Research and Reviews: Journal of Statistics and Mathematical Science

Some Prime Labeling of Graph

B Kavitha[1*], C Vimala[2]

[1]Department of Mathematics, Pondicherry University, Pondicherry, India

[2]Department of Mathematics, Vallam University, Tamil Nadu, India

Research Article

Received: 03-Mar-2022, Manuscript No. JSMS-22-55261; **Editor assigned:** 07-Mar-2022, Pre QC No. JSMS-22-52261(PQ); **Reviewed:** 21-Mar-2022, QC No. JSMS-22-55261; **Revised:** 02-May-2022, Manuscript No. JSMS-22-55261 (R); **Published:** 17-May-2022, DOI: 10.4172/JSMS.8.5.006.

***For Correspondence :** B. Kavitha Department of Mathematics, Pondicherry University Pondicherry, India

Email:kavithajai152014@gmail.com

Keywords: Labeling; Prime labeling; Prime labeling of path graph; Cycle graph; Crown graph; Fan graph; Star graph; Wheel graph

ABSTRACT

A graph G=(V(G), E(G)) with vertex set V is said to have a prime labeling if its vertices can be labeled with distinct positive integer 1,2,3 V such that for edge u v ∈ E(G), the labels assigned to u and v are relatively prime. A graph which admits prime labeling is called a prime graph. Graph labeling is an important area of research in Graph theory. There are many kinds of graph labeling such as graceful labeling, Magic labeling, Prime labeling, and other different labeling techniques. In this paper we discuss prime labeling for some graphs.

We also discuss prime labeling in the related of some graph operations namely cycle, path, crown, Fan, star and wheel graph.

INTRODUCTION

In this article, we consider only finite simple undirected graph [1]. The graph G has vertex set V=V(G) and edge set E=E(G). The labeling of a graph G is an assigning of integers either to the vertices or edges or both subject to certain conditions. The notion of a prime labeling was introduced by Roger Entringer and was discussed in a paper [2] for notations and terminology, [3]. Many researchers have studied prime graph for example in Fu [4]. H has proved that the path P_n on n vertices is a prime graphhave proved that the Cn on n vertices is a prime graph [5]. We refer to have proved Edge Vertex Prime Labeling for Wheel, Fan and Friendship Graph [6]. have proved that wheel Wn is a prime graph [7]. have proved the prime labeling for some Fan related graphs [8]. For latest survey on graph labeling, we refer to [9] have proved the prime labeling for some cycle related graphs [10] have proved the Prime labeling for some fan related graphs. The following definitions and notations are used in main results [11].

MATERIALS AND METHODS

- Let G=(V (G), E (G)) be a graph with p vertices. A bijection f: V (G)→{1,2, … p} is called a prime labeling if for each edge e={u, v} belongs to E, we have gcd { (u), f (v) } =1. A graph which admits prime labeling is called a prime graph.
- A simple graph of 'n' vertices (n ≥ 3) and n edges forming a cycle of length 'n' is called as a cycle graph. In a cycle graph, all the vertices are of degree is 2. By adding the path, the new vertices of $v_1,v_2,...v_m$, and the new graph G is denoted by C_n P_m.
- The crown graph on 2n vertices is an undirected graph with two set of vertices $\{u_1,u_2, ... u_n\}$ and $\{v_1,v_2,...v_n\}$ and with on edge from ui to vj whenever i ≠ j. By adding the path, the new vertices of $w_1,w_2,...w_m$, and the new graph G is denoted by crown C_n P_m.
- The Friendship graph F n is a graph which consists of η −triangles with a common vertex. If V (G) = 2n+1 and E(G) = 3_n by adding the path, the new vertices of $v_1,v_2,...v_m$, and the new graph G is denoted by F_n P_m.
- The star graph S_n is special type of graph in which n-1 vertices have degree 1 and single vertex have n-1 degree. This look like n-1 vertex is connected to central vertex. A star graph wit total n vertex is termed as S_n. By adding the path, the new vertices of $v_1,v_2,...v_m$, and the new graph G is denoted by S_n P_m.
- The wheel graph W_n is obtained by joining all vertices of a cycle C_n to a further vertex is called center. If V(G)=n+1 and E (G)=2n by adding the path, the new vertices of $v_1,v_2,...v_m$, and the new graph G is denoted by W_n P_m.
- Gear graph G_n also known as a bipartite wheel graph is a wheel graph with a vertex added between each pair of adjacent vertices of the outer cycle. Gear graph G_n has 2r+1 vertices and 3r edges. By adding the path, the new vertices of $w_1,w_2,...w_m$, and the new graph G is denoted by G_n Pm.

