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Solutions to Problem of Plates With Variable Rigidity Using Singularity Functions

SUJATHA A[1], V JAGANNATH[2] & SATISH KUMAR[3]

Assistant Professor, Department of Mathematics, R. V. College of Engineering, Bangalore, Karnataka, India[1]

Retired Professor of Civil Engineering, UVCE, Bangalore, Karnataka, India[2]

Former Post Graduate Student of UVCE, Bangalore, Karnataka, India[3]

Abstract: Method of solving problems of rectangular plates of variable stiffness using singularity functions is proposed. It is a novel method and the working methods are presented.Here the solution can be obtained with or without the use of computers, in general method has got certain distinct advantages over FEM.

Keywords: Singularity function, finite difference method, plate analysis.

Notations:

∇	*Nabla*
w	*Deflection*
q	*Loading*
D	*Flexular rigidity*
E	*Modulus of Elasticity*
μ	*Poisson's ratio*
M_x, M_y	*Bending Moments*
V_x, V_y	*Shear Forces*
h	*mesh or Spacing size*
M_{xy}, M_{yx}	*Twisting Moments*
ϖ, n	*Space Co – ordinates*
δ	*Dirac Delta Function*
η	*Doublet Function*
M_{xx}	*Bending moment at x - axis*

I INTRODUCTION

The working methods and manifestations of the use of singularity functions in the analysis of plates having constant rigidity were presented[1] . Here, plates of variable stiffness for certain types of loading and boundary conditions is presented to exemplify the technique of use of singularity functions. The study reveals that finite difference method [2],[3],[4] becomes cumbersome and tedious for problems of Analysis of plates with variable stiffness unless the singularity functions are used in the finite difference equations. In contrast, the need for a computer for even a simple problem in the finite element method required[13] in general. The solutions obtained by the proposed method agrees with the values obtained wholy by finite difference method.

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II SINGULARITY FUNCTION

Dirac delta function and unit step function are discussed in [1]. The third function is doublet function.

Doublet function:

We now introduce the third singularity function, the doublet function, in order to express concentrated couples in terms of a load distribution function.

The definition of doublet function is arrived by limiting process as in case of Dirac delta functions. A force is considered over an interval delta such that the resultant of the distribution is force $F = (C/\Delta)$. Adjoining this interval delta an equal but opposite distribution of force is also considered as shown in the fig 1.

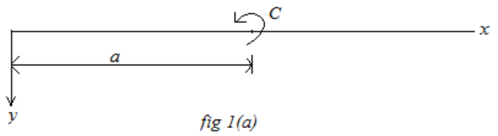

fig 1(a)

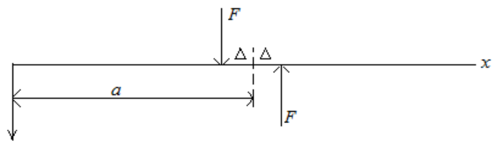

fig 1(b)

The coordinate "a" is the position along the beam at the interface between the two intervals. The resultant force F for each interval is at the centre of the interval so that the distribution for a couple equal to F then the doublet function η is defined as

$$\eta(x-a) = \lim_{\substack{\Delta\to 0\\ F\to 0}} \begin{cases} 0 & when \quad x < a-\Delta; x > a+\Delta \\ +F/\Delta & when \quad a-\Delta < x < a \\ -F/\Delta & when \quad a < x < a+\Delta \end{cases}$$

Thus, a loading distribution which gives a unit point couple at position (x=a) can be obtained with the condition that in the limit, the product F becomes equal to unity.

Properties of doublet function:

1. Integration of a doublet function leads to delta function

$$\int_{-\infty}^{x} \eta\chi(t-a)dt = [\delta(x-a)]$$

2. $\int_0^x (t-a)dx = \lim_{\Delta\to 0} \begin{cases} 0 & when \quad x < a-\Delta \\ F & when \quad x = a \\ 0 & when \quad x > a+\Delta \end{cases}$

3. $\int_0^x k\,\eta(t-a)dt = k\,\delta(x-a), k$ is a constant.

III WORKING METHOD

Plate equation for variable rigidity plate $\nabla^2\left(D\nabla^2 w\right)-(1-\upsilon)\left[L^4\left(D,W\right)\right]=q$, [6],[8], [13] where L^4 is differential operator

$$L^4\left(D,W\right)=\frac{\partial^2 D}{\partial x^2}\frac{\partial^2 W}{\partial y^2}-2\frac{\partial^2 D}{\partial x\partial y}\frac{\partial^2 W}{\partial x\partial y}+\frac{\partial^2 D}{\partial y^2}\frac{\partial^2 W}{\partial x^2}$$

Where $D = D(x,y)$ & $\nabla^2 = \frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$

Here $D = D(x,y)$ is expressed in terms of singularity function and its derivatives are also obtained by taking help of the properties of singularity function.

A certain plate problems of variable rigidity are solved using singularity functions to exemplify the new direction of analysis of plates.

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PROBLEM-1

The plate having ends *pq* and *rs* are fixed & *qr* and *ps* are simply supported

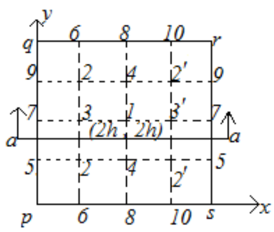

fig 2(a)

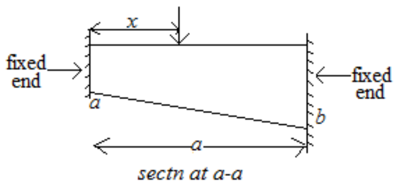

fig 2(b)

Consider the origin at p.

Flexural rigidity of the plate considered here, varies linearly along x and is constant along the direction of y.

