

Reliability of Dynamic Multi-State Oil Supply System by Structure Function

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ABSTRACT: Application of mathematical model of multi state system for oil supply system is considered in this paper. This paper describes a method for estimation of changes of one component states and its influence on the multistate system reliability by dynamic reliability indices. We examine the oil supply system to allow a different number of states for the oil supply system and for each pipeline by proposing dynamic reliability indices for investigation of this system. These indices estimate the influence upon the multi-state system reliability, which is the oil supply system from an oil source to three stations through several oil pipelines, say four. The mathematical approach of logical differential calculus is used for analysing the change in the multi-state system reliability that is caused by modification of some system components states.

KEYWORDS: Multistate system reliability, Structure function, Dynamic reliability indices, Direct partial logic derivatives.

I. INTRODUCTION

Hardware reliability has been increased by technological advances in large complex systems such as nuclear power plants, aircraft, and offshore oil plants, [16]. The basic concepts of MSS reliability were primarily introduced in the mid of the 1970's by Murchland in 1975, [11], [9], [3] and Ross in 1979, [14]. Barlow and Wu in 1978 [1] characterize component state criticality as a measure of how a particular component state affects a specific system state. Griffith in 1980 [6] formalized the concept of MSS performance, and studied the impact of component improvement on the overall system reliability behaviour. The concepts of MSS important were also discussed in Block and Savits in 1982 [2], where a decomposition theorem for MSS structure function was proved. Since that time, MSS reliability began with an intensive development. Essential achievements that were attained up to the mid of 1980's were reflected by Natvig in 1985 [12], and by El-Neveih and Prochan in 1984 [4], where it can be found the state of the art in the field of MSS reliability at this stage.

Readers that are more interested in the history and more ideas related to MSS reliability theory for the later work can find the corresponding overview in Lisnianski and Levitin [9] and Natvig [13].

There are some mathematical models in the reliability analysis. Firstly, it is the Binary System: the system and its components are allowed to have only two possible states (completed failure and perfect functioning). Secondly, it is the Multi-State System (MSS). In a MSS, both the system and its components may experience more than two states, for example, completely failed, partially functioning and perfect functioning. The reliability analysis of the binary system has served as a foundation for the mathematical treatment of the reliability theory. Many problems of the binary system have been settled. But this approach fails to describe many situations where the system can have more than two distinct states [9], [8]. Many practical components and systems have more than two different performance levels. For example, a power generator in a power station can work at full capacity, which is its nominal capacity, say 10MW, when there is no failure at all, [9]. Certain types of failures can cause the generator to be completely failed, while other failures will lead to the generator working at a reduced capacity say at 4 MW.

On the system level, let us consider a power generating system consisting of several power generators. The abilities of the system to meet high power load demand, normal power load demand and lower power load demand can be regarded as different system states.

Another example of multi-state components is an oil transmission pipeline [10]. The pipeline is used to transmit oil from the source to spots A, B and C aligned in order along the pipeline. We say that the pipeline is in state 0 when it cannot transmit oil to any of the spots; it is in state 1 if the oil can reach spot A; it is in state 2 if the oil can reach up to

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spot B, i.e., spot A and B; it is in state 3 if the oil can reach up to spot C. In the paper [19] authors had substantiated basic conceptions to apply the direct partial logic derivatives (it is part of logical differential calculus) for the reliability analysis of MSS. The direct partial logic derivatives reflect changing the value of investigation function when the values of its variables change. In [17] the new class of reliability indices was determined and was named dynamic reliability indices (DRI). The dynamic character of DRI consists to determine states of the system failure or, in other case, to the repair system is caused by a change of state the system component. In other words, these indices define the boundary states of MSS. In paper [21] three groups of DRI were obtained the dynamic reliability indices which define the boundary states of MSS are given and the conditions of being and changing of these states depending on the change of the system component states have been considered. However, these indices have high dimensionality and there are problems in their application for real-word engineering problems include three groups of probabilistic indices, [23]: Dynamic deterministic reliability indices (DDRI's), component dynamic reliability indices (CDRI's) and dynamic integrated reliability indices (DIRI's). The DDRI's evaluate an influence of a level change of a component state on system reliability. Numerical DDRI is defined as sets of the system states. These sets are calculated by direct partial logic derivatives. The CDRI's allows measuring an influence of each individual component to the system reliability. The DIRI's represent how change of one of system components impacts to the system reliability. But these indices permits to analyses the influence of one component state change to MSS reliability only. In this paper we evolve an application of MSS reliability analysis methods for representation and estimation of system with "oil supply system".

