

Regular α Generalized Open Sets In Intuitionistic Fuzzy Topological Spaces

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ABSTRACT: The purpose of this paper is to introduce and study the concept of intuitionistic fuzzy regular α generalized open sets in intuitionistic fuzzy topological spaces. We investigate some of their properties and also we introduce intuitionistic fuzzy regular α $T_{1/2}$ space and obtain some characterizations and several preservation theorems.

KEYWORDS: Intuitionistic fuzzy set, Intuitionistic fuzzy topology, Intuitionistic fuzzy topological space, Intuitionistic fuzzy regular α generalized open set, Intuitionistic fuzzy regular α $T_{1/2}$ space.

I. INTRODUCTION

The theory of fuzzy sets was introduced by Zadeh [10] in 1965. Later, Chang [2] proposed fuzzy topology in 1967. The concept of intuitionistic fuzzy sets, introduced by Atanassov [1] is a generalization of fuzzy sets. Using the notion of intuitionistic fuzzy sets, Coker [3] introduced the notion of intuitionistic fuzzy topological spaces in 1997. In this paper, we introduce intuitionistic fuzzy regular α generalized open set. We investigate some of their properties. We also introduce intuitionistic fuzzy regular α $T_{1/2}$ space and obtain some characterizations and several preservation theorems.

II. PRELIMINARIES

Definition 2.1: [1] An intuitionistic fuzzy set (IFS in short) A in X is an object having the form $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ where the function $\mu_A : X \rightarrow [0,1]$ and $\nu_A : X \rightarrow [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A , respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.

Definition 2.2: [1] Let A and B be two IFSs of the form $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ and $B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle / x \in X \}$. Then

- (i) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$
- (ii) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$
- (iii) $A^c = \{ \langle x, \nu_A(x), \mu_A(x) \rangle / x \in X \}$
- (iv) $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle / x \in X \}$
- (v) $A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle / x \in X \}$

For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A \rangle$ instead of $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$. The IFS $0 \sim = \{ \langle x, 0, 1 \rangle / x \in X \}$ and $1 \sim = \{ \langle x, 1, 0 \rangle / x \in X \}$ are respectively the empty set and the whole set of X .

Definition 2.3: [3] An intuitionistic fuzzy topology (IFT in short) on X is a family τ of IFS in X satisfying the following axioms:

- (i) $0 \sim, 1 \sim \in \tau$
- (ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$
- (iii) $\cup G_i \in \tau$ for any family $\{ G_i / i \in J \} \subseteq \tau$

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X . The complement A^c of an IFOS A in (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X .

Definition 2.4: [3] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X . Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by

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(i) $\text{int}(A) = \cup \{ G / G \text{ is an IFOS in } X \text{ and } G \subseteq A \}$

(ii) $\text{cl}(A) = \cap \{ K / K \text{ is an IFCS in } X \text{ and } A \subseteq K \}$

Note that for any IFS A in (X, τ) , we have $\text{cl}(A^c) = (\text{int}(A))^c$ and $\text{int}(A^c) = (\text{cl}(A))^c$.

Definition 2.5: [5] An IFS A in an IFTS (X, τ) is said to be

(i) intuitionistic fuzzy semiopen if $A \subseteq \text{cl}(\text{int}(A))$

(ii) intuitionistic fuzzy α open if $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$

(iii) intuitionistic fuzzy preopen if $A \subseteq \text{int}(\text{cl}(A))$

(iv) intuitionistic fuzzy regular open if $A = \text{int}(\text{cl}(A))$

Definition 2.6: [4] An intuitionistic fuzzy point (IFP in short), written as $p_{(\alpha, \beta)}$, is defined to be an IFS of X given by

$$p_{(\alpha, \beta)}(x) = \begin{cases} (\alpha, \beta) & \text{if } x = p, \\ (0, 1) & \text{otherwise.} \end{cases}$$

An IFP $p_{(\alpha, \beta)}$ is said to belong to a set A if $\alpha \leq \mu_A$ and $\beta \geq \nu_A$

Definition 2.7: [7] An IFTS (X, τ) is said to be an $\text{IFT}_{1/2}$ space if every IFGCS in (X, τ) is an IFCS in (X, τ) .