It is D_a at $x=0$, D_b at $x=a$, and D at any distance x, it is given by $D=D(x)=(D_b-D_a)x/a$

The plate equation is

$$\nabla^2\left(D\,\nabla^2 w\right)-(1-\upsilon)L^4(D,w)=q \text{------------------------}(1)$$

$$L^4(D,w)=\left(\frac{\partial^2 D}{\partial x^2}\right)\left(\frac{\partial^2 w}{\partial y^2}\right)+2\left(\frac{\partial^2 D}{\partial x\,\partial y}\right)\left(\frac{\partial^2 w}{\partial x\,\partial y}\right)+\left(\frac{\partial^2 D}{\partial y^2}\right)\left(\frac{\partial^2 D}{\partial x^2}\right)$$

In this problem D is constant along Y, $\frac{\partial D}{\partial Y}=0$

$$\Rightarrow\quad L^4(D,w)=\left(\frac{\partial^2 D}{\partial x^2}\right)\left(\frac{\partial^2 w}{\partial y^2}\right)$$

Equation (1) transforms into $\nabla^2\left(D\;\;\nabla^2 w\right)-(1-\upsilon)\left(\frac{\partial^2 D}{\partial x^2}\right)\left(\frac{\partial^2 w}{\partial y^2}\right)=q$

$$\text{Or,}\quad D\nabla^4 w+2\frac{\partial D}{\partial x}\left(\frac{\partial^3 w}{\partial x^3}+\frac{\partial^3 w}{\partial x\,\partial y^2}\right)+\upsilon\left(\frac{\partial^2 D}{\partial y^2}\right)\left(\frac{\partial^2 w}{\partial x^2}\right)=q$$

$$D=D(x)=Cx\,,\quad \frac{\partial D}{\partial x}=C$$

$$or,\quad Cx\nabla^4 w+2C\left[\left(\partial^3 w/\partial x^3\right)+\left(\partial^3 w/\partial x\,\partial y^2\right)\right]=q$$

$$or,\quad \frac{x_i}{h^4}[20w_{i,j}-8(w_{i,j+1}+w_{i,j-1}+w_{i-1,j})+2(w_{i+1,j+1}+w_{i+1,j-1}+w_{i-1,j-1}+w_{i-1,j+1})$$

$$+(w_{i,j+2}+w_{i+2,j}+w_{i,j-2}+w_{i-2,j})+2(w_{i+1,j}-2(w_{i+1,j}+2w_{i-1,j}-w_{i-2,j})$$

$$+w_{i+1,j+1}-w_{i-1,j+1}-2w_{i+1,j}+2w_{i-1,j+1}-w_{i+1,j-1}-w_{i-1,j-1}]$$

$$\text{RHS}=\frac{q}{c}\,\delta(x-a)\,\delta(y-b)$$

Then, <u>At nodal point 1:</u>

The above equation becomes,

$$\frac{2h}{h^4}[20w_1-16w_4-8w_3+8w_2+2w_7+2w_3)]+\frac{2}{h^3}(0)=\frac{q}{c}\delta(x-h)\quad\delta(y-2h)\text{----------}(2)$$

<u>At nodal point</u> $3'$: $x=3h,\ y=2h$

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$$\frac{2}{h^3}[22w_3-16w_2+w_1]+\frac{2}{h^3}[4w_1-4w_7+w_9-w_2]=\frac{q}{c}\delta(x-a)\quad \delta(y-b)--------------(3)$$

<u>At nodal point 4:</u> $x=2h,\ y=h$

$$\frac{2}{h^3}[20w_4-16w_2-8w_1+4w_3+w_4]=\frac{q}{c}\delta(x-2h)\ \delta(y-h)-----------(4)$$

<u>At nodal point 3</u>: $x=h,\ y=2h$

$$h[22w_3+4w_4-16w_2-8w_1+4w_4-8w_1]=\frac{q}{c}\delta(x-a)\ \delta(y-b)-----------(5)$$

<u>At nodal point 2</u>: $x=h,\ y=3h$

$$h[12w_3-8w_4+2w_1+7w_2-8w_4+w_1]=\frac{q}{c}\delta(x-a)\ \delta(y-b)-----------(6)$$

<u>At nodal point</u> $2'$: $x=3h,\ y=h$

$$3h[23w_2-8w_3-8w_4+2w_1+4w_4+w_1]=\frac{q}{c}\delta(x-a)\ \delta(y-b)-----------(7)$$

Applying condition.

$$w_6=w_8=w_{10}=w_9=w_7=w_5=0$$

One can solve the above set of equations to obtain deflection

<u>PROBLEM 2:</u>

A non-prismatic plate pqrs with one of the parallel edges fixed and other simply supported is subjected to concentrated load. Derive expressions to evaluate deflection and other elastic quantities.

Plate with ends *pq* and rs are fixed; *qr* and ps are simply supported.

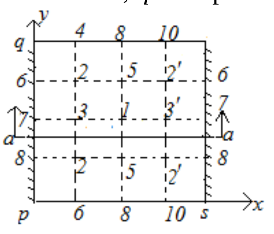

fig 3(a)

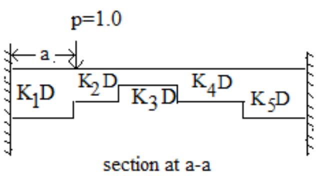

fig 3(b)

Consider the origin at *p*

Flexural rigidity of the plate considered here, has abrupt discontinuities as shown in section at *a-a*

The plate equation is $\nabla^2\left(D\,\nabla^2 w\right)-(1-\upsilon)L^4(D,w)=q-------------(8)$

$$L^4(D,W)=\frac{\partial^2 D}{\partial x^2}\frac{\partial^2 W}{\partial y^2}-2\frac{\partial^2 D}{\partial x\partial y}\frac{\partial^2 W}{\partial x\partial y}+\frac{\partial^2 D}{\partial y^2}\frac{\partial^2 W}{\partial x^2}$$

