4=0$

As in the radial case, the functions Ψ_d^f for the angular component of the spatial frequency k are obtained from the Bernoulli's equation of probability. Using these functions, the general form for the projections of orbitals and periods for electrons and nucleons can be obtained. While implementing the orthogonality condition for the directions of the coordinate system, for construction it is enough to set each of the functions to one of the coordinates in the plane. The 2D projections are considered because of the disadvantages of the spherical coordinate system (see below). Since the correspondence of the function for a particular axis is chosen arbitrarily, the condition of direction independence in space should be implemented. Therefore, to obtain the projection of the orbital for a given plane, the spatial conjugation of functions, for which the axes are interchanged, should be shown **(figure 7)**. Considering that the electron energy is determined by the Coulomb interaction and the distance from the nucleus, and the orbital size (scale) is determined by the main quantum number, it is enough to construct the angular part of the wave function Ψ_d^f to reveal the form of the shell[6].

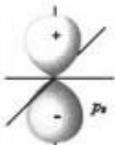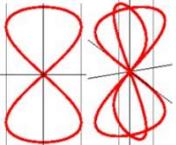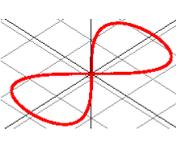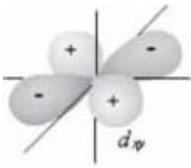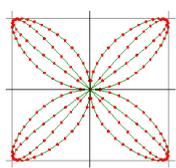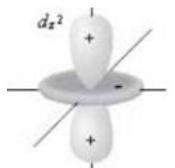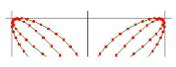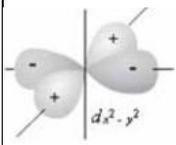**Figure 3.** The dexy- electron orbital (a); spatial conjugations of the de-orbital (XY-b; YX-c) for the Ψ_d^f functions **(40)** (Tables 2 and 3); the pexy - electron orbital (XY-d) and spatial conjugations of the pe-orbital (XY and YX-d).

The coefficients A are calculated by applying the normalization condition (19) in accordance with the solutions of the characteristic equation for the given Ψ_d^f. Spatial conjugation for the pe-orbital is the orbital projection along the perpendicular axes. It corresponds to the "tendency" of the system to the state of maximum multiplicity in accordance with the Hund rule.

Table 3: Comparison of the electron angular function types, which are of greatest interest in chemistry, for the classical case (l, m)[5] and binomial interpretation of quantum numbers (d, f).

l	m	Orbital	φ)	Form	d	f		Form
0	0	s	$\dfrac{1}{2\sqrt{\pi}}$		1	0	{ (36)	
					1	1		
	±1*	p_y	$\dfrac{\sqrt{3}}{2\sqrt{\pi}}\sin\theta\cdot\sin\varphi$		1	0	{	
					1	2	(37)	
					1	0		

1	0	p_z	$\dfrac{\sqrt{3}}{2\sqrt{\pi}}\cos\theta$		1	2	{ (38)	
	±1*	p_x	$\dfrac{\sqrt{3}}{2\sqrt{\pi}}\sin\theta\cdot\cos\varphi$		1 / 1	0 / 2	{ (39)	
2	±2*	d_{xy}	$\dfrac{\sqrt{15}}{4\sqrt{\pi}}\sin^2\theta\cdot\sin^2\varphi$		2 / 3	1 / 2	{ (40)	
	±1*	d_{yz}	$\dfrac{\sqrt{15}}{4\sqrt{\pi}}\sin^2\theta\cdot\sin\varphi$		2 / 3	1 / 2	{ (41)	
	0		$\dfrac{\sqrt{5}}{4\sqrt{\pi}}(3\cos^2\theta-1)$					
	±1*	d_{xz}	$\dfrac{\sqrt{15}}{4\sqrt{\pi}}\sin^2\theta\cdot\cos\varphi$		2 / 3	1 / 2	{ (42)	
	±2*		$\dfrac{\sqrt{15}}{4\sqrt{\pi}}\sin^2\theta\cdot\cos^2\varphi$					

* In the classical case, the quantum number m is indefinite, as the reducible real-value functions are obtained artificially by a linear combination of imaginary functions with the selected values of m.

As it is shown in (Table 3), the form of the probability functions for the se, pe, de-orbitals, obtained due to the quantum numbers l, m and spherical coordinate system (spherical harmonics are the classical case), coincides with the projections of the probability functions for the same shells obtained due to the binomial interpretation of the quantum numbers d and f constructed in a Cartesian coordinate system.

The solutions for characteristic equations that arise when solving the steady-state Schrödinger equation for various shells of electrons and nucleons in the binomial interpretation should be analyzed. First, the solutions for electron orbitals should be analyzed. In accordance with the principle of multilevel periodicity (PMP), the number of

particles for each shell consists of two summands Ψ_df (subshells). The equations for all the summands have solutions; for the se and pe orbitals there are only real roots. All other summands have both real and imaginary roots.

