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Solving Single Machine Scheduling Problem Using Type-2 Trapezoidal Fuzzy Numbers

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ABSTRACT: The objective of this paper is to find the single machine scheduling problem of n-jobs which minimizes the sum of the total tardiness of each job using dynamic programming method by type-2 trapezoidal fuzzy numbers. The effectiveness of the proposed method is illustrated by means of an example.

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KEYWORDS: Machine Scheduling, Dynamic Programming Method, Type-2 Trapezoidal Fuzzy Numbers.

I. INTRODUCTION

The single machine scheduling problem with single processor (Machine) consists of single machine to process n-jobs. The jobs may be independent or dependent. An single machine scheduling problems can provide help and insight into resolving, understanding, managing and modeling more complex multi-machine scheduling problems. In this paper, we propose n-jobs to be processed on single machine scheduling problem (SMSP) involving type-2 trapezoidal fuzzy processing times and type-2 trapezoidal fuzzy due date. The different due date for each job be considered which meet the demand of customer with more satisfaction level. The main objective of this paper is to scheduling the job and minimizing the total tardiness. This method is become lucrative to make decision. In most of the real life problem, there are elements of uncertainty in process. In practical situation processing times and due date are not always deterministic. So, we have associated with fuzzy environment. Dynamic programming problem is the main concept of number of jobs is equal to number of stages.

The concept of a type-2 fuzzy set, which is an extension of the concept of an arbitrary fuzzy set, was introduced by Zadeh [15]. A fuzzy set is two dimensional and a type-2 fuzzy set is three dimensional, type-2 fuzzy sets can better improve certain kinds of inference than do fuzzy sets with increasing imprecision, uncertainty and fuzziness in information. A type-2 fuzzy set is characterized by a membership function, i.e., the membership value for each element of this set is a fuzzy set in [0,1], unlike an ordinary fuzzy set where the membership value is a crisp number in [0,1].

1.1REVIEW OF LITERATURE

Various researchers have done a lot of work in different directions. Ishii and Tada [6] considered a single machine scheduling problem minimizing the maximum lateness of jobs with fuzzy precedence relations. Hong et.al., [5] introduced a single machine scheduling problem with fuzzy due date. Itoh and Ishii [7] proposed a single machine scheduling problem dealing with fuzzy processing times and due date. Gawiejnowiez et.al.,[2] deals with a single machine time dependent scheduling problem. Lawler [8] applied a dynamic programming approach to the single machine total tardiness problem. Uzsoy and Velasquez [13] addressed the problem of scheduling a single machine subject to family dependent set-up times in order to minimize maximum lateness. Graham and Lawler [4] introduced on single machine scheduling problem with non-zero ready times. Emmons [1] studied the single machine scheduling problem to minimize the total tardiness. Ghorhanali Mohammad [3] introduced the concept of single machine and the processing times of the jobs in fuzzy environment.

The paper is organized as follows: In section 2, deals with the preliminaries. In section 3, arithmetic operations on type-2 trapezoidal fuzzy number and ranking function are discussed. In section 4, we introduced a brief note on

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single machine scheduling problem. In section 5, the effectiveness of the proposed method is illustrated by means of an example.

II. PRELIMINARIES

2.1 Definition: Fuzzy Set

A fuzzy set is characterized by a membership function mapping the elements of domain, space or universe of discourse X to the unit interval [0,1].

A fuzzy set $\tilde{A}$ is set of ordered pairs { (x, $\mu_{\tilde{A}}$ (x)) / x ε R} where $\mu_{\tilde{A}}(x): R \rightarrow [0,1]$ is upper semi continuous function, $\mu_{\tilde{A}}(x)$ is called a membership function of the fuzzy set.

2.2 Definition: Fuzzy Number

A fuzzy number f in the real line R is a fuzzy set f: R→[0,1] that satisfies the following properties.

(i) f is piecewise continuous.

(ii) There exists an x ε R such that f(x) =1.

(iii) f is convex (i.e) if x_1 , x_2 ε R and then λ ε [0,1] then
$f(\lambda x_1 + (1-\lambda) x_2) \geq f(x_1) \wedge f(x_2)$.

