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Some Common Fixed Point Theorems in Cone Rectangular Metric Space under T – Kannan and T – Reich Contractive Conditions

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ABSTRACT: The purpose of this paper is to establish some common fixed point theorems for two self mappings which satisfy T- Kannan and T- Reich contractive conditions in cone rectangular metric space.

KEYWORDS: cone rectangular metric space, common fixed point theorem, coincidence point, contractive condition.

I. INTRODUCTION

Recently, Huang and Zhang [5] introduced the notion of cone metric space. They have replaced real number system by an ordered Banach space and established some fixed point theorems for contractive type mappings in a normal cone metric space. The study of fixed point theorems in such spaces is followed by some other mathematicians; see [1], [5], [8], [11], [14]. In 2009, Azam, Arshad and Beg [2] extended the notion of cone metric spaces by replacing the triangular inequality by a rectangular inequality and they proved Banach contraction Principle in a complete normal cone rectangular metric space. Several authors proved some fixed point theorems in such spaces see; [6], [9], [10], [12], [15]. In 2009, Jleli, Samet [6] extended the Kannan's fixed point theorem in a complete normal cone rectangular metric space. In 2012, R. A. Rashwan and S. M. Saleh [12] extended Banach contraction principle in cone rectangular metric space with two self mappings and proved common fixed point theorem for T- contractive condition in cone rectangular metric space. In 2013, Malhotra et al. [10] generalized the result of Azam et al. [2] in ordered cone rectangular metric space and proved some fixed point results for ordered Reich type contractions. In this paper, we prove some common fixed point theorems for two self mappings which satisfy T – Kannan and T – Reich contractive conditions in cone rectangular metric space. Our results generalize and extend the results of M. Jleli et al. [6] and Malhotra et al. [10] on cone rectangular metric spaces.

II. PRELIMINARIES

First, we recall some standard definitions and other results that will be needed in the sequel.

2.1. Definition [5]: A subset P of a real Banach space E is called a *'cone'* if it has following properties:

(1) P is non empty, closed and $P \neq \{\theta\}$;

(2) $a, b \in R,\ a,\ b \geq 0,\ x,\ y \in P$ implies $ax + by \in P$;

(3) $P \cap (-P) = \{\theta\}$.

For a given cone $P \subseteq E$, we can define a partial ordering $\leq$ on E with respect to P by $x \leq y$ if and only if $y - x \in P$. We shall write $x < y$ if $x \leq y$ and $x \neq y$, while $x << y$ will stands for $y - x \in \operatorname{int}(P)$, where $\operatorname{int}(P)$ denotes the interior of P.

2.2. Definition [5]: The cone P is called *'normal'* if there is a number k > 1 such that for all x, y ∈ X $0 \le x \le y$ implies $\|x\| \le k\|y\|$.

The least positive number k satisfying the above condition is called the *normal constant* of P.

2.3. Definition [5]: Let X is a non empty set, E is a real Banach space and P is a cone in E with $\operatorname{int} P \ne \Phi$ and $\le$ is a partial ordering with respect to P. Suppose the mapping $d : X \times X \to E$ satisfies:

(1) $\theta \le d(x, y)$, for all $x, y \in X$ and $d(x, y) = \theta$ if and only if $x = y$;

(2) $d(x, y) = d(y, x)$, for all $x, y \in X$;

(3) $d(x, y) \le d(x, z) + d(z, y)$, for all $x, y, z \in X$.

Then d is called a *'cone metric'* on X and (X, d) is called a *'cone metric space'*.

2.4. Definition [2]: Let X is a non empty set, E is a real Banach space and P is a cone in E with $\operatorname{int} P \ne \Phi$ and $\le$ is a partial ordering with respect to P. Suppose that the mapping $d : X \times X \to E$ satisfies:

(1) $\theta \le d(x, y)$, for all $x, y \in X$ and $d(x, y) = \theta$ if and only if $x = y$;

(2) $d(x, y) = d(y, x)$, for all $x, y \in X$;

(3) $d(x, y) \le d(x, w) + d(w, z) + d(z, y)$, for all x, y $\in X$ and for all distinct points $w, z \in X - \{x, y\}$ [rectangular property].