Here in the given problem D is constant along y

Therefore,

$$\frac{\partial D}{\partial Y}=0,\qquad \therefore\ L^4(D,w)=\frac{\partial^2 D}{\partial x^2}\ \frac{\partial^2 w}{\partial y^2}$$

Equation 8 transforms in to

$$D\nabla^4 w+2\frac{\partial D}{\partial x}\left(\frac{\partial^3 w}{\partial x^3}+\frac{\partial^3 w}{\partial x\,\partial y^2}\right)+\upsilon\frac{\partial^2 D}{\partial x^2}\ \frac{\partial^2 w}{\partial y^2}=q$$

Let , $A_1=D\nabla^4 w\quad A_2=2\frac{\partial D}{\partial x}\left(\frac{\partial^3 w}{\partial x^3}+\frac{\partial^3 w}{\partial x\,\partial y^2}\right)\qquad A_3=\upsilon\frac{\partial^2 D}{\partial x^2}\ \frac{\partial^2 w}{\partial y^2}$

<u>First term</u> A_1

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$$D\nabla^4 w = C[K_1U(x) - K_1U(x-1_1) + K_2U(x-1_1) - K_2U(x-1_2 + K_3U(x-1_2) - K_3U(x-1_3) + K_4U(x-1_3)$$
$$-K_4U(x-1_4) + K_5U(x-1_5)] \quad --------------8(a)$$

Secondterm A_2

$$2\frac{\partial D}{\partial x}\left(\frac{\partial^3 w}{\partial x^3} + \frac{\partial^3 w}{\partial x\,\partial y^2}\right) = 2C[K_1\delta(x) - K_1\delta(x-1_1) + K_2\delta(x-1_1) - K_2\delta(x-1_2) + K_3\delta(x-1_2)$$
$$-K_3\delta(x-1_3) + K_4\delta(x-1_3) - K_4\delta(x-1_4) + K_5\delta(x-1_5)] + [(w_{i+2,j} - 2w_{i+1,j} + 2w_{i-1,j} - w_{i-2,j})$$
$$+(w_{i+1,j+1} - w_{i-1,j+1} - 2w_{i+1,j} + 2w_{i-1,j} + w_{i+1,j} - w_{i-1,j-1})] -----8(b)$$

Third term A_3

$$\upsilon\frac{\partial^2 D}{\partial x^2}\quad\frac{\partial^2 w}{\partial y^2} = C[K_1\eta(x) - K_1\eta(x-1_1) + K_2\eta(x-1_1) - K_2\eta(x-1_2) + K_3\eta(x-1_2) - K_3\eta(x-1_3)$$
$$+K_4\eta(x-1_3) - K_4\eta(x-1_4) + K_5\eta(x-1_5)][w_{i,j+1} - 2w_{i,j} + w_{i,j+1}] --------------------8(c)$$

Therefore $A_1 + A_2 + A_3 = q \quad \delta(x-h) \quad \delta(y-2h)$

At nodal point 3: x = h, y = 2h

First term $A_1 = C[K_1][20w_3 - 8(w_1 + w_2) + 2(2w_s) + 1w_3 + w_3)] -------------8(d)$

$\delta(x) = 1$ Singularity function at origin.

Second term

$$A_2 = 2C[K_1][\delta(x) - K_1\delta(x-1_1) + K_2\delta(x-1_1) - K_2\delta(x-1_2)$$
$$+K_3\delta(x-1_2) - K_1\delta(x-1_3) + K_4\delta(x-1_3) - K_4\delta(x-1_4) + K_5\delta(x-1_5)]$$
$$[(w_3 - 2w_1 + 2w_7 - w_3 + w_5 - w_6 - 2w_1 + 2w_7 + w_5 - w_8)] = 2C[K_1][w_2 - 2w_3 + w_2] -----8(e)$$

Third term

$$A_3 = C[K_1][w_2 - 2w_3 + w_2] ---------------------------------8(f)$$

$$A_1 + A_2 + A_3 = CK_1[20w_3 - 8(w_1 + w_2) + 4w_5 + (w_3 + w_3)] + 2CK_1[w_3 - rw_1 - w_3 + 2w_5] + \quad CK_1[2w_2 - 2w_3]$$
$$= \quad q\,\delta(x-h)\delta(y-2h) = \quad 0(\because x = h, y = 2h) \quad ---------------------------------8(g)$$

For nodal point at 2: *x=h, y=3h)*

First term $A_1 = C[K_1][20w_2 - 8(w_3 + w_5) + 2w_1 + w_2 + w_2] ------------------9(a)$

Second term $A_2 = 2C[K_1][w_2 - 2w_5 + 2w_6 - w_2 + w_8 - 2w_5 + 2w_6 + w_1 - w_7] -----9(b)$

Third term $A_3 = C[K_1][w_4 - 2w_2 + w_3] -----------------9(c)$

$$= CK_1[21w_2 - 8(w_3 + w_5) + 2w_1 + w_2] + 2CK_1[w_2 - 4w_5 - w_2 + w_1] + CK_1[-2w_2 + w_3] = q\,\delta(x-h)\,\delta(y-2h) ------9(d)$$

For nodal point at $2'$ *x=3h, y=3h*

Applying boundary conditions:

First term $A_1 = C[K_4][20w_2 - 8(w_3 + w_6) + 2(w_8 + w_7 + w_1 + w_7) + w_2 + w_2] --------10(a)$

Second term $A_2 = 2C[K_4][w_2 - 2w_6 + 2w_5 - w_2 + w_r - w_8 - 2w_6 + 2w_5 + w_7 - w_{11}] --------10(b)$