RESULTS AND DISCUSSION

Theorem

The cycle and path graph are a prime graph. Then the graph C_n P_m prime labeling of the graph.

Proof

Let G be the graph obtained by joining cycle C_n and a path P_m, then the graph C_n P_m admit to prime labeling of the graph.

Let $u_1,u_2,...$ un be the vertices of cycle C_n and $v_1,v_2,...v_n$ be the vertices of path P_m.

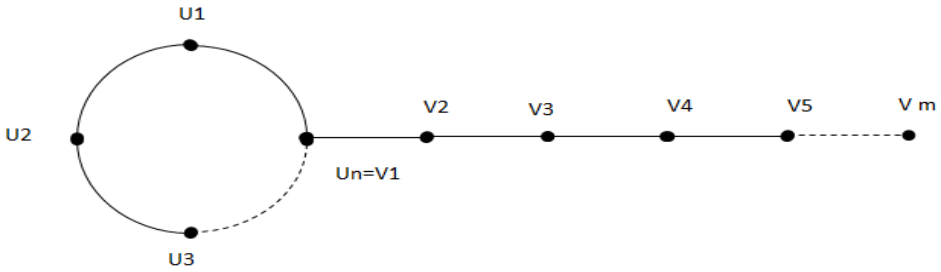
$u_i = \mathrm{i}; 1 \le i \le \mathrm{n}$

Assume $u_n = \mathrm{v}_1$

$v_{i+1} = n + \mathrm{i}; 1 \le i \le (\mathrm{m}-1)$

Clearly vertex labels are distinct. Then f admits prime labeling. Thus, C_n P_m is a prime graph.

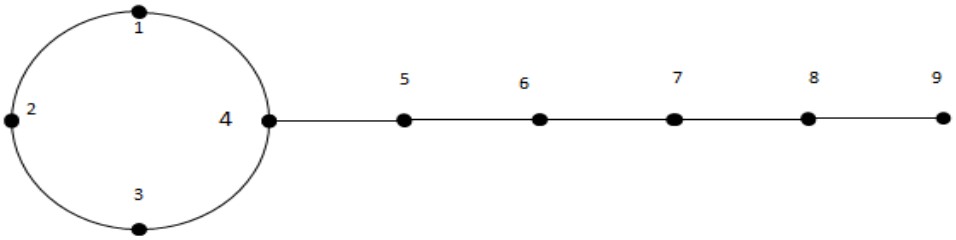
Figure 1. Prime labeling of graph C_n P_m.



Prime labeling of graph C_4 P_6.

The prime labeling of the graph is presented in the following graph.

Figure 2. Prime labeling of graph C_4 P_6.



Theorem

The crown and path graph are a prime graph. Then the graph C_n P_m prime labeling of the graph.

Proof

Let G be the graph obtained by joining crown C_n by a path Pm admit to prime labeling of the graph.

Let u_1, u_2,... un be the vertices of crown C_n and v_1, v_2, ... v_n be the vertices of cycle C_n, then

w_1, w_2,... w_n be the vertices of path P_m.