2.3 Definition: Type-2 Fuzzy Set

The type-2 fuzzy sets are defined by functions of the form $\mu_A : X \rightarrow \tilde{\lambda}([0,1])$ where $\tilde{\lambda}([0,1])$ denotes the set of all ordinary fuzzy sets that can be defined within the universal set [0,1]. An example of a membership function of this type is given in fig-1.

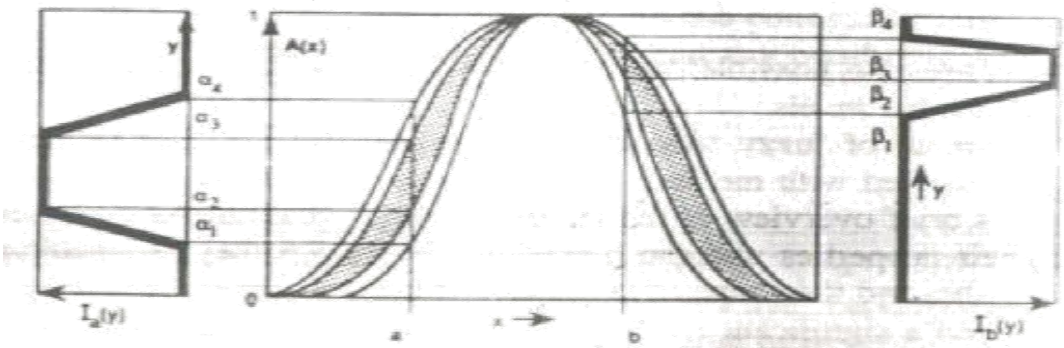

Illustration of the concept of a fuzzy set of type 2.

2.4 Definition: Type-2 Fuzzy Number

Let $\tilde{\tilde{A}}$ be a type-2 fuzzy set defined in the universe of discourse R. If the following conditions are satisfied.

(i) $\tilde{\tilde{A}}$ is normal.

(ii) $\tilde{\tilde{A}}$ is a convex set.

(iii) The support of $\tilde{\tilde{A}}$ is closed and bounded, then $\tilde{\tilde{A}}$ is called a type-2 fuzzy number.

2.5 Definition: Trapezoidal Fuzzy Number

A trapezoidal fuzzy number A = (a_1, a_2, a_3, a_4) whose membership function is given by

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$$\mu_A(x) = \begin{cases} 0 & , x < a_1 \ \& \ x > a_4 \\ \dfrac{x-a_1}{a_2-a_1} & , a_1 \le x \le a_2 \\ 1 \ , & a_2 \le x \le a_3 \\ \dfrac{a_4-x}{a_4-a_3} & , a_3 \le x \le a_4 \end{cases}$$

2.6 Definition: Type-2 Trapezoidal Fuzzy Number

A type-2 trapezoidal fuzzy number $\tilde{\tilde{A}}$ on R is given by

$\tilde{\tilde{A}}$ = { $x, \mu_{\tilde{A}_1}(x), \mu_{\tilde{A}_2}(x), \mu_{\tilde{A}_3}(x), \mu_{\tilde{A}_4}(x))$, xεR } and $\mu_{\tilde{A}_1}(x) \le \mu_{\tilde{A}_2}(x) \le \mu_{\tilde{A}_3}(x) \le \mu_{\tilde{A}_4}(x)$

for all xεR. (ie) $\tilde{\tilde{A}} = \left(\tilde{A}_1, \tilde{A}_2, \tilde{A}_3, \tilde{A}_4\right)$ where

$\tilde{\tilde{A}}$ = ($(a_1^L, a_1^M, a_1^N, a_1^U)$, $(a_2^L, a_2^M, a_2^N, a_2^U)$, $(a_3^L, a_3^M, a_3^N, a_3^U)$, $(a_4^L, a_4^M, a_4^N, a_4^U)$).