Then d is called a *'cone rectangular metric'* on X and (X, d) is called a *'cone rectangular metric space'*.

2.5. Definition [2]: Let (X, d) be a cone rectangular metric space. Let $\{x_n\}$ be a sequence in (X, d) and $x \in X$. If for every $c \in E$, with $\theta << c$ there is $n_0 \in N$ such that for all $n > n_0$, $d(x_n, x) << c$, then $\{x_n\}$ is said to be convergent, $\{x_n\}$ converges to x and x is the limit of $\{x_n\}$. We denote this by $\lim_{n\to\infty} x_n = x$ or $x_n \to x$, as $n \to \infty$.

2.6. Definition [2]: Let (X, d) be a cone rectangular metric space. Let $\{x_n\}$ be a sequence in (X, d). If for every $c \in E$, with $\theta << c$ there is $n_0 \in N$ such that for all $m, n > n_0$, $d(x_n, x_m) << c$, then $\{x_n\}$ is called a Cauchy sequence in (X, d).

2.7. Definition [2]: Let (X, d) be a cone rectangular metric space. If every Cauchy sequence is convergent in (X, d), then (X, d) is called a complete cone rectangular metric space.

2.8. Lemma [2]: Let (X, d) be a cone rectangular metric space and P be a normal cone with normal constant k, let $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ converges to x if and only if $\|d(x_n, x)\| \to 0$, as $n \to \infty$.

2.9. Lemma [2]: Let (X, d) be a cone rectangular metric space, P be a normal cone with normal constant k. Let $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ is a Cauchy sequence if and only if $\|d(x_n, x_m)\| \to 0$, *as* $n, m \to \infty$.

2.10. Definition [3]: A self map T: X→X on a metric space (X, d) is called *'Banach contraction'* if for all $x, y \in X$, there exists $\lambda \in [0, 1)$ such that,

$$d(Tx, Ty) \le \lambda\, d(x, y).$$

2.11. Definition [7]: A self map T: X→X on a metric space (X, d) is called *'Kannan contraction'* if for all $x, y \in$ X, there exists $\lambda \in [0, 1/2)$ such that,

$$d(Tx, Ty) \le \lambda\, [d(x, Tx) + d(y, Ty)].$$

2.12. Definition [4]: Let T and S be two self mapping of a metric space (X, d). The self mapping S of X is said to be a *'T – contraction'* if for all $x, y \in X$ there exists a real number $0 \le \lambda < 1$ such that

$$d(TSx, TSy) \le \lambda\, d(Tx, Ty).$$

2.13. Definition [13]: A self map T: X→X on a metric space (X, d) is called *'Reich contraction'* if for all $x, y \in X$, there exists $\lambda, \mu, \delta \in [0, 1)$ with $\lambda + \mu + \delta < 1$ such that,

$$d(Tx, Ty) \le \lambda d(x, y) + \mu\, d(x, Tx) + \delta\, d(y, Ty).$$

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III. MAIN RESULTS

First we give definitions of T- Kannan contractive and T-Reich contractive mappings on cone rectangular metric spaces which are based on the ideas of Morales and Rojas [11].

3.1. Definition: Let (X, d) be a cone rectangular metric space and T, S: X→X two functions:

(i) A mapping S is said to be '*T – Kannan contraction*', if there is $\lambda \in [0, ½)$ such that,

$$d\left(TSx, TSy\right) \le \lambda\left[d\left(Tx, TSx\right) + d\left(Ty, TSy\right)\right],$$

for all $x, y \in X$.

(ii) A mapping S is said to be '*T – Reich contraction*', if there is $\lambda_1, \lambda_2, \lambda_3 \ge 0$ with $\lambda_1 + \lambda_2 + \lambda_3 < 1$ such that,

$$d\left(TSx, TSy\right) \le \lambda_1 d\left(Tx, TSx\right) + \lambda_2 d\left(Ty, TSy\right) + \lambda_3 d\left(Tx, Ty\right),$$

for all $x, y \in X$.