II. PROBLEM FORMULATION AND DESCRIPTION OF SYSTEM MODEL

The MSS and each of n -components can be in one of m possible states: from the complete failure (it is 0) to the perfect functioning (it is $m - 1$) and has m possible values. A structure function is one of typical representations of MSS and defines correlation of system performance level depending on MSS components states, [9]. Every system component $x_i, \forall i = 1, 2, \dots, n$; is characterized by probability of the performance rate:

$$p_{i,s_i} = Pr\{x_i = S_i\} \quad (1)$$

where $i = 1, 2, \dots, n$ and $S_i = 0, 1, \dots, m_i - 1$.

The system reliability (system state) depends of its components state and is defined by the structure function:

$$\Phi(x) = \Phi(x_1, x_2, \dots, x_n): \{0, \dots, m_1 - 1\}^n \rightarrow \{0, \dots, m_1 - 1\} \quad (2)$$

The following assumptions are used for structure functions as shown in Equation (2) that are peculiar to reliability analysis, [7], [8]:

- a) The structure function is non-decreasing and so: $\Phi(s, s, \dots, s) = s, \Phi(0, 0, \dots, 0) = 0$ and $\Phi(x) \leq \Phi(x')$ if $x \leq x'$
- b) The behaviour of each component is mutually s-independent;
- c) The every component is relevant to the system

The structure functions of parallel, series and k-out-of-n MSS terms are declared by OR (\vee) and AND (\wedge), [18]:

$$\Phi_P(x) = \vee_{i=1}^n x_i \quad (3)$$

$$\Phi_{\emptyset_S}(x) = \wedge_{i=1}^n x_i \quad (4)$$

$$\Phi(x) = \vee(\wedge_{z \geq k} x_{i_z}) \quad (5)$$

where $\vee_{i=1}^n x_i = \text{Max}_i(x_1, x_2, \dots, x_n); \wedge_{i=1}^n x_i = \text{Min}_i(x_1, x_2, \dots, x_n)$

The mathematical model of MSS k-out-of-n (5) can simplify by:

$$x_{i_1} x_{i_2} \dots x_{i_k} \vee x_{i_1} x_{i_2} \dots x_{i_k} x_{i_{k+1}} = x_{i_1} x_{i_2} \dots x_{i_k}$$

and define as:

$$\Phi(x) = \vee(\wedge_{z=1}^k x_i) \quad (6)$$

For example, the structure functions MSS 2-out-of-3 is presented by:

$$\Phi(x) = x_1 x_2 \vee x_1 x_3 \vee x_2 x_3 \vee x_1 x_2 x_3$$

The MSS 2-out-of-3 structure function is declare in this case:

$$\Phi(x) = x_1 x_2 \vee x_1 x_3 \vee x_2 x_3$$

A parallel system structure function is 1-out-of-n system:

$$\Phi(x) = \vee(\wedge_{z=1}^1 x_i) = \vee_{z=1}^n x_i \quad (7)$$

And a series system structure function is n-out-of-n MSS:

$$\Phi(x) = \vee(\wedge_{z=1}^n x_{i_z}) = \wedge_{z=1}^n x_z \quad (8)$$

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III. DIRECT PARTIAL LOGIC DERIVATIVE FOR MSS MODEL

The direct partial logic derivative of function $\Phi(x)$ with respect to the component $x_i, \forall i = 1, 2, \dots, n$ which is termed $\partial\Phi(j \rightarrow h)/\partial x_i(a \rightarrow b)$ reflects the fact of changing of the function Φ from state j to state h , when the value of component x_i changing from a to b [22]:

$$\partial\Phi(j \rightarrow h)/\partial x_i(a \rightarrow b) = \Phi(a_i, x) \cdot \Phi(b_i, x) \quad (9)$$

where:

$$\Phi(a_i, x) = \Phi(x_1, \dots, x_{i-1}, \dots, a, x_{i+1}, \dots, x_n); \text{ and } \Phi(b_i, x) = \Phi(x_1, \dots, x_{i-1}, \dots, b, x_{i+1}, \dots, x_n);$$