$u_i = (2\mathrm{i}-1); 1 \le \mathrm{i} \le \mathrm{n}$

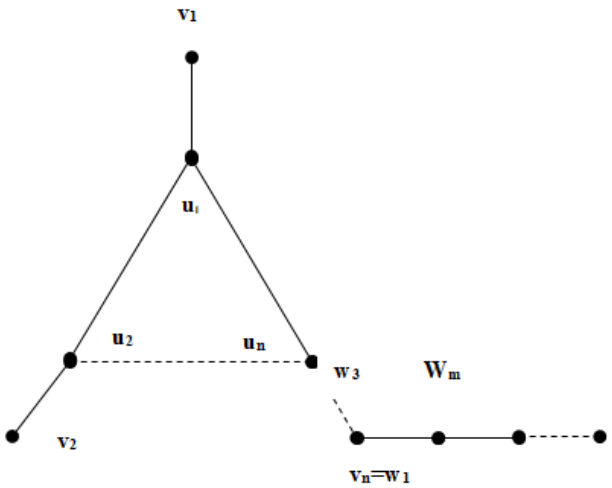
$v_i = 2\mathrm{i}; 1 \le \mathrm{i} \le \mathrm{n}$

$v_n = w_1 = 2n$

$w_{i+1} = (2n + \mathrm{i}); 1 \le \mathrm{i} \le (\mathrm{m}-1)$

Clearly vertex labels are distinct. Then f admits prime labeling. Thus, C_n P_m is a prime graph.

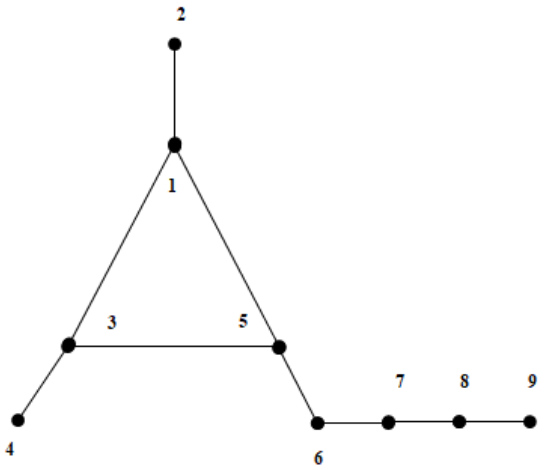
Figure 3. Prime labeling of crown graph C_n P_m.



Prime labeling of graph crown C_3 P_4.

The prime labeling of the graph is presented in the following graph.

Figure 4. Prime labeling of crown graph C_3 P_4.



Theorem

The friendship and path graph are a prime graph. Then the graph F_n P_m prime labeling of the graph.

Proof

Let G be the graph obtained by joining friendship F_n by a path Pm admit to prime labeling of the graph.

Let u_1, u_2,... u_n be the vertices of friendship F_n and v_1,v_2,... v_n be the vertices of path P_m.

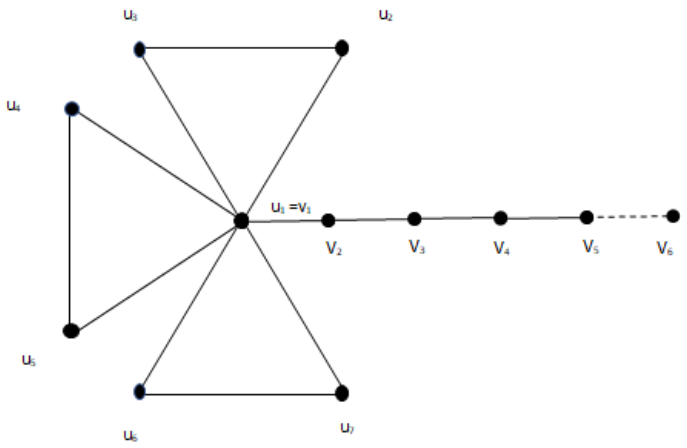
$$u_i = \mathrm{i}\ ;\ \ 1 \le i \le (2\mathrm{n}+1)$$

Assume $u_1 = v_1$

$v_{i+1} = (2n+1)+i\ ;\ \ 1 \le i \le (m-1)$

Clearly vertex labels are distinct. Then f admits prime labeling. Thus, F_n P_m is a prime graph.

