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Robust PID Control System Design Using ITAE Performance Index (DC Motor Model)

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ABSTRACT: One of the most important issues in control system design is to ensure the stability of the plant. PID controller used in industrial solutions still represents the most common controller in industry. However PID can only guess stability area and indicates stability zone by trial and error together with the experience of the designer. Decrement of system performance index leads to easier and better control system stability. Integral time absolute error (ITAE) is one of the most criterion used to reduce system error and give the best PID gain values for a desired system response requirements. This paper discusses the steps used to obtain a proper PID gain parameters values using ITAE method.

KEYWORDS: Robust control, Integral Time Error, PID controller, Step response.

I. INTRODUCTION

In modern control theory its assume that required system performance can easily be specified properly, where the performance index is calculated and measured or used to obtain the whole system behaviour quantitatively [1]. We consider control system with feedback shown in Fig (1) bellow where the close loop transfer function is:

$$\frac{C(s)}{R(s)} = T(s) = \frac{G_c(s)G(s)}{1+G_c(s)G(s)}. \tag{1}$$

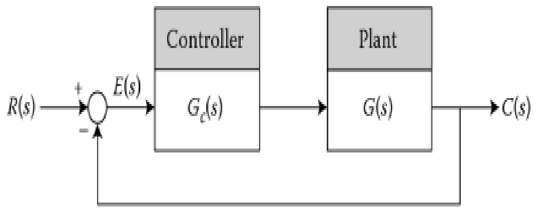

Fig. 1 Closed loop control system

The selection of PID controllers is basically a search problem in a three dimensional space, and by choosing different points of parameter space we can produce different step response for a step input. PID controller can be determined by moving this search point by trial and error basis[2].
The main problem in the selection of PID coefficients is that they do not meet the desired performance index or robust control system that the designer requires, in this section we will produce one of several robust PID controller design method which are[3]:

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- **Integral Squire Error (ISE):**

  It integrates squire error over time and penalizes large errors more than smaller ones. Control systems specified to minimize ISE will tend to eliminate large error quickly, for that it presents fast response but with low amplitude and oscillation, the ISE defined as:

$$ISE = \int_0^T e^2(t)\,dt. \qquad (2)$$

where t is the time and e(t) is the difference between set point and controlled variable.

- **Integral Absolute Error (IAE):**

  Integrates the absolute error over time, and it does not add Weight to any of the errors in the system response, it tends to produce slower response than ISE but with less sustained oscillation, described as:

$$IAE = \int_0^T |e|dt. \qquad (3)$$

II. INTEGRAL TIME ABSOLUTE ERROR (ITAE) METHOD

Integrates the absolute error multiplied by time over time, it weights errors which exist after a long time much more heavily than those at start of the response, its relationship is[4]:

$$ITAE = \int_0^T t|e(t)|\,dt \qquad (4)$$

ITAE produces system with settling time much more quickly than the above two methods. The method of ITAE can be used as one of the best robust PID control system design. The ITAE performance criterion for a step input has been determined for the general closed loop transfer function as [3]:

$$T(s) = \frac{C(s)}{R(s)} = \frac{b_0}{s^n + b_{n-1}s^{n-1} + \cdots + b_1 s + b_0} \qquad (5)$$

The transfer function in equation (5) has a steady state error equal zero for a step input with n poles and no zeros, the optimum coefficients for the ITAE criterion are carried out for equation (1) as shown below[3]:

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$$s+\omega_n$$
$$s^2+1.4\omega_n s+\omega_n^2$$
$$s^3+1.75\omega_n s^2+2.15\omega_n^2 s+\omega_n^3$$
$$s^4+2.1\omega_n s^3+3.4\omega_n^2 s^2+2.7\omega_n^3 s+\omega_n^4$$
$$s^5+2.8\omega_n s^4+5.0\omega_n^2 s^3+5.5\omega_n^3 s^2+3.4\omega_n^4 s+\omega_n^5$$
$$s^6+3.25\omega_n s^5+6.6\omega_n^2 s^4+8.6\omega_n^3 s^3+7.45\omega_n^4 s^2+3.95\omega_n^5 s+\omega_n^6$$