$j, h \in \{0, 1, \dots, M - 1\}$ and $a, b \in \{0, 1, \dots, m_i - 1\}$; and “ \cdot ” is the symbol of a comparison operation, defined by:

$$\frac{\partial\Phi(j \rightarrow h)}{\partial x_i(a \rightarrow b)} = \begin{cases} M - 1, & \text{if } \Phi(a_i, x) = j \text{ and } \Phi(b_i, x) = h \\ 0, & \text{otherwise} \end{cases}$$

So, the direct partial logic derivative of the structure function allows to examine the influence of the state change of the $i - th$ component into the system reliability. In other words this derivative discovers the system states that are transformed as a result of the change of the component state, [23].

IV. MULTI STATE SYSTEM FAILURE AND REPAIR OF MSS, [23]

Direct partial logic derivative of the structure function allows to examine the influence of the $i - th$ component state change into the system reliability. In other words this derivative discovers the system states that are transformed as a result of the change of the component state.

Consequences of the direct partial logic derivatives are of interest for reliability analysis of the MSS for this purpose, consider the following two partial derivatives:

$$\partial\Phi(j \rightarrow 0)/\partial x_i(a \rightarrow b) \text{ for } j, a \in \{1, 2, \dots, m_i - 1\} \text{ and } b \in \{0, 1, \dots, m_i - 1\}, b < a$$

$$\partial\Phi(0 \rightarrow h)/\partial x_i(c \rightarrow d) \text{ for } h \in \{1, 2, \dots, m_i - 1\} \text{ and } c, d \in \{0, 1, \dots, m_i - 1\}, c < d$$

The first derivative is a mathematical representation for the model of the system failure if the $i - th$ component state changes from a to b . Because the structure function $\Phi(x)$ is non-decreasing, this derivative is $\partial\Phi(j \rightarrow 0)/\partial x_i(a \rightarrow a - 1)$; where $j, a \in \{1, 2, \dots, m_i - 1\}$.

The second derivative permits the mathematical description of the system renewal. There are two variants of investigation for the system repairing. First it is the system repairing by the replacement of the failure component. This situation is determined by the direct partial logic derivative $\partial\Phi(0 \rightarrow h)/\partial x_i(0 \rightarrow m_i - 1)$. Second, it is the increase of component state that is described as $\Phi(0 \rightarrow h)/(\partial x_i(c \rightarrow c + 1))$. However, the first variant is more important for applications. Because the structure function of the MSS is non-decreasing, this derivative can be assigned as $\partial\Phi(0 \rightarrow h)/\partial x_i(0 \rightarrow m_i - 1)$.

It is remarkable that direct partial logic derivatives allow to analyze dynamic properties of the MSS, which is submitted as structural function.

V. DYNAMIC RELIABILITY INDICES

Dynamic reliability indices characterize the change of the MSS reliability that is caused by the change of a component state and include three groups of probabilistic indices: dynamic deterministic reliability indices (DDRI's), component dynamic reliability indices (CDRI's) and dynamic integrated reliability indices (DIRI's). Therefore, we will explain next each of these concepts in details.

Definition (1) (Dynamic Deterministic Reliability Indices), [20], [21]:

Dynamic deterministic reliability indices are sets of the boundary state of the system $\{G_f\}$ (for system failure) and $\{G_r\}$ (for system repairing).