3.2. Theorem: Let (X, d) be a cone rectangular metric space, P be a normal cone with normal constant k and let the mappings S and T: X →X satisfy the following:

$$d\left(TSx, TSy\right) \le \lambda\left[d\left(Tx, TSx\right) + d\left(Ty, TSy\right)\right], \tag{1}$$

for all x, y ∈ X, where $\lambda \in [0, ½)$. Suppose T is one to one and T(X) is a complete subspace of X, then the mapping S has a unique fixed point in X. Moreover, if S and T are commuting at the fixed point of S, then S and T have a unique common fixed point in X.

Proof: Let x_0 be an arbitrary point in X. Define a sequence $\{x_n\}$ in X such that $x_{n+1} = Sx_n$, for all $n = 0,1,2, \ldots$.
If $x_m = x_{m+1}$, for some $m \in$ N, then $x_m = Sx_m$. That is, S has a fixed point x_m in X.
Assume $x_n \neq x_{n+1}$, for all $n \in$ N. Then from (1) it follows that,

$$\begin{aligned} d(Tx_n, Tx_{n+1}) &= d(TSx_{n-1}, TSx_n) \\ &\le \lambda\left[d\left(Tx_{n-1}, TSx_{n-1}\right) + d\left(Tx_n, TSx_n\right)\right] \\ &= \lambda\left[d\left(Tx_{n-1}, Tx_n\right) + d\left(Tx_n, Tx_{n+1}\right)\right] \end{aligned}$$

which implies that,

$$d(Tx_n, Tx_{n+1}) \le \frac{\lambda}{1-\lambda} d(Tx_{n-1}, Tx_n), \text{ for all n} = 0,1,2, \ldots.$$

Thus, $d(Tx_n, Tx_{n+1}) \le \mu d(Tx_{n-1}, Tx_n) \le \mu^2 d(Tx_{n-2}, Tx_{n-1}) \le \ldots \le \mu^n d(Tx_0, Tx_1)$, (2)

For all $n \ge 0$, where $\mu = \dfrac{\lambda}{1-\lambda} < 1$.

From (1), (2) and using the facts $\lambda \le \mu$ and $0 \le \lambda < ½ < 1$, we get,

$$\begin{aligned} d(Tx_n, Tx_{n+2}) &= d(TSx_{n-1}, TSx_{n+1}) \\ &\le \lambda\left[d\left(Tx_{n-1}, TSx_{n-1}\right) + d\left(Tx_{n+1}, TSx_{n+1}\right)\right] \\ &= \lambda\left[d\left(Tx_{n-1}, Tx_n\right) + d\left(Tx_{n+1}, Tx_{n+2}\right)\right] \\ &\le \lambda\left[\mu^{n-1} d\left(Tx_0, Tx_1\right) + \mu^{n+1} d\left(Tx_0, Tx_1\right)\right] \\ &\le \mu^n d\left(Tx_0, Tx_1\right) + \mu^{n+1} d\left(Tx_0, Tx_1\right) \\ &= (1+\mu)\mu^n d\left(Tx_0, Tx_1\right), \text{ for all } n \ge 0 \end{aligned} \tag{3}$$

For the sequence $\{Tx_n\}$ we consider $d(Tx_n, Tx_{n+p})$ in two cases.
If p is odd say $2m+1$, for $m \ge 1$, then by using rectangular inequality and (2) we get,

$$d(Tx_n, Tx_{n+2m+1}) \le d(Tx_{n+2m+1}, Tx_{n+2m}) + d(Tx_{n+2m}, Tx_{n+2m-1}) + d(Tx_{n+2m-1}, Tx_n)$$