The responses using optimum coefficients for a unit step input are given as in figure (2) bellow for ITAE, where the responses are given for normalized time $\omega_n t$:

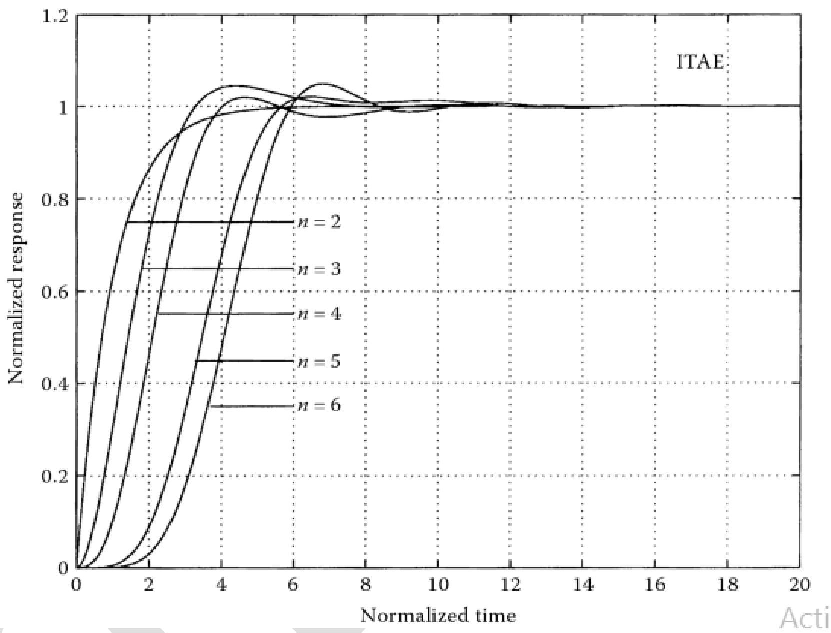

Fig. 2: Optimum coefficients responses for a step input

The transfer function *T(s)* has to normalized when determining the optimum coefficients.. For the transfer function of a third order:

$$T(s)=\frac{\omega_n^3}{s^3+\alpha\omega_n s^2+\beta\omega_n^2 s+\omega_n^3}. \quad (6)$$

Dividing numerator and denominator by ω_n

$$T(s)=\frac{1}{\frac{s^3}{\omega_n^3}+\alpha\frac{s^2}{\omega_n^2}+\beta\frac{s}{\omega_n}+1}. \quad (7)$$

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Equation (7) is the normalized third order transfer function, the same procedures are followed to obtain the higher order.

III. ROBUST PID CONTROLLED SYSTEM OF A DC MOTOR

Consider a DC motor with a transfer function as in equation (8) and block diagram as shown in figure (5) bellow, its assume that the input of the system is the voltage source (V) applied to the motor's armature, where the output is the rotational speed of the shaft $\frac{d\theta}{dt}$ [5]. The rotor and shaft are assumed to be static. We also assume a viscous friction model, that is, the friction torque is proportional to shaft angular velocity as in the following[6] [7]:

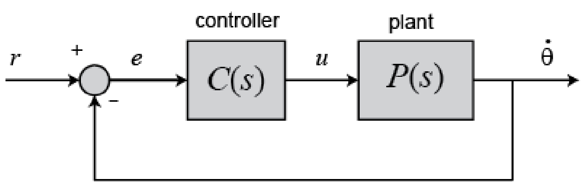

Fig. 3 DC motor speed control

$$P(s) = \frac{\dot{\Theta}(s)}{V(s)} = \frac{K}{(Js+b)(Ls+R)+K^2} \qquad \left[\frac{rad/sec}{V}\right] \tag{8}$$

Typical values used are:
(J) moment of inertia of the rotor 0.01 kg.m^2
(b) motor viscous friction constant 0.1 N.m.s
(Ke) electromotive force constant 0.01 V/rad/sec
(Kt) motor torque constant 0.01 N.m/Amp.
(R) electric resistance 1 Ohm.
(L) electric inductance 0.5 H.