Definition 2.8: [8] Let A be an IFS in an IFTS (X, τ) . Then

(i) $\alpha\text{int}(A) = \cup \{ G / G \text{ is an IF}\alpha\text{OS in } X \text{ and } G \subseteq A \}$

(ii) $\alpha\text{cl}(A) = \cap \{ K / K \text{ is an IF}\alpha\text{CS in } X \text{ and } A \subseteq K \}$

Result 2.9: [8] Let A be an IFS in (X, τ) . Then

(i) $\alpha\text{cl}(A) = A \cup \text{cl}(\text{int}(\text{cl}(A)))$

(ii) $\alpha\text{int}(A) = A \cap \text{int}(\text{cl}(\text{int}(A)))$

Definition 2.10: [9] An IFS A in an IFTS (X, τ) is intuitionistic fuzzy α generalized closed set (IF α GCS in short) if $\alpha\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X .

Definition 2.11: [6] An IFS A of an IFTS (X, τ) is called intuitionistic fuzzy regular α generalized closed set (IFR α GCS in short) if $\alpha\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFROS in X .

III. INTUITIONISTIC FUZZY REGULAR α GENERALIZED OPEN SETS

In this section we introduce the notion of intuitionistic fuzzy regular α generalized open sets and study some of their properties.

Definition 3.1: An IFS A of an IFTS (X, τ) is called intuitionistic fuzzy regular α generalized open set (IFR α GOS in short) if $\alpha\text{int}(A) \supseteq U$ whenever $A \supseteq U$ and U is an IFRCS in X .

The family of all IFR α GOSs of an IFTS (X, τ) is denoted by $\text{IFR}\alpha\text{GO}(X)$.

Note that the complement A^c of an IFR α GCS A in an IFTS (X, τ) is an IFR α GOS in X .

Example 3.2: Let $X = \{a, b\}$ and let $\tau = \{0\sim, G_1, G_2, 1\sim\}$ where $G_1 = \langle x, (0.6, 0.7), (0.4, 0.2) \rangle$ where $\mu_a=0.6, \mu_b=0.7, \nu_a=0.4, \nu_b=0.2$ and $G_2 = \langle x, (0.1, 0.2), (0.7, 0.7) \rangle$ where $\mu_a=0.1, \mu_b=0.2, \nu_a=0.7, \nu_b=0.7$ let $A = \langle x, (0.6, 0.7), (0.3, 0.1) \rangle$ be any IFS in (X, τ) . Then $A \supseteq U$ where $U = \langle x, (0.4, 0.2), (0.6, 0.7) \rangle$ is an IFRCS in X . Now $\alpha\text{int}(A) = \langle x, (0.6, 0.7), (0.4, 0.2) \rangle \supseteq U$. Therefore A is an IFR α GOS in (X, τ) .

Theorem 3.3: Every IFOS, IFROS and IF α OS is an IFR α GOS but the converses are not true in general.

Proof: Straight forward.

Example 3.4: Let $X = \{a, b\}$ and let $\tau = \{0\sim, G_1, G_2, 1\sim\}$ where $G_1 = \langle x, (0.6, 0.5), (0.4, 0.2) \rangle$ and $G_2 = \langle x, (0.2, 0.1), (0.8, 0.8) \rangle$. Let $A = \langle x, (0.6, 0.6), (0.3, 0.2) \rangle$ be any IFS in X . Then $A \supseteq U$ where $U = \langle x, (0.4, 0.2), (0.6, 0.5) \rangle$ is an IFRCS in X . Now $\alpha\text{int}(A) = \langle x, (0.6, 0.5), (0.4, 0.2) \rangle \supseteq U$. A is an IFR α GOS but not an IFOS in X , since $\text{int}(A) = \langle x, (0.6, 0.5), (0.4, 0.2) \rangle \neq A$.

Example 3.5: Let $X = \{a, b\}$ and let $\tau = \{0\sim, G_1, G_2, 1\sim\}$ where $G_1 = \langle x, (0.6, 0.7), (0.4, 0.2) \rangle$ and $G_2 = \langle x, (0.1, 0.2), (0.8, 0.8) \rangle$. Let $A = \langle x, (0.6, 0.7), (0.3, 0.2) \rangle$ be any IFS in (X, τ) . Then $A \supseteq U$ where $U = \langle x, (0.4, 0.2), (0.6, 0.7) \rangle$ is an IFRCS in X . Now $\alpha\text{int}(A) = \langle x, (0.6, 0.7), (0.4, 0.2) \rangle \supseteq U$. A is an IFR α GOS but not an IFROS in X , since $\text{int}(\text{cl}(A)) = \langle x, (0.6, 0.7), (0.4, 0.2) \rangle \neq A$.