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$$
\begin{aligned}
&\le d(Tx_{n+2m},Tx_{n+2m+1})+d(Tx_{n+2m-1},Tx_{n+2m})+d(Tx_{n+2m-1},Tx_{n+2m-2})\\
&\quad +d(Tx_{n+2m-2},Tx_{n+2m-3})+\ \ldots\ +d(Tx_{n+2},Tx_{n+1})+d(Tx_{n+1},Tx_{n})\\
&= d(Tx_{n},Tx_{n+1})+d(Tx_{n+1},Tx_{n+2})+\ \ldots\ +d(Tx_{n+2m-1},Tx_{n+2m})+d(Tx_{n+2m},Tx_{n+2m+1})\\
&\le \mu^{n}d(Tx_0,Tx_1)+\mu^{n+1}d(Tx_0,Tx_1)+\ \ldots\ +\mu^{n+2m-1}d(Tx_0,Tx_1)+\mu^{n+2m}d(Tx_0,Tx_1)\\
&\le \left[1+\mu+\mu^2+\mu^3+\ \ldots\ \right]\mu^{n}d(Tx_0,Tx_1)
\end{aligned}
$$

Hence, $d(Tx_n,Tx_{n+2m+1})\le \dfrac{\mu^n}{1-\mu}d(Tx_0,Tx_1),$ for all $n\ge 1, m\ge 1.$

If p is even say $2m$, for $m \ge 1$, then by using rectangular inequality, (2), (3) and the fact that $\mu<1$ we get,

$$
\begin{aligned}
d(Tx_n,Tx_{n+2m}) &\le d(Tx_{n+2m},Tx_{n+2m-1})+d(Tx_{n+2m-1},Tx_{n+2m-2})+d(Tx_{n+2m-2},Tx_{n})\\
&\le d(Tx_{n+2m-1},Tx_{n+2m})+d(Tx_{n+2m-2},Tx_{n+2m-1})+\ldots+d(Tx_{n+4},Tx_{n+3})+d(Tx_{n+3},Tx_{n+2})+d(Tx_{n+2},Tx_{n})\\
&= d(Tx_{n},Tx_{n+2})+d(Tx_{n+2},Tx_{n+3})+d(Tx_{n+3},Tx_{n+4})+\ldots+d(Tx_{n+2m-2},Tx_{n+2m-1})+d(Tx_{n+2m-1},Tx_{n+2m})\\
&\le (1+\mu)\mu^{n}d(Tx_0,Tx_1)+\left[\mu^{n+2}d(Tx_0,Tx_1)+\mu^{n+3}d(Tx_0,Tx_1)\right.\\
&\qquad\qquad \left.+\ \ldots\ +\mu^{n+2m-2}d(Tx_0,Tx_1)+\mu^{n+2m-1}d(Tx_0,Tx_1)\right]\\
&\le \left[1+\mu+\mu^2+\mu^3+\ \ldots\ \right]\mu^{n}d(Tx_0,Tx_1)
\end{aligned}
$$

Hence, $d(Tx_n,Tx_{n+2m})\le \dfrac{\mu^n}{1-\mu}d(Tx_0,Tx_1),$ for all $n\ge 1, m\ge 1.$

Thus, combining the above two cases we have,

$$d(Tx_n,Tx_{n+p})\le \frac{\mu^n}{1-\mu}d(Tx_0,Tx_1),\ \text{for all } n\in N,\ p\in N.$$

Since, P is a normal cone with normal constant k and $\mu<1$, we have

$$\left\|d(Tx_n,Tx_{n+p})\right\|\le \frac{k\mu^n}{1-\mu}\left\|d(Tx_0,Tx_1)\right\|\to 0,\ \text{as } n\to\infty,$$

$$i.e.,\ \left\|d(Tx_n,Tx_{n+p})\right\|\to 0,\ \text{as } n\to\infty,\quad \forall\, p\in N. \tag{4}$$

Therefore, $\{Tx_n\}$ is a Cauchy sequence in X.
Since, $T(X)$ is a complete subspace of X, then there exists a point z in $T(X)$ such that

$$\lim_{n\to\infty} Tx_{n+1}=\lim_{n\to\infty} TSx_n=z \tag{5}$$

Also, we can find $x\in X$ such that $z = Tx$
We shall show that $TSx = z$.
Using Rectangular property and (1) we get,

$$
\begin{aligned}
d(z,TSx) &\le d(z,Tx_n)+d(Tx_n,TSx_n)+d(TSx_n,TSx)\\
&\le d(z,Tx_n)+d(Tx_n,Tx_{n+1})+\lambda\left[d\left(Tx_n,TSx_n\right)+d\left(Tx,TSx\right)\right]\\
&\le d(z,Tx_n)+d(Tx_n,Tx_{n+1})+\lambda\left[d\left(Tx_n,Tx_{n+1}\right)+d\left(z,TSx\right)\right]
\end{aligned}
$$