The states system failure $\{G_f\}$ and the states system repairing $\{G_r\}$ are defined by:

$$\{G_f\} = \{G_f|x_1\} \cup \{G_f|x_2\} \cup \dots \cup \{G_f|x_n\} \quad (10)$$

$$\{G_r\} = \{G_r|x_1\} \cup \{G_r|x_2\} \cup \dots \cup \{G_r|x_n\} \quad (11)$$

where the subsets $\{G_f|x_i\}$ and $\{G_r|x_i\}$ are defined by:

$$\{G_f|x_i\} = \{G_f | \partial\Phi(j \rightarrow 0)/\partial x_i(j \rightarrow 0) \neq 0\} \quad (12)$$

$$\{G_r|x_i\} = \{G_r | \partial\Phi(0 \rightarrow h)/\partial x_i(0 \rightarrow m_{i-1}) \neq 0\} \quad (13)$$

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where $\{G_f|x_n\}$ and $\{G_r|x_n\}$ are subsets of the boundary state of the system for everyone of system component $x_i, \forall i = 1, 2, \dots, n$.

Definition (2) (Component Dynamic Reliability Indices), [21], [18]:

Component dynamic reliability indices are probabilities of MSS failure and repairing at a modification of a state of $i - th$ system component

$$P_f = p(i)_{a \rightarrow a-1}^{j \rightarrow 0} \times p_a(i) \quad (14)$$

$$P_r = p(i)_{0 \rightarrow m_{i-1}}^{0 \rightarrow h} \times p_0(i) \quad (15)$$

where $p(i)_{a \rightarrow a-1}^{j \rightarrow 0}$ is the probability of the $i - th$ component state modification from a to $(a - 1)$ where the system fail; $p_a(i)$ is the probability of state a of $i - th$ component; $p(i)_{0 \rightarrow m_{i-1}}^{0 \rightarrow h}$ is the probability of $i - th$ component replace for system repairing; $p_0(i)$ is the probability of $i - th$ component failure.

$$p(i)_{1 \rightarrow 0}^{1 \rightarrow 0} = \frac{\rho(i)_{a \rightarrow a-1}^{j \rightarrow 0}}{m_1 \times m_2 \times \dots \times m_n} \quad (16)$$

$$p(i)_{0 \rightarrow m_{i-1}}^{0 \rightarrow h} = \frac{\rho(i)_{0 \rightarrow m_{i-1}}^{0 \rightarrow h}}{m_1 \times m_2 \times \dots \times m_n} \quad (17)$$

$\rho(i)_{a \rightarrow a-1}^{j \rightarrow 0}$ is the number of system states when a change $i - th$ component state from a to $(a - 1)$ forces the system failure; and $\rho(i)_{0 \rightarrow m_{i-1}}^{0 \rightarrow h}$ is number of system states when system repairing bring about by to replace $i - th$ component.

Noting that, the numbers $\rho(i)_{a \rightarrow a-1}^{j \rightarrow 0}$ and $\rho(i)_{0 \rightarrow m_{i-1}}^{0 \rightarrow h}$ are obtained as numbers of values of direct partial logic derivative $\frac{\partial \Phi(j \rightarrow 0)}{\partial x_i(a \rightarrow a-1)}$ and $\frac{\partial \Phi(0 \rightarrow h)}{\partial x_i(0 \rightarrow m_{i-1})}$ with respect to the $i - th$ variable, which are not equal 0.

In other words numbers $\rho(i)_{a \rightarrow a-1}^{j \rightarrow 0}$ and $\rho(i)_{0 \rightarrow m_{i-1}}^{0 \rightarrow h}$ are the cardinality of the set $\{G_f|x_i\}$ in eq. (12) and the set $\{G_r|x_i\}$ in eq. (13) accordingly.

Definition (3) (Dynamic Integrated Reliability Indices), [19], [21]:

Dynamic integrated reliability indices is probability of the system failure or repairing if one of system component fails or restores, the failure and repair probability defined by:

$$P_f = \sum_i^n P_f(i) \times \prod_{q=1, q \neq i}^n (1 - P_f(i)) \quad (18)$$

$$P_r = \sum_i^n P_r(i) \times \prod_{q=1, q \neq i}^n (1 - P_r(i)) \quad (19)$$

where $P_f(i)$ and $P_r(i)$ is determined in equations (14) and (15).