Example 3.6: Let $X = \{a, b\}$ and let $\tau = \{0\sim, G_1, G_2, 1\sim\}$ where $G_1 = \langle x, (0.5, 0.7), (0.4, 0.2) \rangle$ and $G_2 = \langle x, (0.2, 0.2), (0.8, 0.8) \rangle$. Let $A = \langle x, (0.5, 0.7), (0.3, 0.2) \rangle$ be any IFS in (X, τ) . Then $A \supseteq U$ where $U = \langle x, (0.4, 0.2), (0.5, 0.7) \rangle$ is an IFRCS in X . Now $\alpha\text{int}(A) = \langle x, (0.5, 0.7), (0.4, 0.2) \rangle \supseteq U$. A is an IFR α GOS but not an IF α OS in X , since $\text{int}(\text{cl}(\text{int}(A))) = \langle x, (0.5, 0.7), (0.4, 0.2) \rangle \not\subseteq A$.

Remark 3.7: Every IFR α GOSs and every IFPOSs are independent to each other.

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Example 3.8: In Example 3.6, let $A = \langle x, (0.6,0.7), (0.2,0.2) \rangle$ be any IFS in X . Then $A \supseteq U$ where $U = \langle x, (0.4,0.2), (0.5,0.7) \rangle$ is an IFRCS in X . Now $\alpha \text{int}(A) = \langle x, (0.5,0.7), (0.4,0.2) \rangle \supseteq U$. A is an IFR α GOS but not an IFPOS in X , since $\text{int}(\text{cl}(A)) = \langle x, (0.5,0.7), (0.4,0.2) \rangle \not\supseteq A$.

Example 3.9: Let $X = \{a,b\}$ and let $\tau = \{0\sim, G_1, G_2, 1\sim\}$ where $G_1 = \langle x, (0.2,0.3), (0.6,0.7) \rangle$ and $G_2 = \langle x, (0.8,0.7), (0.1,0.1) \rangle$. Let $A = \langle x, (0.7,0.8), (0.1,0.2) \rangle$ be any IFS in (X, τ) . Then $\text{int}(\text{cl}(A)) = 1\sim \supseteq A$. Therefore A is an IFPOS in X but not an IFR α GOS in X , since $A \supseteq U$ where U is an IFRCS in X but $\alpha \text{int}(A) = \langle x, (0.2,0.3), (0.6,0.7) \rangle \not\supseteq U$.

Remark 3.10: Every IFR α GOSs and every IFSOSs are independent to each other.

Example 3.11: Let $X = \{a,b\}$ and let $\tau = \{0\sim, G_1, G_2, 1\sim\}$ where $G_1 = \langle x, (0.6,0.7), (0.4,0.2) \rangle$ and $G_2 = \langle x, (0.1,0.2), (0.7,0.8) \rangle$. Let $A = \langle x, (0.8,0.7), (0.2,0.1) \rangle$ be any IFS in (X, τ) . Then $A \supseteq U$ where $U = \langle x, (0.4,0.2), (0.6,0.7) \rangle$ is an IFRCS in X . Now $\alpha \text{int}(A) = \langle x, (0.6,0.7), (0.4,0.2) \rangle \supseteq U$. A is an IFR α GOS but not an IFSOS in X , since $\text{cl}(\text{int}(A)) = \langle x, (0.7,0.8), (0.1,0.2) \rangle \not\supseteq A$.

Example 3.12: Let $X = \{a,b\}$ and let $\tau = \{0\sim, G_1, G_2, 1\sim\}$ where $G_1 = \langle x, (0.5,0.2), (0.5,0.8) \rangle$ and $G_2 = \langle x, (0.2,0.2), (0.8,0.8) \rangle$. Let $A = \langle x, (0.5,0.8), (0.5,0.2) \rangle$ be any IFS in (X, τ) . Then $\text{cl}(\text{int}(A)) = \langle x, (0.5,0.8), (0.5,0.2) \rangle = A$. Therefore A is an IFSOS in X but not an IFR α GOS in X , since $A = U$ where $U = \langle x, (0.5,0.8), (0.5,0.2) \rangle$ is an IFRCS in X but $\alpha \text{int}(A) = \langle x, (0.5,0.2), (0.5,0.8) \rangle \not\supseteq U$.

Remark 3.13: The intersection of two IFR α GOS in an IFTS (X, τ) need not be IFR α GOS in general.