Third term $A_3 \quad A_3 = \upsilon[K_4][w_{10} - 2w_2 + w_3] ------------------------------10(c)$

$$= CK_4[21w_2 - 8w_3 + 2w_1 + w_2] + 2CK_4[w_2 + 4w_5 - 2w_6 - w_1] + CK_4[-2w_2 + w_3] = q\,\delta(x-h)\,\delta(y-2h) -----------------10(d)$$

$$\therefore A_1 + A_2 + A_3 = \frac{CK_4}{h^4}[21w_2 - 8w_3 + 2w_1 + w_2] + \frac{2CK_4}{h^3}[w_2 + 4w_5 - 2w_6 - w_1] + \frac{CK_4}{h^2}[-2w_2 + w_3] = q -------10(e)$$

For nodal point at 5: *x=2h, y=3h*

Applying boundary conditions:

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$$\text{First term } A_1 = \frac{2C[K_3]}{h^4}[20w_5 - 8(w_2 + w_2 + w_1) + 2(w_3 + w_3) +] ----------11(a)$$

$$\text{Second term } A_2 = \frac{2C[K_3]}{h^4}[w_6 - 2w_2 + 2w_6 - w_6 + w_{10} - w_4 - 2w_2 + 2w_2 + w_3 - w_{13}] ------11(b)$$

$$\text{Third term } A_3 = \frac{CK_3}{h^2}[w_8 - 2w_5 + w_1] --------------------11(c)$$

$$\therefore A_1 + A_2 + A_3 = \frac{CK_3}{h^4}[20w_5 - 8(w_2 + w_2 + w_1] + 2[w_3 + w_3] + \frac{CK_3}{h^2}[-2w_5 + w_1] = q(x-h)\ (y-2h) ----------11(d)$$

$$\frac{CK_3}{h^4}[20w_5 - 8(w_2 + w_2 + w_1] + 2[w_2 + w_3] + \frac{2CK_3}{h^3}[4w_2 + w_3 - w_3]\frac{\upsilon CK_3}{h^2}[-2w_5 + w_1] = q -------------11(e)$$

For nodal point 1: *x=2h, y=2h*
Applying boundary conditions

$$\text{First term } A_1 = \frac{CK_3}{h^4}[20w_1 - 8(w_5 + w_3 + w_3 + w_5) + 2(w_2 + w_2 + w_2 + w_2) + 2w_7 + 2w_8] \_\_\_\_ 12(a)$$

$$\text{Second term } A_2 = 2CK_3[(w_7 - 2w_3 + w_3 - w_7) + w_2 - w_2 - 2w_2 + 2w_2 + w_2 - w_2] --------12(b)$$

$$\text{Third term } A_3 = \frac{CK_3}{h^2}[w_5 - 2w_1 + w_5] -----------------12(c)$$

$$\therefore A_1 + A_2 + A_3 = \frac{CK_3}{h^4}[20w_1 - 16w_5 - 8(w_3 + w_3) + 4w_2 + 4w_2] + \frac{2CK_3}{h^3}[+w_3 + 2w_2 - 2w_3] + \frac{\nu CK_3}{h^2}[2w_5 - 2w_1] = q\,\delta(x-h)\,\delta(y-2h) = 0 -----12(d)$$

applying $w_q = w_6 = w_7 = w_8 = w_r = w_4 = w_{10} = w_5 = 0$ one can solve the above set of equations.

PROBLEM 3:

A non-prismatic plate pqrs with one of the parallel edges fixed and other simply supported is subjected to concentrated load. Derive expressions to evaluate deflection and other elastic quantities.
Plate with Ends *pq* and *rs* are fixed, *qr* and *ps* are simply supported, Tapered haunches as shown in section at *a-a*

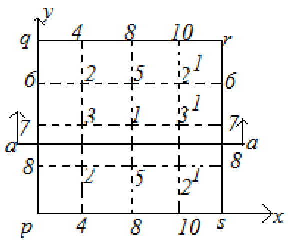

fig 4(a)

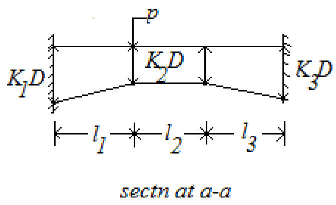

fig 4(b)

Consider origin at *P*:
Flexural rigidity varies as shown in section at *a-a*

(i) Up to length flexural rigidity is varying linearly along *x*, it is K_1D at *x = 0*& K_2D at $x = l_1$ and at any distance *x* where $0 < x \le l_1$.

$$D = D(x) = K_2D + \frac{(K_1 - K_2)}{l_1}D(l_1 - x)$$

(ii) For length l_2, Flexural rigidity = K_2D

(iii) For length l_3, Flexural rigidity is varying linearly along x, it is K_2D $at\ x = l_1 + l_2$

K_3D $at\ x = l_1 + l_2 + l_3$

And at any distance *x* where $(l_1 + l_2)(l_1 + l_2 + l_3)$

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$$D = D(x) = K_2 D + \frac{(K_3 - K_2)}{l_3} D\,[x - (l_1 \_ l_2)]$$

Taking origin at p

$$D_x = D_1 U(x) - D_1 U(x - l_1) + D_2 U(x - l_1) - D_2 U(x - l_2) + D_3 U(x - l_2)$$