III. ARITHMETIC OPERATIONS

3.1 Arithmetic Operations on Type-2 Trapezoidal Fuzzy Numbers:

Let $\tilde{\tilde{A}} = (\tilde{A}_1, \tilde{A}_2, \tilde{A}_3, \tilde{A}_4)$

$= ((a_1^L,a_1^M,a_1^N,a_1^U),(a_2^L,a_2^M,a_2^N,a_2^U),(a_3^L,a_3^M,a_3^N,a_3^U),(a_4^L,a_4^M,a_4^N,a_4^U))$

& $\tilde{\tilde{B}} = \left(\tilde{B}_1, \tilde{B}_2, \tilde{B}_3, \tilde{B}_4\right)$

$= ((b_1^L, b_1^M, b_1^N,b_1^U),(b_2^L,b_2^M,b_2^N, b_2^U),(b_3^L,b_3^M,b_3^N,b_3^U),(b_4^L,b_4^M,b_4^N,b_4^U))$

be two type-2 trapezoidal fuzzy numbers. Then, we define

(i)Addition:

$\tilde{\tilde{A}} + \tilde{\tilde{B}}$ = $\{(a_1^L + b_1^L, a_1^M + b_1^M, a_1^N + b_1^N, a_1^U + b_1^U), (a_2^L + b_2^L, a_2^M + b_2^M, a_2^N + b_2^N, a_2^U + b_2^U), (a_3^L + b_3^L, a_3^M + b_3^M, a_3^N + b_3^N, a_3^U + b_3^U), (a_4^L + b_4^L, a_4^M + b_4^M, a_4^N + b_4^N, a_4^U + b_4^U)\}$.

(ii)Subtraction:

$\tilde{\tilde{A}} - \tilde{\tilde{B}}$ = $\{(a_1^L - b_1^U, a_1^M - b_1^N, a_1^N - b_1^M, a_1^U - b_1^L), (a_2^L - b_2^U, a_2^M - b_2^N, a_2^N - b_2^M, a_2^U - b_2^L), (a_3^L - b_3^U, a_3^M - b_3^N, a_3^N - b_3^M, a_3^U - b_3^L), (a_4^L - b_4^U, a_4^M - b_4^N, a_4^N - b_4^M, a_4^U - b_4^L)\}$.

(iii)Multiplication:

$\tilde{\tilde{A}} x \tilde{\tilde{B}}$ = $\{(a_1^L * b_1^L, a_1^M * b_1^M, a_1^N * b_1^N, a_1^U * b_1^U), (a_2^L * b_2^L, a_2^M * b_2^M, a_2^N * b_2^N, a_2^U * b_2^U), (a_3^L * b_3^L, a_3^M * b_3^M, a_3^N * b_3^N, a_3^U * b_3^U). (a_4^L * b_4^L, a_4^M * b_4^M, a_4^N * b_4^N, a_4^U * b_4^U)\}$.

(iv)Division:

$$\frac{\tilde{\tilde{A}}}{\tilde{\tilde{B}}} = \left\{\left(\frac{a_1^L}{b_1^U}, \frac{a_1^M}{b_1^N}, \frac{a_1^N}{b_1^M}, \frac{a_1^U}{b_1^L}\right), \left(\frac{a_2^L}{b_2^U}, \frac{a_2^M}{b_2^N}, \frac{a_2^N}{b_2^M}, \frac{a_2^U}{b_2^L}\right), \left(\frac{a_3^L}{b_3^U}, \frac{a_3^M}{b_3^N}, \frac{a_3^N}{b_3^M}, \frac{a_3^U}{b_3^L}\right), \left(\frac{a_4^L}{b_4^U}, \frac{a_4^M}{b_4^N}, \frac{a_4^N}{b_4^M}, \frac{a_4^U}{b_4^L}\right)\right\}.$$

3.2 Ranking on Type-2 Trapezoidal Fuzzy Number:

Let F(R) be the set of all type-2 normal trapezoidal fuzzy numbers. One convenient approach for solving numerical valued problem is based on the concept of comparison of fuzzy numbers by use of ranking function. An effective approach for ordering the elements of F (R) is to define a linear ranking function R: F (R) → R which maps each fuzzy number into R.