Example 3.14: Let $X = \{a,b\}$ and let $\tau = \{0\sim, G_1, G_2, G_3, G_4, G_5, G_6, 1\sim\}$ is an IFT on (X, τ) . Where $G_1 = \langle x, (0.5,0.7), (0.3,0.2) \rangle$, $G_2 = \langle x, (0.4,0.2), (0.4,0.8) \rangle$, $G_3 = \langle x, (0.2,0.2), (0.4,0.8) \rangle$, $G_4 = \langle x, (0.4,0.2), (0.4,0.7) \rangle$, $G_5 = \langle x, (0.4,0.2), (0.5,0.8) \rangle$ and $G_6 = \langle x, (0.2,0.2), (0.5,0.8) \rangle$. Let $A = \langle x, (0.4,0.2), (0.5,0.2) \rangle$ and $B = \langle x, (0.3,0.2), (0.3,0.7) \rangle$ be any two IFS in (X, τ) . Then $A \supseteq U$ where $U = \langle x, (0.3,0.2), (0.5,0.7) \rangle$ is an IFRCS in X . $\alpha \text{int}(A) = \langle x, (0.4,0.2), (0.5,0.7) \rangle \supseteq U$ and then $B \supseteq U$ where $U = \langle x, (0.3,0.2), (0.5,0.7) \rangle$ is an IFRCS in X . $\alpha \text{int}(B) = \langle x, (0.3,0.2), (0.4,0.7) \rangle \supseteq U$. Therefore A and B are IFR α GOS in X but $A \cap B = \langle x, (0.3,0.2), (0.5,0.7) \rangle$ is not an IFR α GOS in X , since $A \cap B \supseteq U$ but $\alpha \text{int}(A \cap B) = \langle x, (0.2,0.2), (0.5,0.8) \rangle \not\supseteq U$.

Theorem 3.15: Let (X, τ) be an IFTS. Then for every $A \in \text{IFR}\alpha\text{GO}(X)$ and for every $B \in \text{IFS}(X)$, $\alpha \text{int}(A) \subseteq B \subseteq A \Rightarrow B \in \text{IFR}\alpha\text{GO}(X)$.

Proof: Let A be any IFR α GOS of X and B be any IFS of X . Let $\alpha \text{int}(A) \subseteq B \subseteq A$. Then A^c is an IFR α GCS and $A^c \subseteq B^c \subseteq \alpha \text{cl}(A^c)$. Then B^c is an IFR α GCS[6] and Therefore B is an IFR α GOS in X . Hence $B \in \text{IFR}\alpha\text{GO}(X)$.

Theorem 3.16: If A is an IFRCS and an IFR α GOS in (X, τ) . Then A is an IF α OS in (X, τ) .

Proof: As $A \supseteq A$, by the hypothesis, $\alpha \text{int}(A) \supseteq A$. But we have $A \supseteq \alpha \text{int}(A)$. This implies $\alpha \text{int}(A) = A$. Hence A is an IFR α GOS.

Theorem 3.17: Let (X, τ) be an IFTS. Then for every $A \in \text{IFS}(X)$ and for every $B \in \text{IFRC}(X)$, $B \subseteq A \subseteq \text{int}(\text{cl}(B)) \Rightarrow A \in \text{IFR}\alpha\text{GO}(X)$.

Proof: Let B be an IFRCS. Then $B = \text{cl}(\text{int}(B))$. By hypothesis, $A \subseteq \text{int}(\text{cl}(B)) \subseteq \text{int}(\text{cl}(\text{cl}(\text{int}(B)))) \subseteq \text{int}(\text{cl}(\text{int}(B))) \subseteq \text{int}(\text{cl}(\text{int}(A)))$. Therefore A is an IF α OS and by Theorem 3.3, A is an IFR α GOS. Hence $A \in \text{IFR}\alpha\text{GO}(X)$.

IV. APPLICATIONS OF INTUITIONISTIC FUZZY REGULAR α GENERALIZED CLOSED SETS

In this section we provide some applications of intuitionistic fuzzy regular α generalized closed sets.

Definition 4.1: If every IFR α GCS in (X, τ) is an IF α CS in (X, τ) , then the space can be called as an intuitionistic fuzzy regular $\alpha T_{1/2}$ space ($\text{IF}_{\tau\alpha}T_{1/2}$ in short).

Theorem 4.2: An IFTS (X, τ) is an $\text{IF}_{\tau\alpha}T_{1/2}$ space if and only if $\text{IF}\alpha\text{O}(X) = \text{IFR}\alpha\text{GO}(X)$.