VI. ILLUSTRATIVE EXAMPLES

In this section, we shall study general example of MSS model for investigation of its dynamic behavior.

Example (1):

The direct partial logic derivative of the structure function with $m_i = 3, i = 1, 2, 3, \partial \Phi(0 \rightarrow 1) / \partial x_1(0 \rightarrow 2)$, which corresponds to description of MSS 2-out-of-3, are given in Table 1.

Table 1: The MSS 2-out-of-3 and the direct partial logic derivative.

x_1, x_2, x_3	$\Phi(x)$	$\frac{\partial \Phi(0 \rightarrow 1)}{\partial x_1(0 \rightarrow 2)}$	x_1, x_2, x_3	$\Phi(x)$	$\frac{\partial \Phi(0 \rightarrow 1)}{\partial x_1(0 \rightarrow 2)}$	x_1, x_2, x_3	$\Phi(x)$	$\frac{\partial \Phi(0 \rightarrow 1)}{\partial x_1(0 \rightarrow 2)}$
0 0 0	0	0	1 0 0	0	0	2 0 0	0	0
0 0 1	0	2	1 0 1	1	0	2 0 1	1	0
0 0 2	0	0	1 0 2	1	0	2 0 2	2	0
0 1 0	0	2	1 1 0	1	0	2 1 0	1	0
0 1 1	1	0	1 1 1	1	0	2 1 1	1	0
0 1 2	1	0	1 1 2	1	0	2 1 2	2	0
0 2 0	0	0	1 2 0	1	0	2 2 0	2	0
0 2 1	1	0	1 2 1	1	0	2 2 1	2	0
0 2 2	2	0	1 2 2	2	0	2 2 2	2	0

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We will consider in this section the MSS of 2-out-of-3, where the structure function $\Phi(x)$ depends on three variables which are the system components $n = 3$ and has the best level of the components $m_i = 3, (i = 1,2,3)$. The used probabilities of the component state supported by expert which are given in Table (2).

Table 2: Component state probability.

Component	State		
	0	1	2
x_1	0.1	0.6	0.3
x_2	0.4	0.5	0.1
x_3	0.2	0.2	0.6

We can calculate the CDRI for a 2-out-of-3 system which are presented in Table (3).

Table 3: CDRI calculation for parallel system (MSS 2-out-of-3).

x_i	$\rho(i)_{1 \rightarrow 0}^{1 \rightarrow 0}$	$\rho(i)_{0 \rightarrow m_{i-1}}^{0 \rightarrow 1}$	$p(i)_{1 \rightarrow 0}^{1 \rightarrow 0}$	$p(i)_{0 \rightarrow m_{i-1}}^{0 \rightarrow 1}$	$P_f(i)$	$P_r(i)$
x_1	4	2	0.148	0.074	0.089	0.022
x_2	4	2	0.148	0.074	0.074	0.007
x_3	4	2	0.148	0.074	0.029	0.044

Therefore the analysis of CDRI may be analyzed as follows:

- a) The system has the maximum probability of failure when the first component is in failure state because its CDRI has the largest value $P_f(1) = 0.089$.
 - b) The system fails with minimum probability if the third component has failed $P_f(3) = 0.029$.
 - c) The MSS repairs with maximum probability by replacement of the third component since $CDRI, P_r(3) = 0.044$.
- Finely, the DIRI permits to obtain the probability of the system failure if one of the system components is breakdown. It is $P_f = 0.17$, while the probability of the system repairing is $P_r = 0.071$ if one of the failure components of the system is replaced.

VII. DYNAMIC RELIABILITY OF OIL SUPPLY SYSTEM MODEL

Many engineering systems can fit into the proposed multi state system model. In this section, we will present on applications that have been identified by Tian, Z., Li, W. and Zuo, M. J., [15], Similar applications can be found in power supply systems and telecommunication systems.

Consider for example an oil supply system, as shown in Figure (1).