Suppose if $\tilde{\tilde{A}} = \left(\tilde{A}_1, \tilde{A}_2, \tilde{A}_3, \tilde{A}_4\right)$

$= ((a_1^L, a_1^M, a_1^N, a_1^U), (a_2^L, a_2^M, a_2^N, a_2^U), (a_3^L, a_3^M, a_3^N, a_3^U), (a_4^L, a_4^M, a_4^N, a_4^U)).$

Then we define

$R(\tilde{\tilde{A}}) = a_1^L + a_1^M + a_1^N + a_1^U + a_2^L + a_2^M + a_2^N + a_2^U + a_3^L + a_3^M + a_3^N + a_3^U + a_4^L + a_4^M + a_4^N + a_4^U / 16.$

Also, we define orders on F (R) by

$R(\tilde{\tilde{A}}) \geq R(\tilde{\tilde{B}})$ if and only if $\tilde{\tilde{A}} \geq \tilde{\tilde{B}}$

$R(\tilde{\tilde{A}}) \leq R(\tilde{\tilde{B}})$ if and only if $\tilde{\tilde{A}} \leq \tilde{\tilde{B}}$

$R(\tilde{\tilde{A}}) = R(\tilde{\tilde{B}})$ if and only if $\tilde{\tilde{A}} = \tilde{\tilde{B}}$

IV. SINGLE MACHINE SCHEDULING PROBLEM

Tardiness:

Tardiness is the lateness of job j if it fails to meet its due date; otherwise, it is zero. It is defined as :

$T_j = \max \{ 0, c_j - d_j \} = \max \{ 0, L_j \}$ which means

$$T_j = \begin{cases} c_j - d_j & , if\ c_j > d_j \\ 0 & , otherwise \end{cases}$$

4.1 Notations:

- n : The total number of independent jobs.
- j : Represents the j^{th} job , j = 1,2,…..N
- t_j : The processing time of the job j.
- d_j : The due date of the job j.
- c_j : The completion time of the job j.
- T_j : The tardiness of the job j.
- NT_j : Number of the tardy jobs.
- $T_{m,n}$: Minimum tardiness.
- N : Total number of jobs to be scheduled.
- σ : The set of scheduled jobs at the end of the sequence.
- σ' : The set of unscheduled jobs (or) complement of σ.
- $q_\sigma = q_\varphi$: The sum of the processing times of unscheduled jobs in σ.
- $S(\sigma)$: The sum of the tardiness values of the job in σ.

4.2 Algorithm:

The processing times of jobs and due date are uncertain. This leads to the use of type-2 trapezoidal fuzzy number for representing these imprecise values.

Step-1: $T_j (q_0 + t_j)$ when j is $1 = c_1 - d_1 = (q_1 + t_1) - d_1$.

If $q_\sigma + t_j \leq d_j$ then $T_j (q_\sigma + t_j) = 0$.

Step-2: For σ = {1,2}, S(σ – {j}) for j = 1 is equal to the minimum of S(σ = 2) in stage 1 where, σ contains the remaining job(s) of σ in the current stage under consideration, that is {2}.

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Step-3: For σ = {1,2,3} in stage3, S(σ – {j}) for j = 1 is equal to the minimum of S(σ = 2,3) in stage2 where , σ contains the remaining jobs of σ in the current stage under consideration, that is {2,3}.
Step-4: For σ = { 1,2,3,5} in stage 4, S(σ – {j}) for j = 2 is equal to the minimum of S(σ = 1,3,5) in stage3 where, σ contains the remaining jobs of σ in the current stage under consideration, that is {1,3,5}.
Step-5: For σ = { 1,2,3,4,5} in stage 5, S(σ – {j}) for j = 5 is equal to the minimum of S(σ = 1,2,3,4) in stage 4 where, σ contains the remaining jobs of σ in the current stage under consideration, that is {1,2,3,4}.