Proof: Necessity: Let A be an IFR α GOS in (X, τ) , then A^c is an IFR α GCS in (X, τ) . By hypothesis, A^c is an IF α CS in (X, τ) and therefore A is an IF α OS in (X, τ) . Hence $\text{IF}\alpha\text{O}(X) = \text{IFR}\alpha\text{GO}(X)$.

Sufficiency: Let A be an IFR α GCS in (X, τ) . Then A^c is an IFR α GOS in (X, τ) . By hypothesis, A^c is an IF α OS in (X, τ) and therefore A is an IF α CS in (X, τ) . Hence (X, τ) is an $\text{IF}_{\tau\alpha}T_{1/2}$ space.

Theorem 4.3: Let an $\text{IF}_{\tau\alpha}T_{1/2}$ space be an IFTS. If A is an IFS of X then the following properties are hold:

- (i) $A \in \text{IFR}\alpha\text{GO}(X)$
- (ii) $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$
- (iii) There exists IFOS G such that $G \subseteq A \subseteq \text{int}(\text{cl}(G))$.

Proof: (i) \Rightarrow (ii): Let $A \in \text{IFR}\alpha\text{GO}(X)$. This implies A is an IF α OS in X , since X is an $\text{IF}_{\tau\alpha}T_{1/2}$ space. Then A^c is an IF α CS in X . Therefore $\text{cl}(\text{int}(\text{cl}(A^c))) \subseteq A^c$. This implies $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$.

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(ii) \Rightarrow (iii): Let $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$. Hence $\text{int}(A) \subseteq A \subseteq \text{int}(\text{cl}(\text{int}(A)))$. Then there exists IFOS G in X such that $G \subseteq A \subseteq \text{int}(\text{cl}(G))$ where $G = \text{int}(A)$.

(iii) \Rightarrow (i): Suppose that there exists IFOS G such that $G \subseteq A \subseteq \text{int}(\text{cl}(G))$. It is clear that $(\text{int}(\text{cl}(G)))^c \subseteq A^c$. That is $(\text{int}(\text{cl}(\text{int}(A))))^c \subseteq A^c$. This implies $\text{cl}(\text{int}(\text{cl}(A^c))) \subseteq A^c$. That is A^c is an IF α CS in X . This implies A is an IF α OS in X . Hence $A \in \text{IFR}\alpha\text{GO}(X)$.

Definition 4.4: An IFTS (X, τ) is said to be an intuitionistic fuzzy regular α $T_{1/2}^*$ space (IF $_{\text{ra}}T_{1/2}^*$ space in short) if every IF α GCS is an IFCS in (X, τ) .

Remark 4.5: Every IF $_{\text{ra}}T_{1/2}^*$ space is an IF $_{\text{ra}}T_{1/2}$ space but not conversely.

Proof: Let (X, τ) be an IF $_{\text{ra}}T_{1/2}^*$ space. Let A be an IF α GCS in (X, τ) . By hypothesis, A is an IFCS. Since every IFCS is an IF α CS, A is an IF α CS in (X, τ) . Hence (X, τ) is an IF $_{\text{ra}}T_{1/2}$ space.

Example 4.6: Let $X = \{a, b\}$ and let $\tau = \{0, G_1, G_2, G_3, 1\}$ where $G_1 = \langle x, (0.5, 0.5), (0.3, 0.1) \rangle$, $G_2 = \langle x, (0.1, 0.1), (0.7, 0.7) \rangle$ and $G_3 = \langle x, (0.5, 0.5), (0.4, 0.2) \rangle$. Let $A = \langle x, (0.4, 0.1), (0.5, 0.5) \rangle$ be any IFS in (X, τ) . Then $A \subseteq U$ where $U = \langle x, (0.5, 0.5), (0.3, 0.1) \rangle$ is an IFROS in X . Now $\alpha\text{cl}(A) = \langle x, (0.4, 0.1), (0.5, 0.5) \rangle \subseteq U$. Therefore A is an IF α GCS in X but not an IFCS in X , since $\text{cl}(A) = \langle x, (0.4, 0.2), (0.5, 0.5) \rangle \neq A$.

Theorem 4.7: For any IFS A in (X, τ) where X is an IF $_{\text{ra}}T_{1/2}^*$ space, $A \in \text{IFR}\alpha\text{GO}(X)$ if and only if every IFP $p_{(\alpha, \beta)} \in A$, there exist an IF α GOS B in X such that $p_{(\alpha, \beta)} \in B \subseteq A$.