By substituting the values above we obtain:

$$P(s) = \frac{1}{s^2+1.5s+1.01} \tag{9}$$

The step response of the system is shown in Fig (6) bellow.

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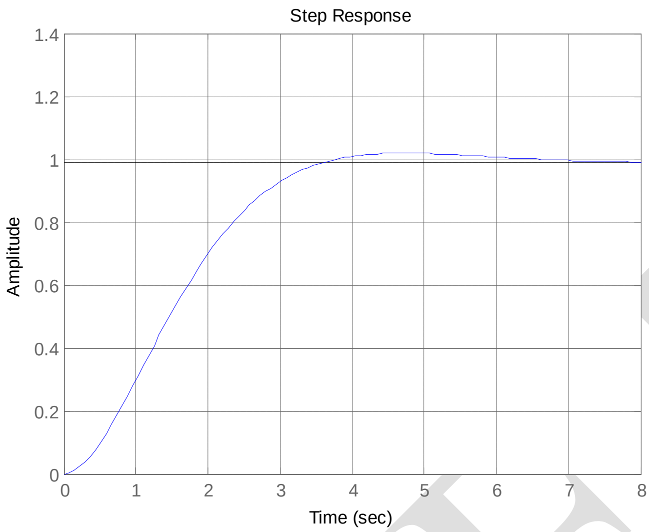

Fig. 4 DC motor open loop step reponse

Now consider we desire to design a PID control system by using ITAE creterion.

If $G(s) = 1$, the settling time is 5.38 seconds for step input.

we want to obtain an optimum ITAE preformance with settling time less than 0.5 seconds.

Using PID controller we have

$$G(s) = \frac{K_d s^2 + K_p s + K_i}{s} \tag{10}$$

Therefore the clsed loop transfer function of Fig 3 is:

$$\frac{\theta(s)}{R(s)} = \frac{K_d s^2 + K_p s + K_i}{s^3 + (K_d + 1)s^2 + (K_p + 1.5)s + (K_i + 1.01)} \tag{11}$$

we will choose ω_n that meets the settling time requirments, since $T_s = \frac{4}{\xi \omega_n}$ and ξ is unknown but near 0.8, we set $\omega_n = 10$, comparing the denominator of close loop PID system with the third order optimum coefficients

$$(s^3 + 1.75\omega_n s^2 + 2.15\omega_n^2 s + \omega_n^3) \tag{12}$$

We obtain $K_d = 16.5$, $K_p = 20$, $K_i \approx 1000$. then the closed loop becomes

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$$\frac{\theta\dot(s)}{R(s)} = \frac{16.5s^2+20s+1000}{s^3+17.5s^2+215s+1000} \quad (11)$$

The response of this system to step input isshown in figure (6).

we have to select a prefilter *G(s)p* so that we can ahieve the desired ITAE response