V.NUMERICAL ILLUSTRATION

Consider a single machine scheduling problem in which each of the 5 jobs can be fully processed in the single machine. The type-2 trapezoidal fuzzy processing times and type-2 trapezoidal fuzzy due date for each jobs are given in the following table:

Table-1:

Job j	Processing time (t_j)	Due date (d_j)
1	(3,4,6,7), (2,4,6,8), (1,4,6,9), (0,4,6,10)	(6,10,14,18), (5,10,14,19), (4,10,14,20) ,(3,10,14,21)
2	(5,6,10,11),(4,6,10,12), (3,6,10,13),(2,6,10,14)	(8,12,16,20),(7,12,16,21), (6,12,16,22),(5,12,16,23)
3	(4,5,9,10),(3,5,9,11), (2,5,9,12),(1,5,9,13)	(6,10,14,18),(5,10,14,19), (4,10,14,20),(3,10,14,21)
4	(5,6,10,11),(4,6,10,12) (3,6,10,13),(2,6,10,14)	(6,8,10,12),(5,8,10,13), (4,8,10,14),(3,8,10,15)
5	(4,8,12,16),(3,8,12,17), (2,8,12,18),(1,8,12,19)	(14,18,22,26),(13,18,22,27), (12,18,22,28),(11,18,22,29)

Table-2

Jobs in different σ at different stages

Stage	Jobs in different σ
1	{1} {2} {3} {4} {5}
2	{1,2} {1,3} {1,4} {1,5} {2,3} {2,4} {2,5} {3,4} {3,5} {4,5}
3	{1,2,3} {1,2,4} {1,2,5} {1,3,4} {1,3,5} {1,4,5} {2,3,4} {2,3,5} {2,4,5} {3,4,5}
4	{1,2,3,4},{1,2,3,5},{1,2,4,5},{1,3,4,5},{2,3,4,5}
5	{1,2,3,4,5}

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Step-1:

σ	{1}	{2}	{3}	{4}	{5}
q_σ	(18,25,41,48), (14,25,41,52), (10,25,41,56), (6,25,41,60)	(16,23,37,44), (12,23,37,48), (8,23,37,52), (4,23,37,56)	(17,24,38,45), (13,24,38,49), (9,24,38,53), (5,24,38,57)	(16,23,37,44), (12,23,37,48), (8,23,37,52), (4,23,37,56)	(17,21,35,39), (13,21,35,43), (9,21,35,47), (5,21,35,51)
jεσ	[1]	[2]	[3]	[4]	[5]
$T_j(q_0 +t_j)$	(3,15,37,49), (-3,15,37,55), (-9,15,37,61), (-15,15,37,67)	(1,13,35,47), (-5,13,35,53), (-11,13,35,59), (-17,13,35,65)	(3,15,37,49), (-3,15,37,55), (-9,15,37,61), (-15,15,37,67)	(9,19,39,49), (3,19.39,55), (-3,19,39,61), (-9,19,39,67)	(-5,7,29,41), (-11,7,29,47), (-17,7,29,53), (-23,7,29,59)
S(σ-{j})	0	0	0	0	0
Min S(σ)	(3,15,37,49), (-3,15,37,55), (-9,15,37,61), (-15,15,37,67)	(1,13,35,47), (-5,13,35,53), (-11,13,35,59), (-17,13,35,65)	(3,15,37,49), (-3,15,37,55), (-9,15,37,61), (-15,15,37,67)	(9,19,39,49), (3,19,39,55), (-3,19,39,61), (-9,19,39,67)	(-5,7,29,41). (-11,7,29,47), (-17,7,29,53), (-23,7,29,59)

Step-2:

σ	{1,2}		{1,3}	
q_σ	(13,19,31,37), (10,19,31,40), (7,19,31,43), (4,19,31,46)		(14,20,32,38), (11,20,32,41), (8,20,32,44), (5,20,32,47)	
jεσ	1	2	1	3
$T_j(q_0 + t_j)$	(-2,9,27,38), (-7,9,27,43), (-12,9,27,48), (-17,9,27,53)	(-2,9,29,40), (-7,9,29,45), (-12,9,29,50), (-17,9,29,55)	(-1,10,28,39), (-6,10,28,44), (-11,10,28,49), (-16,10,28,54)	(0,11,31,42), (-5,11,31,47), (-10,11,31,52) (-15,11,31,57)
S(σ – {j})	(1,13,35,47), (-5,13,35,53), (-11,13,35,59), (-17,13,35,65)	(3,15,37,49), (-3,15,37,55), (-9,15,37,61), (-15,15,37,67)	(3,15,37,49), (-3,15,37,55), (-9,15,37,61), (-15,15,37,67)	(3,15,37,49), (-3,15,37,55), (-9,15,37,61), (-15,15,37,67)
Min S(σ)	(-1,22,62,85), (-12,22,62,96), (-23,22,62,107), (-34,22,62,118)	-	(2,25,65,88), (-9,25,65,99), (-20,25,65,110), (-31,25,65,121)	-

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σ	{1,4}		{1,5}	
q_σ	(13,19,31,37), (10,19,31,40), (7,19,31,43), (4,19,31,46)		(14,17,29,32), (11,17,29,35), (8,17,29,38), (5,17,29,41)	
jεσ	1	4	1	5
T_j ($q_0 + t_j$)	(-2,9,27,38), (-7,9,27,43), (-12,9,27,48), (-17,9,27,53)	(6,15,33,42), (1,15,33,47), (-4,15,33,52), (-9,15,33,57)	(-1,7,25,33), (-6,7,25,38), (-11,7,25,43), (-16,7,25,48)	(-8,3,23,34), (-13,3,23,39), (-18,3,23,44), (-23,3,23,49)
S(σ – {j})	(9,19,39,49), (3,19,39,55), (-3,19,39,61), (-9,19,39,67)	(3,15,37,49), (-3,15,37,55), (-9,15,37,61), (-15,15,37,67)	(-5,7,29,41), (-11,7,29,47), (-17,7,29,53), (-23,7,29,59)	(3,15,37,49), (-3,15,37,55), (-9,15,37,61), (-15,15,37,67)
Min S(σ)	(7,28,66,87), (-4,28,66,98), (-15,28,66,109), (-26,28,66,120)	-	(-6,14,54,74), (-17,14,54,85), (-28,14,54,96), (-39,14,54,107)	-
σ	{2,3}		{2,4}	
q_σ	(12,18,28,34), (9,18,28,37), (6,18,28,40), (3,18,28,43)		(11,17,27,33), (8,17,27,36), (5,17,27,39), (2,17,27,42)	
jεσ	2	3	2	4
T_j ($q_0 + t_j$)	(-3,8,26,37), (-8,8,26,42), (-13,8,26,47), (-18,8,26,52)	(-2,9,27,38), (-7,9,27,43), (-12,9,27,48), (-17,9,27,53)	(-4,7,25,36), (-9,7,25,41), (-14,7,25,46), (-19,7,25,51)	(4,13,29,38), (-1,13,29,43), (-6,13,29,48), (-11,13,29,53)
S(σ – {j})	(3,15,37,49), (-3,15,37,55), (-9,15,37,61), (-15,15,37,67)	(1,13,35,47), (-5,13,35,53), (-11,13,35,59), (-17,13,35,65)	(9,19,39,49), (3,19,39,55), (-3,19,39,61), (-9,19,39,67)	(1,13,35,47), (-5,13,35,53), (-11,13,35,59), (-17,13,35,65)
Min S(σ)	-	(-1,22,62,85), (-12,22,62,96), (-23,22,62,107), (-34,22,62,118)	(5,26,64,85), (-6,26,64,96) (-17,26,64,107), (-28,26,64,118)	(5,26,64,85), (-6,26,64,96), (-17,26,64,107), (-28,26,64,118)