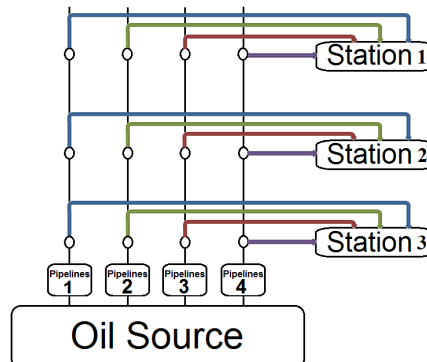


Figure (1) An oil supply system.

International Journal of Innovative Research in Science, Engineering and Technology

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The oil is delivered from the oil source to three stations through four oil pipelines. A pipeline is considered to be a multi-state component (thus $n = 4$). A failure might occur at any part of a pipeline. Take pipeline 1 for example, if there is a failure in section S_{01}^1 , the section of pipeline 1 between the oil source and station 1, the oil will not be able to reach any station via pipeline 1. If there are no failures in section S_{01}^1 , but there is a failure in section S_{12}^1 , the section of pipeline 1 between station 1 and station 2, the oil will be able to reach station 1 but will not be able to reach station 2 or beyond. Similarly, if there is no failure in section S_{01}^1 or section S_{12}^1 , but there is a failure in section S_{23}^1 , the oil will be able to reach station 1 and station 2, but will not be able to reach station 3. Based on the possible failures in different sections of a pipeline, four states of a pipeline can be defined as follows:

1. State 0: oil cannot reach any stations.
2. State 1: oil can reach only station 1.
3. State 2: oil can reach station 1 and 2.
4. State 3: oil can reach station 1, 2 and 3.

Each station has different demands on the oil.

1. Station 1: requires at least one pipelines working to meet its demand.
2. Station 2: requires at least two pipelines working to meet its demand.
3. Station 3: requires at least four pipelines working to meet its demand.

At the system level, we are interested in whether the demands of up to a certain station can be met. Thus, four states of the oil supply system can be defined as follows:

1. System state 0: it cannot meet the oil demand of station 1.
2. System state 1: it can meet the oil demand of up to station 1. That is, the system can meet the demand of station 1, but cannot meet the demand of station 2.
3. System state 2: it can meet the oil demands of up to station 2. That is, the system can meet the demands of station 1 and station 2, but cannot meet the demand of station 3.
4. System state 3: it can meet the oil demands of up to station 3. That is, the demands of station 1, 2 and 3 can all be met.

In practice, we may be interested in the probability of the oil supply system in states 0, 1, 2 or 3.

If $x_i, i = 1, 2, 3, 4$ are used for the pipeline, then, the component state probability are given in table (4)

Table 4: component state probability of the oil source system.

component	state			
	0	1	2	3
x_1	0.0500	0.0950	0.0684	0.7866
x_2	0.0500	0.0950	0.0684	0.7866
x_3	0.0300	0.0776	0.0446	0.8478
x_4	0.0300	0.0776	0.0446	0.8478

Note the structure function of MSS in this example has dimension is $m^n = 4^4 = 256$.

The structure function released to this system and the direct partial logic derivative $\partial\Phi(1 \rightarrow 0)/\partial x_i(1 \rightarrow 0)$ are found using computer programs and the subsets failure system $\{G_f|x_i\}$ are therefore found to be:

- a) $\{G_f|x_1\} = \{1000\}$, if the section S_{01}^1 is a failure, the oil will not be able to reach any station via pipeline 1.
- b) $\{G_f|x_2\} = \{0100\}$, if the section S_{01}^2 is a failure, the oil will not be able to reach any station via pipeline 2.
- c) $\{G_f|x_3\} = \{0010\}$, if the section S_{01}^3 is a failure, the oil will not be able to reach any station via pipeline 3.
- d) $\{G_f|x_4\} = \{0001\}$, if the section S_{01}^4 is a failure, the oil will not be able to reach any station via pipeline 4.