For all the se, pe and de summands, the real part of the roots for the characteristic equation is symmetric with respect to the origin of coordinates. Between the symmetric roots $k\alpha$, the straight lines intersecting at the origin can be drawn. It is shown that in the configuration space of this shell, there are directions of axial symmetry in the central field of the atom. The number of directions for the unit circle is equal to the binomial coefficient C_d^f or number d corresponding to this shell and equation. Upon reaching the number of directions $d \geq 3$, i.e. more than the number of dimensions for 3D space, the central symmetry is broken. It should be assumed that the roots of the Schrödinger equation have central symmetry at d being equal to the number of coordinates analyzed in the equation. Also, it should be assumed that the location of the axes in the coordinate system with d(n)=4 is different from the Cartesian coordinate system. Therefore, the central symmetry is broken. This result can be taken into account as a confirmation of the assumption that d and f have the geometric sense for the degrees of freedom.

The angular part of the spatial frequency k depends on the direction α. The dependence of k on α is nonlinear. However, for all the summands and shells, it has a similar hyperbolic and layered nature (fig. 8), where the hyperbole family is distanced from the origin as n is increased. n is the analogue of the main quantum number for the binomial interpretation. For electrons and nucleons, the principal quantum number should be divided into ne and nn by the correspondence of radius and scale of the shell distance. On the basis of the graph, there is a mutually proportional fluctuation of the values for the ratio of k and α. Near $\alpha = 0$, the value of the spatial frequency k is increased significantly, and theoretically it can be equal to infinity. At the same time, the value of the angle α for the spatial cycle of particle fluctuations is finite. Therefore, the graph for the dependence of α on k jumps abruptly through the ordinate axis at the point: $\alpha = n\pi$, where $k \rightarrow 0$ in the one-dimensional case and $\alpha = n\pi/2$ in the two- dimensional case. The value of the vector k has spatial anisotropy. It follows that to calculate the spatial frequency, it is important to select the coordinate system, location of the axes and, at the same time, differentiation directions when choosing the operator. For objective estimation, it is important that the coordinate system corresponds to the structure of the configuration space.

Thus, for the binomial interpretation, the particle position, without taking into account the spin, can be set using 3 quantum numbers: n is the analogue of the main quantum number; d is a number showing the total number of degrees of freedom in the configuration space of the subshell for the orbital or period; taking into account that f varies from shell to shell: f = 0 ... d, then f is a number showing the number of allowed degrees of freedom in the configuration space of a particular subshell for the orbital or period. It should be assumed that the periodic change in the atomic shells is a consequence of the atomic configuration space evolution from f to d. As the number is increased, the division of periods into a larger number of orbitals should also be considered as a consequence of the configuration space evolution. Thus, if the atom is considered to be the system with varying degrees of freedom, the atomic electron shells exist in the configuration space with the dimensions from 0 to 5, and nucleons from 2 to 7.

DISCUSSION AND CONCLUSION

The preliminary results should be summarized. As a result of calculations and constructions, it is shown that:

the principle of multilevel periodicity (PMP) suggests a single algorithm for determining the size of electron and nucleon shells for both periodic systems of elements and isotopes.

For the binomial interpretation of quantum numbers, the expression for radial probability function can be obtained not only by solving the differential equation, but also directly from the Bernoulli's equation of probability using

the probability multiplication theorem. The Bernoulli's equation to a greater extent corresponds to the spatial (geometric) formalism, when the exponent determines the degree of freedom for the system, and the same formula corresponds to the probability multiplication theorem. Therefore, it should be assumed that the distribution of particles in the shells can be caused to a greater extent by the properties of the configuration space inside the atom.

If the atom is considered to be the system with varying degrees of freedom, then in accordance with the principle of multilevel periodicity, the shells of atomic electrons exist in the configuration space from 0 to 5 dimensions, and the shells of nucleons exist in the configuration space from 2 to 7 dimensions.

It is shown that the spherical function of the degree n and the binomial probability function have one type of expression and are the homogeneous polynomials: $(x + y + z)$ n and $(p + q)$ d. Due to the spherical function of the degree n, the probability distribution for finding a particle (orbital) is constructed. In the expression for the spherical function, the value of the shell is determined by the coefficient $C_{(i,j,k)}$. It follows that the principle of multilevel periodicity, in which the binomial coefficients C_d^f are used for $(p+q)d$, when determining the size and shape of the electron and nucleon shells, is presented in the same format of reasoning as the classical case. Therefore, it has a predictive potential not only in the empirical aspect as the periodic system of isotopes, but also in the theoretical aspect.

The projections of the graphs for the probability functions of the se, pe, de-orbitals obtained using the quantum numbers l, m in the spherical coordinate system (spherical harmonics are the classical case) coincide with the projections of the probability functions for the same shells obtained using the binomial interpretation of the quantum numbers d and f, in the Cartesian coordinate system. This shows the viability of this interpretation.

Using the binomial interpretation of the quantum numbers n, d, f and the principle of multilevel periodicity, the projections of the wave functions for orbitals and nucleon periods are simulated. The form of 2D projections for the nucleon orbitals is similar to that of the projections for the electronic orbitals; the only exception is the sn-orbital.

The results of calculations and the assumption that the distribution of particles on shells is due to the properties of configuration spaces inside the atom makes it necessary to search for the causes of the emergence of such spaces, as well as their manifestations in nature ("Multidimensional interference model of intra- atomic space. Hypotheses of the principle multilevel periodicity"). As a result of the analysis, it is shown that for CSCS, there is the specified Z axis and the plane of the horizontal XOY angle. Due to this, the system is anisotropic and this affects the simulation results. Two alternative variants of the spherical coordinate systems are proposed. The construction of 3D models for orbitals and periods in these coordinate systems is a rather extensive study. These models will be presented separately.

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