σ	{2,5}		{3,4}	
q_σ	(12,15,25,28), (9,15,25,31), (6,15,25,34), (3,15,25,37)		(12,18,28,34), (9,18,28,37), (6,18,28,40), (3,18,28,43)	
jεσ	2	5	3	4
$\mathbf{T}_j (q_0 + t_j)$	(-3,5,23,31), (-8,5,23,36), (-13,5,23,41), (-18,5,23,46)	(-10,1,19,30), (-15,1,19,35), (-20,1,19,40), (-25,1,19,45)	(-2,9,27,38), (-7,9,27,43), 9-12,9,27,48), (-17,9,27,53)	(5,14,30,39), (0,14,30,44), (-5,14,30,49), (-10,14,30,54)
S(σ – {j})	(-5,7,29,41), (--1,7,29,47), (17,7,29,53), (-23,7,29,59)	(1,13,35,47), (-5,13,35,53), (11,13,35,59), (-17,13,35,65)	(9,19,39,49), (3,19,39,55), (-3,19,39,61), (-9,19,39,67)	(3,15,37,49), (-3,15,37,55), (-9,15,37,61), (-15,15,37,67)
Min S(σ)	(-8,12,52,72), (-19,12,52,83), (-30,12,52,94), (-41,12,52,105)	-	(7,28,66,87), (-4,28,66,98), (-15,28,66,109), (-26,28,66,120)	-
σ	{3,5]		{4,5}	
q_σ	(13,16,26,29), (10,16,26,32), (7,16,26,35), (4,16,26,38)		(12,15,25,28), (9,15,25,31), (6,15,25,34), (3,15,25,37)	
jεσ	3	5	4	5
$\mathbf{T}_j (q_0 + t_j)$	(-1,7,25,33), (-6,7,25,38), (-11,7,25,43), (-16,7,25,48)	(-9,2,20,31), (-14,2,20,36), (-19,2,20,41), (-24,2,20,46)	(5,11,27,33), (0,11,27,38), (-5,11,27,43), (-10,11,27,48)	(-10,1,19,30), (-15,1,19,35), (-20,1,19,40), (-25,1,19,45)
S(σ – {j})	(-5,7,29,41), (-11,7,29,47), (-17,7,29,53), (-23,7,29,59)	(3,15,37,49), (-3,15,37,55), (-9,15,37,61), (-15,15,37,67)	(-5,7,29,41), (-11,7,29,47), (-17,7,29,53), (-23,7,29,59)	(9,19,39,49), (3,19,39,55), (-3,19,39,61), (-9,19,39,67)
Min S(σ)	(-6,14,54,74), (-17,14,54,85), (-28,14,54,96), (-39,14,54,107)	-	(0,18,56,74), (-11,18,56,85), (-22,18,56,96), (-33,18,56,107)	-

Proceeding in this way, we get

σ	{1,2,3,4,5}				
q_σ	0				
jεσ	1	2	3	4	5
$\mathbf{T}_j (q_0 + t_j)$	0	0	0	0	0
S(σ – {j})	(-22,11,75,108), (-40,11,75,126), (-58,11,75,144), (-76,11,75,162)	(-16,15,77,108), (-34,15,77,126), (-52,15,77,144), (-70,15,77,162)	(-22,11,75,108), (-40,11,75,126), (-58,1175,144), (-76,11,75,162)	(-24,9,73,106), (-42,9,73,124), (-60,9,73,142), (-78,9,73,160)	(-16,26,90,132), (-34,26,90,150), (-52,26,90,168), (-70,26,90,186)
Min S(σ)	(-22,11,75,108), (-40,11,75,126), (-58,11,75,144), (-76,11,75,162)	(-16,15,77,108), (-34,15,77,126), (-52,15,77,144), (-70,15,77,162)	(-22,11,75,108), (-40,11,75,126), (-58,11,75,144), (-76,11,75,162)	(-24,9,73,106), (-42,9,73,124), (-60,9,73,142), (-78,9,73,160)	(-16,26,90,132), (-34,26,90,150), (-52,26,90,168), (-70,26,90,186)

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Optimal Sequence of Jobs = 4 → 1 →3 → 2 → 5
Minimum Total Tardiness = 41.

V. CONCLUSION

We considered a single machine scheduling problem (SMSP) with fuzzy processing times and fuzzy due date to minimize the total tardiness. This method is very easy to understand each stage that will help the decision maker in determining a best schedule for a given set of jobs effectively. This method has significant use of practical results in industries.

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