Which implies that,

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$$d(z,TSx) \le \frac{1}{1-\lambda} d(z,Tx_n) + \frac{1+\lambda}{1-\lambda} d(Tx_n,Tx_{n+1}), \text{ for all } n \ge 1.$$

Since, P is a normal cone with normal constant *k*, using (4) and (5) we get,

$$\|d(z,TSx)\| \le \frac{k}{1-\lambda}\|d(z,Tx_n)\| + \frac{1+\lambda}{1-\lambda} k \|d(Tx_n,Tx_{n+1})\| \to 0, \text{as n} \to \infty,$$

$$i.e., \ \|d(z,TSx)\| = 0$$

Therefore, $TSx = Tx = z$. Since T is one to one, $x = Sx$.
Hence, x is a fixed point of S in X.
Now, we prove the uniqueness of the fixed point of S.
Let y be another fixed point of S, that is $y = Sy$, then,

$$d(Tx,Ty) = d(TSx,TSy) \le \lambda\left[d\left(Tx,\ TSx\right) + d\left(Ty,TSy\right)\right] = \lambda\left[d\left(Tx,\ Tx\right) + d\left(Ty,Ty\right)\right] = \theta.$$

Hence, $Tx = Ty$. Since T is one to one, we conclude that $x = y$.
Since, S and T are commuting at the fixed point of S, $TSx = STx = Tx$. Therefore Tx is a fixed point of S. Since S has a unique fixed point, $Tx = x$. Hence $Tx = Sx = x$, x is a unique common fixed point of S and T in X.

3.3. Corollary [6]: Let (X, d) be a complete cone rectangular metric space, P be a normal cone with normal constant *k*. Suppose the mapping $\mathrm{T} : \mathrm{X} \to \mathrm{X}$ satisfies the contractive condition:

$$d\left(Tx,Ty\right) \le \lambda\left[d\left(Tx,x\right) + d\left(Ty,y\right)\right],$$

for all $x, y \in X$ where $\lambda \in [0, ½)$. Then T has a unique fixed point in X.

Proof: If we put $S = I$, in Theorem (3.2), where I is the identity mapping in X, then we obtain the corollary.

3.4. Theorem: Let (X, d) be a cone rectangular metric space, P be a normal cone with normal constant *k* and let the mappings S and T: X →X satisfy the following:

$$d\left(TSx,TSy\right) \le \lambda_1 d\left(Tx,TSx\right) + \lambda_2 d\left(Ty,TSy\right) + \lambda_3 d\left(Tx,Ty\right), \tag{6}$$

for all $x, y \in$ X and $\lambda_1, \lambda_2, \lambda_3 \ge 0$ with $\lambda_1 + \lambda_2 + \lambda_3 < 1$. Suppose T is one to one and T(X) is a complete subspace of X, then the mapping S has a unique fixed point in X. Moreover, if S and T are commuting at the fixed point of S, then S and T have a unique common fixed point in X.

Proof: Let x_0 be an arbitrary point in X. Define a sequence $\{x_n\}$ in X such that $x_{n+1} = Sx_n$, for all $n = 0,1,2, \ldots$.
If $x_m = x_{m+1}$, for some $m \in$ N, then $x_m = Sx_m$. That is, S has a fixed point x_m in X.
Assume $x_n \neq x_{n+1}$, for all $n \in$ N. Then from (6) it follows that,

$$\begin{aligned} d(Tx_n,Tx_{n+1}) &= d(TSx_{n-1},TSx_n) \\ &\le \lambda_1 d\left(Tx_{n-1},TSx_{n-1}\right) + \lambda_2 d\left(Tx_n,TSx_n\right) + \lambda_3 d\left(Tx_{n-1},Tx_n\right), \\ &= \lambda_1 d\left(Tx_{n-1},Tx_n\right) + \lambda_2 d\left(Tx_n,Tx_{n+1}\right) + \lambda_3 d\left(Tx_{n-1},Tx_n\right), \end{aligned}$$

which implies that,

$$d(Tx_n,Tx_{n+1}) \le \frac{\lambda_1+\lambda_3}{1-\lambda_2} d(Tx_{n-1},Tx_n), \text{ for all n} = 0,1,2, \ldots.$$