Region AC

$$D_1 = K_2 D + \frac{(K_1 - K_2)}{l_1} D(l_1 - x)$$

$$= \frac{[K_2 D l_1 + K_1 D l_1 - K_2 D l_1 - K_1 D x + K_2 D x]}{l_1}$$

$$D_1 = K_1 D + \frac{(K_1 - K_2)}{l_1} D x$$

Let $A = K_1 D$, $B = \frac{K_1 - K_2}{l_1}$, $D_1 = [A - Bx] D$

$$D_3 = K_2 D + \left[\frac{(K_3 - K_2)x}{l_3} - \frac{K_3 - K_2}{l_3}(l_1 + l_2) \right] D$$

$$= \left[K_2 - \frac{(K_3 - K_2)}{l_3}(l_1 + l_2) + \frac{K_3 - K_2}{l_3} x \right] D$$

where $C = K_2 - \frac{(K_3 - K_2)\,(l_1 - l_2)}{l_3}$, $\quad E = \frac{(K_3 - K_2)}{l_3}$

$$D_3 = [C + Ex]\, D$$

$$D_x = [A - Bx] D U(x) - [A - Bx] D\, U(x - l_1) + K_2 D\, U(x - l_1) - K_2 D\, U(x - l_2) + [C + Ex] D\, U(x - l_2)]$$

Flexural rigidity is constant along y direction.

The plate equation is

$$i.e, \quad \nabla^2 \left(D\, \nabla^2 w \right) - (1 - \nu) L^4 (D, w) = q \text{---------------} (13)$$

Where,

$$L^4 (D, w) = \frac{\partial^2 D}{\partial x^2} \frac{\partial^2 w}{\partial y^2} - \frac{2\partial^2 D}{\partial x\, \partial y} \frac{\partial^2 w}{\partial x\, \partial y} + \frac{\partial^2 D}{\partial y^2} \frac{\partial^2 D}{\partial x^2}$$

Here in the given problem D is constant along y, $\frac{\partial D}{\partial y} = 0, \quad L^4 (D, w) = \frac{\partial^2 D}{\partial y} \frac{\partial^2 w}{\partial y^2}$

Equation (13) transforms into

$$\nabla^2 \left(D\, \nabla^2 w \right) - (1 - \nu) \frac{\partial^2 D}{\partial x^2} \frac{\partial^2 w}{\partial y^2} = q \text{-----------------------} (2)$$

First term:

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$$D\nabla^4 w=[A-Bx]DU(x)-[A-Bx]D\,U(x-+[C+Ex]DU(x-(l_2+l_1)]\left(\frac{\partial^4 w}{\partial x^4}+\frac{2\partial^4 w}{\partial x^2\,\partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)l_1)-K_2 D\,U(x-l_2)$$

Second term: $\frac{2\partial D}{\partial x}\left(\frac{\partial^3 w}{\partial x^3}+\frac{\partial^3 w}{\partial x\,\partial y^2}\right)$

where $D_x=[A-Bx]D[U(x)-U(x-l_1)]+K_2 D[U(x-l_1)-U(x-l_2)]+[C+Ex]DU(x-l_2)$

$$\frac{\partial^2 D_x}{\partial x^2}=[A-Bx]D[\delta(x)-\delta(x-l_1)-B(D[\,U(x)-U(x-l_1)+K_2 D[\,\delta(x-l_1)-\delta(x-l_2)$$

$$+[C+Ex]D\,\delta(x-l_2)+E\,D\,U(x-l_2)$$

$$\frac{\partial^2 D_x}{\partial x^2}=[-B]\,D[\delta(x)-\delta(x-l_1)]+(A-Bx)(D[\,\eta(x)-\eta(x-l_1)]-BD[\delta(x)-\delta(x-l_1)]+$$

$$K_2 D[\,\eta(x)-\eta(x-l_2)]+[E]D\,\delta(x-l_2)+[E]\,D\,\delta(x-l_2)$$

$$=-2\,BD[\delta(x)-\delta(x-l_1)]+(A-Bx+K_2)D[\,\eta(x)]+(Bx-A)D\,\eta(x-l_1)-K_2 D\eta(x-l_2)$$

Secondterm:

$$2\frac{\partial D}{\partial x}\left(\frac{\partial_3^3 w}{\partial x_3^3}+\frac{\partial_3^3 w}{\partial x\,\partial y^2}\right)=2[(A-Bx)]D[(x)-(x-l_1)]-B(D[\,U(x)-U(x-l)]+K_2 D[\,\delta(x-l_1)-\delta(x(l_2+l_1)]$$

$$+[C+Ex]D\,\delta(x-(l_2-l_1)+E\,D\,U(x-(l_2+l_1)][w_{i+2,j}-2w_{i+1,j}+2w_{i+,j}-w_{i-2,j}]-------13(a)$$

Third term:

$$\frac{\partial^2 Dx}{\partial x^2}\frac{\partial^2 w}{\partial y^2}=y[-2\,BD[\delta(x)-\delta(x-l_1)]+(A-Bx+K_2)D[\,\eta(x)]+(Bx-A)D\,\eta(x(l_2+l_1))$$

$$-K_2 D\eta(x-l_2+l_1)][w_{i,j+1}-2w_{i,j}+w_{i,j-1}]$$

$$\frac{2\partial D}{\partial x^2}\left(\frac{\partial^3 w}{\partial x^3}+\frac{\partial^3 w}{\partial x\,\partial y^2}\right)=[2(A-Bh)D][w_3-2w_1+2w_7-w_3]$$

$$+(w_5-w_6-2w_1+2w_7+w_5--w_8)]-------------------------13(c)$$

For nodal point 3: $x=h, y=2h$

Applying boundary conditions:

$$D\nabla^4 w=[A-Bh]D(20w_3-8(2w_2+w_1)+2(2w_5)+w_3+w_3--------------13(b)$$

Second term:

$$D_x\nabla_4^4 w+\frac{2\partial D}{\partial}\left(\frac{\partial^3 w}{\partial x^3}+\frac{\partial^3 w}{\partial x\,\partial y^2}\right)+\frac{\partial^2 D}{\partial x^2}\;\frac{\partial^2 w}{\partial y^2}\;=\;q$$

Third term:

Adding all the three terms we obtain.

$$v\frac{\partial^2 D}{\partial x^2}\frac{\partial^2 w}{\partial y^2}=v[-2BD+(A-Bh+K_2)\,D(w_2-2w_3+w_2)]-------------13(d)$$

$$[A-Bh]D\,(20w_3-8(2w_2+w_1)+4w_5+w_3+w_3+[2(A-Bh)\,D(w_3-2w_1)$$

$$-w_3+2w_5-2w_1)]+v[-2BD+(A-Bh+K_2)D]\;[2w_2-2w_2]$$

$$=p\delta(h-l_1)\,\delta(y-2h)---------------------------------13(e)$$

$\delta(h-l_1)$ is neglected when $h-l_1$ is -ve

$\delta(y-2h)$ is zero when y-$2h$ = 0

In the above problem as $h<l_1, h-l_1$ is –ve

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Right hand term $p\delta(h-l_1)\ \delta(y-2h)=0-------------------13(f)$

Equation 13(e) transforms in to

$[A-Bh]D\,(21w_3-16w_2-8w_1+4w_5+w_3+w_3)+[2(A-Bh)\,D\,(w_3-4w_1-w_3)$

$+2w_5)]+\nu\,[-2BD+(A-Bh+K_2)D]\,[2w_2-2w_3]=0----------------14$

For nodal point 2: *x=h, y=3h*

Applying boundary conditions

First term:

$D\nabla^4 w=[A-Bh]\,D\,[(20w_2-8(w_5+w_3)+2(w_1)+w_2+w_2]-------------14(a)$

Second term:

$$\frac{2\partial D}{\partial x}\left(\frac{\partial^3 w}{\partial x^3}+\frac{\partial^3 w}{\partial x\,\partial y^2}\right)=2[(A-Bh)D-BD]\,[w_2-2w_5+2w_6-w_2+w_8-w_q-2w_5+2w_6+w_1-w_7]----14(b)$$

Third term:

For any value of *x*, $\delta(x)=1,\eta(x)=1$

Adding all the three terms:

$$\nu\frac{\partial^2 D}{\partial x^2}\frac{\partial^2 w}{\partial y^2}=\nu\,[-2BD+(A-Bh+K_2)\ D]\,[w_4-2w_2+w_3]---------------14(c)$$

$[A-Bh[D](21w_2+w_2+2w_1-8w_3-8w_5)-2[(A-Hb-B)\,D\,](w_2-4w_5+w_6+w_8-w_q+w_1-w_7-w_2)]$

$+\nu\,[(-2B+A-Bh+K_2)D]\ \ [w_4-2w_2+w_3]=P\delta(x-l_1)\ \delta(y-2h)---------14(d)$

For *x=hand y=2h*

$\delta(x-l_1)=\delta(y-2h)=0$

R.H.S of equation 14(d) = 0

For nodal point5: *x=2h, y=3h*

Applying boundary conditions:

First term:

$D\nabla^4 w=[A-2h8]\,D-[(A-2hB)D+K_2D]\,[20w_5-8(w_2+w_2+w_1)+2(w_3+w_3)+0]-----------14(e)$

Second term:

$$\left(\frac{\partial^3 w}{\partial x^3}+\frac{\partial^3 w}{\partial x\,\partial y^2}\right)=2[K_2D\ ]\,[w_6-2w_2+2w_2-w_6]+w_{10}-w_4-2w_2+2w_2+w_3-w_3]-----14(f)$$

Third term:

$$\nu\frac{\partial^2 D}{\partial x^2}\frac{\partial^2 w}{\partial y^2}=\nu\,[-2BD\,[0]+(A-Bx+K_2)\ D\,(Bx-A)D]\,[-2w_5+w_1]------14(g)$$

On adding the above terms:

$K_2D\,[20w_5-8(w_2+w_2+w_1)+2(w_3+w_3)]+wK_2D\,[4w_2-4w_2+w_3-w_3]$

$+\nu\,[+K_2D]\,[-2w_5+w_3]\ =\ P\delta(x-l_1)\,\delta(y-2h)$

For *x=2h, y=3h*

$\delta(x-l_1)=\delta(y-2h)=1$

$K_2D\,[20w_5-8(w_2+w_2+w_1)+2(w_3+w_3)]+2K_2D\,[4w_2+w_3-w_3]$

$+\nu\,[+K_2D]\,[-2w_5+w_1]\ =\ P$

For nodal point 2: *x=3h, y=3h*

First term:

$D\nabla^4 w=[C+Ex]\,D\,[20w_2-8(w_5+w_3)+2w_1+w_2+w_2]$

Second term:

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$$2\frac{\partial D}{\partial x}\left(\frac{\partial^3 w}{\partial x^3}+\frac{\partial^3 w}{\partial x\,\partial y^2}\right)=[(C+E\,x)\,D+E\,D]\,[w_2-2w_6+2w_5-w_2]+(w_5-w_6-2w_1+2w_7+w_5-w_8)]$$

Third term:

$$v\frac{\partial^2 D}{\partial x^2}\frac{\partial^2 w}{\partial y^2}=v\,[-K_2D]\,[w_{10}-2w_2+w_3]$$

Adding all the three terms:

$$C+E(3h)\,D\,[21w_2+w_2+2w_1-8w_5-8w_3]+(C+E3h+E)\,D\,[w_2+4w_5-w_2-2w_1+2w_1-w_8]$$
$$-vK_2D\,[-2w_2+w_3]\;=P\delta(x-l_1)\,\delta\,(y-2h)-----------14(h)$$