Also, the direct partial logic derivative $\partial\Phi(1 \rightarrow 2)/\partial x_i(0 \rightarrow m_{i-1})$ are calculated using computer program for ($i = 1, 2, 3, 4$) and the analysis of this derivative permits to obtain states of the $x_1x_2x_3$ system failure for which the replacement of the broken component restores the system.

The states for repairing the system are calculated by a similar method for failure system. The subsets if the repairing system $\{G_r|x_i\}$ are:

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- a) $\{G_r|x_1\} = \{0002,0003,0012,0013,0020,0021,0030,0031,0102,0103,0112,0113,0120,0121,0131,0200,0201,0210,0211,0300,0301,0310,0311\}$, if there is no failure in sections $S_{01}^1, S_{12}^1,$ and S_{23}^1 .
- b) $\{G_r|x_2\} = \{0002,0003,0012,0013,0020,0021,0030,0031,1002,1003,1012,1013,1020,1021,1030,1031,2000,2001,2010,2011,3000,3001,3010,3011\}$, if there is no failure in sections $S_{01}^2, S_{12}^2,$ and S_{23}^2 .
- c) $\{G_r|x_3\} = \{0002,0003,0102,0103,0200,0201,0300,0301,1002,1003,1102,1103,1200,1300,1301,2000,2001,2101\}$, if there is no failure in section $S_{01}^3, S_{12}^3,$ and S_{23}^3 .
- d) $\{G_r|x_4\} = \{0020,0030,0120,0130,0200,0210,0300,0310,1120,1130,1200,1210,1300,1310,2000,2010,2100,2110,3000,3010,3100,3110\}$, if there is no failure in sections $S_{01}^4, S_{12}^4,$ and S_{23}^4 .

VIII. SIMULATION AND RESULTS

The results of the CDRI for the oil supply system are presented in Table (5).

Table 5: CDRI calculation for oil supply system.

x_i	$\rho(i)_{1 \rightarrow 0}^{1 \rightarrow 0}$	$\rho(i)_{0 \rightarrow m_i - 1}^{1 \rightarrow 2}$	$p(i)_{1 \rightarrow 0}^{1 \rightarrow 0}$	$p(i)_{0 \rightarrow m_i - 1}^{0 \rightarrow 1}$	$P_f(i)$	$P_r(i)$
x_1	1	23	0.003	0.089	0.00028	0.07000
x_2	1	24	0.003	0.093	0.00028	0.07315
x_3	1	18	0.003	0.07	0.00023	0.05934
x_4	1	22	0.003	0.085	0.00023	0.07206

So, the breakdown of the section S_{01}^1 and section S_{01}^2 causes the maximum probability of system failure ($P_f(1) = 0.00028$) and ($P_f(2) = 0.00028$). Section S_{01}^3 and S_{01}^4 have an influence on the system failure least of all ($P_f(3) = P_f(4) = 0.00023$). The system repairing has its most value probable if there is no failures in sections $S_{01}^2, S_{12}^2,$ and in S_{23}^2 , (i.e. $P_r(2) = 0.07315$).

Also, The DIRI are probabilities of the change of the system reliability if the state of one of the system components is changed. The probability of the system failure, if one of the components breaks down, is $P_f = 0.001$ in accordance to eq. (18). The probability of system repairing obtained by eq. (19) and is $P_r = 0.2219$ if one of the failed components of the system is replaced.

IX. CONCLUSION AND DISCUSSION

In this paper a new measure for MSS reliability is presented, which is calculated by using the structure function of MSS model for certain system network. This model allows to determine some level of system availability in contrast to the binary system. The MSS model is improved moreover: the system has a different number of discrete states for the system and for each component. This measure, which is labeled as DRI, involves the probabilities of the changes of the system states that are assigned by changes of component states. We consider two system changes: the system failure and system repair. But the suggesting method for reliability analysis of the MSS can be used to estimate the other changes in the system state. The dynamic reliability approach has been used successfully to evaluate the probability of the failure and repairing of the oil supply system as a real life application of dynamic multi-state k-out-of-n system model where the components and the system have multiple performance levels. All computational results are made by MATLAB program.

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International Journal of Innovative Research in Science, Engineering and Technology

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