$$\frac{\theta\dot(s)}{R(s)} = \frac{1000}{s^3+17.5s^2+215s+1000} \quad (12)$$

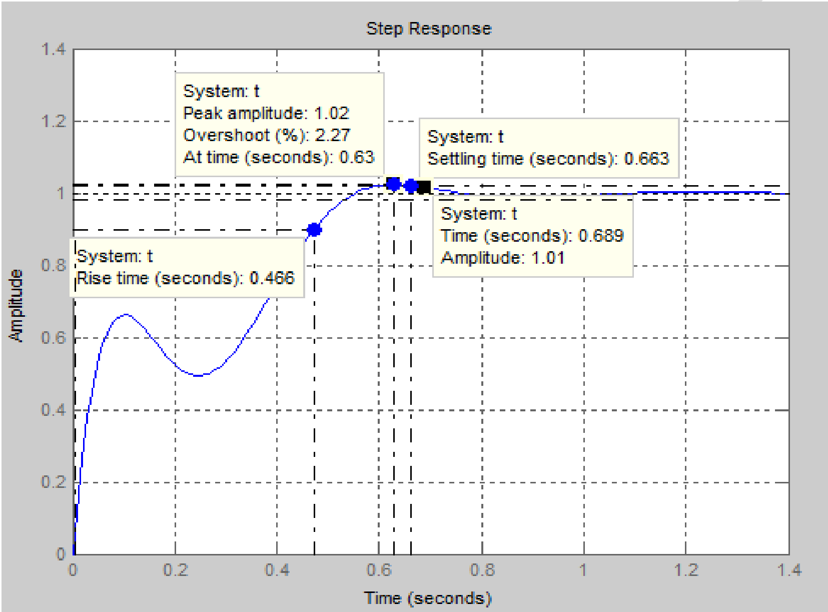

Fig. 5 Step response of un filtered close loop

in oerder to eleminate the zeros in equation (7) and bring overall numerator to 1000, we require

G(s)p =

$$\frac{16.5\ s^5 + 308.8\ s^4 + 3898\ s^3 + 20800\ s^2 + 20000\ s}{s^6 + 35\ s^5 + 736.3\ s^4 + 9525\ s^3 + 81225\ s^2 + 430000\ s + 1e06}$$

With step response as shown in Fig. 6 bellow.

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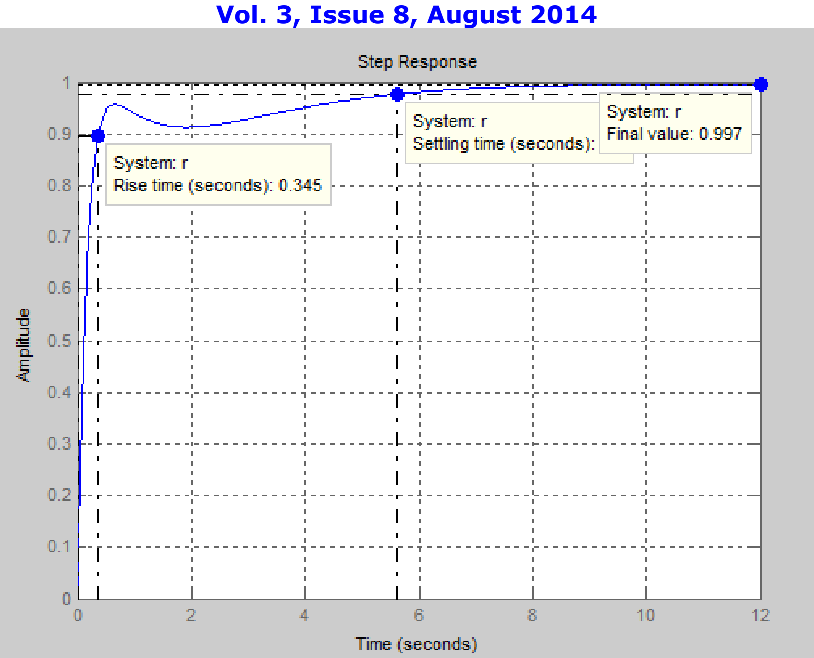

Fig. 6 System response after prefiltering

VI.RESULTS DISCUSSION

Control system design using ITAE gives predictable response and indicates clear system behaviar, specially in determining the desired settling time. Beside that ITAE guesses optimum PID gain value and desiralbe time respone paramters. Table(2) bellow shows DC motor response parameters with and without PID with prefilter gain. The table shows clearley the benefits of using the ITAE with a prefilter controller to achieve the desired response values.

Table 1 unit step response characteristics of the DC motor with and without controller

Controller	**P(s)**	**P(s) PID with $G(s)_p$ prefilter**
Precent overshoot	2.5%	2.27%
Steady state	0.99	1
Settling time	5.75	0.66

V. CONCLUSION

In control system design the controler is necessary to obtain the desired performance. The PID controller was used in this paper to obtain the design criteria. ITAE peformance index was used to determine the values of Kp, Kd, and Ki of the PID controller. Results have shown that the ITAE method is an optimum in finding the appropriate robust PID parameters.

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