When $x=3h$, $y=3h$

$\delta(x-l_1)\,\delta\,(y-2h)=1$

Equation 14(h) reduces to

$$C+3Eh)\,D\,[21w_2+w_2+2w_1-8w_5-8w_3]+(C+3Eh+E)\,D\,[w_2+4w_5-w_2-2w_1+2w_1-w_8]$$
$$-vK_2D\,[-2w_2+w_3]\;=P$$

For nodal point3: $x=3h$, $y=2h$

Applying boundary condition:

First term:

$$D\nabla^4 w=[C+E(3h)]\,D\,[20w_3-8(w_1+w_2+w_2)+2(w_5+w_5+w_3+w_3)]$$

Second term:

$$2[(C+E(3h)\,D+E\,D]\,[w_3-2w_7+2w_1-w_3+w_6-w_5-2w_7+2w_1+w_8-w_5]$$

Third term:

$$v\frac{\partial^2 D}{\partial x^2}\frac{\partial^2 w}{\partial y^2}=v\,[-K_2D]\,[w_2-2w_3+w_2]$$

Adding the above three terms:

$$[(C+3Eh)\,D\,[21w_2-8(w_1+2w_2)+4w_5+w_3+w_3]+2[(C+3Eh)D+ED]$$
$$[w_3-4w_7+4w_1-w_3+w_6-2w_5]+v\,[-K_2D]\,[w_2-w_3+w_2]\quad=\;P\delta(x-l_1)\;\delta\,(y-2h)\;=\;0$$

As, $x=3h$, $y=2h$

For nodal point 1: $x=2h$, $y=2h$

First term:

$$D\nabla^4 w=[K_2D]\,[20w_1-8(2w_5+w_3+w_3)+2(2w_2+2w_5)+]--------------15(a)$$

Second term:

$$\frac{2\partial D}{\partial x}\left(\frac{\partial^3 w}{\partial x^3}+\frac{\partial^3 w}{\partial x\,\partial y^2}\right)=2[K_2D]\,[w_7-2w_3+2w_3+w_7+w_2-2w_3+2w_7+w_2-w_2)+]------15(b)$$

Third term:

$$v\frac{\partial^2 D}{\partial x^2}\frac{\partial^2 w}{\partial y^2}=v\,[+(A-Bx+K_2)D(Bx-A)D]\,[w_5-2w_1+w_5]-----------13(c)$$

Adding the above three terms:

$$K_2D\,[20w_1-16w_5-8w_3-8w_3+4w_2+4w_2)+2K_2D[2w_7-4w_3+2w_3+2w_2-2w_2]+v\,[K_2D]\,[2w_5-2w_1]$$
$$=\;P\delta(x-l_1)\;\delta\,(y-2h)=\;0-----------------16(a)$$

As, $x=h$, $y=2h$

Or,

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$$K_2 D\,[20w_1 - 16w_5 - 8w_3 - 8w_3 + 4w_2 + 4w_2)$$

$$+2K_2 D[2w_7 - 4w_3 + 2w_3 + 2w_2 - 2w_2] + \nu\, K_2 D[2w_5 - 2w_1] \;=\; 0$$

Problem 4:

A non-prismatic plate pqrs with one of the parallel edges fixed and other simply supported is subjected to concentrated load. Derive expressions to evaluate deflection and other elastic quantities

Plate with, ends *pq* and *rs* are fixed, *qr* and *ps* are simply supported, parabolic varying haunch as shown in the section at *a-a*

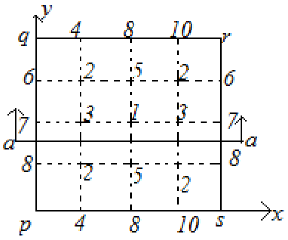

fig 5(a)

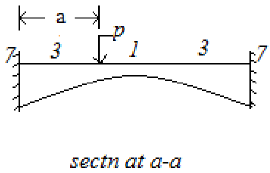

fig 5(b)

Consider the origin at *p*:

Flexural rigidity varies as shown in section *a-a*

$$D_x = \frac{4(K_1 - K_2)}{L^2}\left[\left(x - \frac{L}{2}\right)^2 + \frac{L^2 K_2}{4(K_1 - K_2)}\right] D$$

Let, $A = \dfrac{4(K_1 - K_2)}{L^2}$, $B = \dfrac{L^2 K_2}{4(K_1 - K_2)}$, $L_1 = \dfrac{L}{2}$

$$D_x = A[(x - L_1)^2 + B]\, D\,[U(x) - U(x - L)]$$

Flexural rigidity is constant along *y* direction. After rearranging the terms in the plate equation we obtain

$$D_x \nabla^4 w + \frac{2\partial D}{\partial x}\left(\frac{\partial^3 w}{\partial x^3} + \frac{\partial^3 w}{\partial x\,\partial y^2}\right) + \nu\,\frac{\partial^2 D}{\partial x^2}\,\frac{\partial^2 w}{\partial y^2} = q ----------------(1)$$

First term in LHS of (1):

$$D_x\, \nabla^4 w = A[(x - L_1)^2 + B]\, D\,[U(x) - U(x - L_1)]\left[\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right]$$

Second term in LHS of (1):

$$2\frac{\partial D}{\partial x}\left(\frac{\partial^3 w}{\partial x^3} + \frac{\partial^3 w}{\partial x\,\partial y^2}\right) = 2[A\,[2(x - L_1)]\,D[U(x) - U(x - L_1)] +$$

$$A[(x - L_1)^2 + B]D[\delta(x) - \delta(X - L_1)]\,[w_{i+2,\,j} - 2w_{\pm 1,\,j} + 2w_{i-1,\,j} - w_{i-2,\,j} +$$

$$w_{i+1,\,j+1} - w_{i-1,\,j+1} - 2w_{i+1,\,j} + 2w_{i-1,\,j} + w_{i+1,\,j-1} - w_{i-1,\,j-1})]$$

Third term in LHS of (1):