Thus, $d(Tx_n,Tx_{n+1}) \le \lambda d(Tx_{n-1},Tx_n) \le \lambda^2 d(Tx_{n-2},Tx_{n-1}) \le \ldots \le \lambda^n d(Tx_0,Tx_1),$ (7)

for all $n \ge 0$, where $\lambda = \dfrac{\lambda_1+\lambda_3}{1-\lambda_2} < 1.$

Using the same argument in the proof of theorem (3.2) to prove that $\{Tx_n\}$ is a Cauchy sequence in X.
Since, $T(X)$ is a complete subspace of X, then there exists a point z in $T(X)$ such that $\lim_{n\to\infty} Tx_{n+1} = \lim_{n\to\infty} TSx_n = z.$
Consequently, we can find $x \in X$ such that $z = Tx$

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Now consider,

$$\begin{aligned} d(z,TSx) &\le d(z,Tx_n)+d(Tx_n,TSx_n)+d(TSx_n,TSx) \\ &\le d(z,Tx_n)+d(Tx_n,Tx_{n+1})+\left[\lambda_1 d\left(Tx_n,TSx_n\right)+\lambda_2 d\left(Tx,TSx\right)+\lambda_3 d\left(Tx_n,Tx\right)\right] \\ &\le d(z,Tx_n)+d(Tx_n,Tx_{n+1})+\left[\lambda_1 d\left(Tx_n,Tx_{n+1}\right)+\lambda_2 d\left(z,TSx\right)+\lambda_3 d\left(Tx_n,z\right)\right] \end{aligned}$$

Which implies that,

$$d(z,TSx) \le \frac{1+\lambda_3}{1-\lambda_2} d(z,Tx_n)+\frac{1+\lambda_1}{1-\lambda_2} d(Tx_n,Tx_{n+1}), \text{ for all } n \ge 1.$$

Since, P is a normal cone with normal constant *k*, using (7) and (8) we get,

$$\left\|d(z,TSx)\right\| \le \frac{1+\lambda_3}{1-\lambda_2} k\left\|d(z,Tx_n)\right\|+\frac{1+\lambda_1}{1-\lambda_2} k\left\|d(Tx_n,Tx_{n+1})\right\| \to 0, \text{as n} \to \infty,$$

$$i.e., \left\|d(z,TSx)\right\| = 0$$

Therefore, $TSx = Tx = z$. Since T is one to one, $x = Sx$.
Hence x is a fixed point of S.
Now, we prove the uniqueness of the fixed point of S.
Let y be another fixed point of S, that is $y = Sy$, then,

$$\begin{aligned} d(Tx,Ty) &= d(TSx,TSy) \\ &\le \lambda_1 d\left(Tx,TSx\right)+\lambda_2 d\left(Ty,TSy\right)+\lambda_3 d(Tx,Ty) \\ &\le \lambda_1 d\left(Tx,Tx\right)+\lambda_2 d\left(Ty,Ty\right)+\lambda_3 d(Tx,Ty) \\ &\le \lambda_3 d(Tx,Ty) \end{aligned}$$

Since, $\lambda_3 < 1$, $d(Tx, Ty) = \Theta$ which implies that $Tx = Ty$. Since T is one to one, we conclude that $x = y$.
Since, S and T are commuting at the fixed point of S, $TSx = STx = Tx$. Therefore Tx is a fixed point of S. Since S has a unique fixed point, $Tx = x$. Hence $Tx = Sx = x$, x is unique common fixed point of S and T in X.

IV. CONCLUSION

In this article we have proved that the existence and uniqueness of common fixed point theorems for T-Kannan and T-Reich contractions in cone rectangular metric spaces. We note that the results of this paper generalize the results of M. Jleli et al. [6] and Malhotra et al. [10] on cone rectangular metric spaces.

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