Proof: Necessity: If $A \in \text{IFR}\alpha\text{GO}(X)$, then we can take $B = A$ so that $p_{(\alpha, \beta)} \in B \subseteq A$. For every IFP $p_{(\alpha, \beta)} \in A$.

Sufficiency: Let A be an IFS in (X, τ) and assume that there exist $B \in \text{IFR}\alpha\text{GO}(X)$ such that $p_{(\alpha, \beta)} \in B \subseteq A$. Since X is an IF $_{\text{ra}}T_{1/2}^*$ space, B is an IFOS. Then $A = \bigcup_{p_{(\alpha, \beta)} \in A} \{p_{(\alpha, \beta)}\} \subseteq \bigcup_{p_{(\alpha, \beta)} \in A} B \subseteq A$. Therefore $A = \bigcup_{p_{(\alpha, \beta)} \in A} B$, which is an IFOS in X . Hence by Theorem 3.3, A is an IF α GOS in X .

Definition 4.8: An IFTS (X, τ) is said to be an intuitionistic fuzzy regular α generalized $T_{1/2}$ (IF $_{\text{rag}}T_{1/2}$ in short) space if every IF α GCS in X is an IF α GCS in X .

Theorem 4.9: If an IFTS (X, τ) is an IF $_{\text{rag}}T_{1/2}$ space, then every IF α GOS is an IF α GOS.

Proof: Let A be an IF α GOS in X . This implies A^c is an IF α GCS in X . Since X is an IF $_{\text{rag}}T_{1/2}$ space, A^c is an IF α GCS in X . Hence A is an IF α GOS in X .

Theorem 4.10: Let an IF $_{\text{rag}}T_{1/2}$ space be an IFTS. If A is an IFS of X then the following properties are hold:

- (i) $A \in \text{IFR}\alpha\text{GO}(X)$
- (ii) $U \subseteq \text{int}(\text{cl}(\text{int}(A)))$ whenever $U \subseteq A$ and U is an IFCS in X
- (iii) There exists IFOSs G and G_1 such that $G_1 \subseteq U \subseteq \text{int}(\text{cl}(G))$.

Proof: (i) \Rightarrow (ii): Let $A \in \text{IFR}\alpha\text{GO}(X)$. This implies A is an IF α GOS in X , since X is an IF $_{\text{rag}}T_{1/2}$ space. Then A^c is an IF α GCS in X . Therefore $\alpha\text{cl}(A^c) \subseteq V$ whenever $A^c \subseteq V$ and V is an IFOS in X . That is $\text{cl}(\text{int}(\text{cl}(A^c))) \subseteq V$. This implies $V^c \subseteq \text{int}(\text{cl}(\text{int}(A)))$ whenever $V^c \subseteq A$ and V^c is IFCS in X . Replacing V^c by U , $U \subseteq \text{int}(\text{cl}(\text{int}(A)))$ whenever $U \subseteq A$ and U is an IFCS in X .

(ii) \Rightarrow (iii): Let $U \subseteq \text{int}(\text{cl}(\text{int}(A)))$ whenever $U \subseteq A$ and U is an IFCS in X . Hence $\text{int}(U) \subseteq U \subseteq \text{int}(\text{cl}(\text{int}(A)))$. Then there exists IFOSs G and G_1 in X such that $G_1 \subseteq U \subseteq \text{int}(\text{cl}(G))$ where $G = \text{int}(A)$ and $G_1 = \text{int}(U)$.

(iii) \Rightarrow (i): Suppose that there exists IFOSs G and G_1 such that $G_1 \subseteq U \subseteq \text{int}(\text{cl}(G))$. It is clear that $(\text{int}(\text{cl}(G)))^c \subseteq U^c$. That is $(\text{int}(\text{cl}(\text{int}(A))))^c \subseteq U^c$. This implies $\text{cl}(\text{int}(\text{cl}(A^c))) \subseteq U^c$, $A^c \subseteq U^c$ and U^c is an IFOS in X . This implies $\alpha\text{cl}(A^c) \subseteq U^c$. That is A^c is an IF α GCS in X . This implies A is an IF α GOS in X . Hence $A \in \text{IFR}\alpha\text{GO}(X)$.

V. CONCLUSION

Thus we have analyzed relationship between intuitionistic fuzzy regular α generalized open sets and the already existing intuitionistic fuzzy open sets and obtain many interesting theorem using the new spaces introduced above.

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