For nodal point 3: $x{=}h$, $y{=}2h$

$$\nu\,\frac{\partial^2 D}{\partial x^2}\,\frac{\partial^2 w}{\partial y^2} = \nu\,[2AD\,[U(x) - U\,(x - L_1)]\,D[\delta(x) - \delta(x - L_1)]$$

$$+A[(x - L_1)]D[\delta(x) - \delta(X - L_1)] + A[x - L_1)^2 + B]D[\eta(x - L_1)]\;\; [w_{i,\,j+1} - 2w_{i,\,j} + w_{i,\,j-1}]$$

Applying boundary conditions, we obtain

Adding the above three terms

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$$[A[(h-L_1)^2+B]\,D[20w_3-16w_2-8w_7-8w_1+2w_6+2w_5+w_8+2w_3]$$
$$+2[A[2(h-L_1)]D+A[(h-L_1)^2+B]\,[-2w_1+2w_5]$$
$$+\nu[2AD+4A(h-L_1)D+A[(h-L_1)^2+B]D]\,[2w_2-2w_3]$$
$$=p\delta(x-l_1)\delta(y-2h)-----------------------------------1(a)$$

Where $x=h$, $y=2h$

R.H.S of equation 1(a)=0

For nodal point 5: $x=2h$, $y=3h$

$$[A[(2h-L_1)^2+B]\,D[20w_5-16w_2-8w_1-8w_1+4w_3]\times\frac{1}{h^4}$$
$$+\frac{\nu}{h^2}[2AD+4AD(2h-L_1)+A[(2h-L_1)^2+B]D]\,[-2w_5+w_1]$$
$$=p\delta(x-L_1)\;\;\delta(y-2h)$$

or,

$$[A[(2h-L_1)^2+B]\,D[20w_5-16w_2-8w_1-8w_1+4w_3]\times\frac{1}{h^4}$$
$$+\frac{\nu}{h^2}[2AD+4AD(2h-L_1)+A[(2h-L_1)+B]D]\,[-2w_5+w_1]\;=\;P$$

As, $\delta(x-l_1)\;\;\delta(y-2h)\;=\;1$

For nodal point 2: x = h, y = 3h

$$[A[(h-L_1)^2+B]\,D[20w_2-8(w_5+w_3)+w_1+2w_2]\times\frac{1}{h^4}$$

$+[2A[2(h-L_1)D+A[(h-L_1)^2+B]D]\,[-4w_5+w_1]+\frac{\nu}{h^2}[2AD+4AD(h-L_1)+A(h-L_1)^2+B]D$ for nodal point 1: x = 2h,

$[-2w_2+w_3)=p\delta(x-l_1)\;\;\delta(y-2h)=0$

y = 2h

$$[A[(2h-L_1)^2+B]\,D[20w_1-16w_5-16w_3+8w_2]\times\frac{1}{h^4}+\frac{\nu}{h^2}[2AD(1+4h-2L)$$
$$+\frac{1}{2A}(2h-L)^2]\,[2w_5-2w_1]=p\delta(x-l_1)\;\delta(y-2h)=0$$

At different nodal points, equations are obtained which are solved to evaluate deflection.

Solution by singularity method for a particular data namely:

Data used for problem 6 with numerical values

A = 0.4375 B = 2.28

h = 2m D = $6\times10^7\;Nm$

$L_1=4m$ $t_{centre}=0.150\,m$

$t_{ends}=0.300\,m$

Substituting the numerical data

From equation 1:

$$-w_1-1.267w_2+1.59w_3+0.35w_5\;\;=\;\;0---------------------1(a)$$

From equation 2:

$$w_1+3.17w_2-0.79w_3-2.79w_5\;\;=\;\;-0.0026-------------------2(a)$$

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From equation 3:

$w_1 + 6.34w_2 - 2.30w_3 - 4w_5 = 0$ --3(a)

From equation 4:

$w_1 + 0.64w_2 - 1.28w_3 - 0.68w_5 = 0$ -------------------------4(a)

Solving equations 1(a), 2(a), 3(a), 4(a) we obtain

$w_5 = -0.0015\,m \quad w_3 = -0.005\,m \quad w_1 = -0.0052\,m \;\&\; w_2 = -0.002\,m$

IV DISCUSSION

Certain discussions included in [1]. The pure finite difference approach [2] ,[3] becomes difficult and not trackable when multiple concentrated loads are present or when number of mathematical discontinuities occur due to loading or geometry or stiffness (E / I variations).

The finite element analysis of plate appears to be the best possible approach to any types of problems of plates [12] [13], in general, but the method requires use of computers and computer time for plate analysis and many times software is not available to the structural designers. Hence cannot get solution certain problems. Also method implies through the knowledge of the method and computer applications including minimum requirement in computer configuration. The proposed method in contrast in ammenable to both computer or both manual calculations. Hence, a boon to a practical designer. In general. Also a closed form solution using Fourier Transform for the basic Partial Differential Equation of rectangular plates of constant stiffness containing singularity functions (where the loarding function q is expressed using singularity functions)[14], [15] can be obtained. For plate problems of variable stiffness the application of Fourier Transform becomes difficult not trackable many times.

V CONCLUSION

The use of singularity function in the analysis of variable stiffness plates not only become superior but also easy to apply and get solution with or without the use of computers. The variation of flexural rigidity D and load variation on the plate q can be handled with ease using singularity functions. It is not essential to have one to one correspondence between the load at nodal point deflection, as in finite difference and finite element approaches. That is, load point and nodal point can be different if singularity functions are used in finite difference method.The application of singularity functions to the analysis of plates is a contribution of mathematics, in the sense, it may portray a new dimension to the potentiality and application of singularity functions. Different types of loads on the plate can be analysed without help of principle of superposition or equivalent representation method, in general.

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