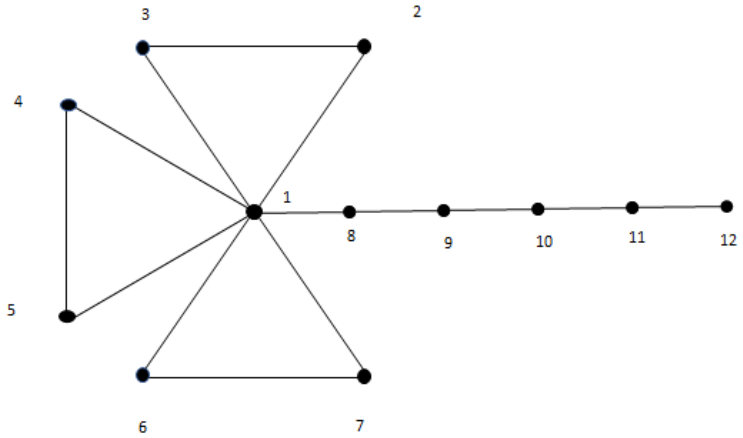
Figure 5. Prime labeling of graph F_n P_m.



Prime labeling of graph F_3 P_6.

The prime labeling of the graph is presented in the following graph.

Figure 6. Prime labeling of graph F_3 P_6.



Theorem

The star and path graph are a prime graph. Then the graph S_n P_m prime labeling of the graph.

Proof

Let G be the graphs obtained by joining star S_n by a path P_m admit to prime labeling of the graph.

Let $u_0,u_1,u_2,\ldots u_n$ be the vertices of star S_n and $v_1,v_2,\ldots v_n$ be the vertices of path P_m.

Assume $u_0 = 1$

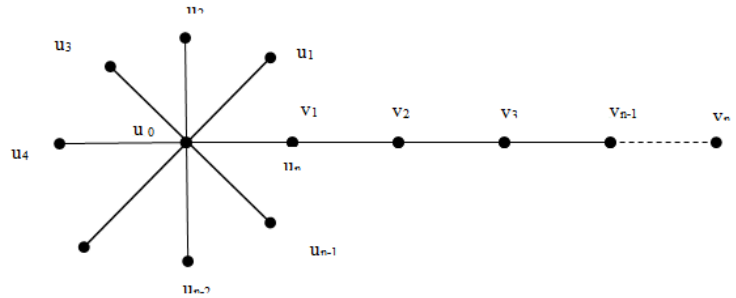
$u_i = 1+\mathrm{i}\ ;\ \ 1 \le i \le \mathrm{n}$

Assume $u_n = v_1$

$v_{i+1} = (n+1)+\mathrm{i}\ ;\ \ 1 \le i \le (\mathrm{m}-1)$

Clearly vertex labels are distinct. Then f admits prime labeling. Thus, S_n P_m is a prime graph.

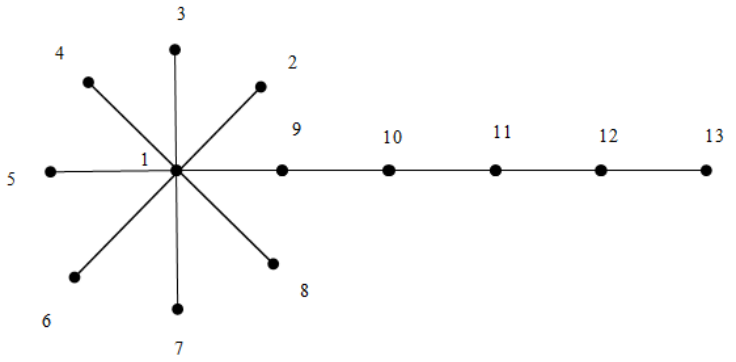
Figure 7. Prime labeling of star graph S_n P_m.



Prime labeling of graph S_8 P_5.

The prime labeling of the graph is presented in the following graph.

Figure 8. Prime labeling of star graph S_8 P_5.



Theorem

The wheel and path graph are a prime graph. Then the graph W_n P_m prime labeling of the graph.

Proof:

Let G be the graph obtained by joining wheel W_n by a path P_m, admit to prime labeling of the graph.

Let $u_0, u_1, u_2 \ldots u_n$ be the vertices of wheel W_n and $v_1, v_2 \ldots v_n$ be the vertices of path P_m.

Assume $u_0 = 1$

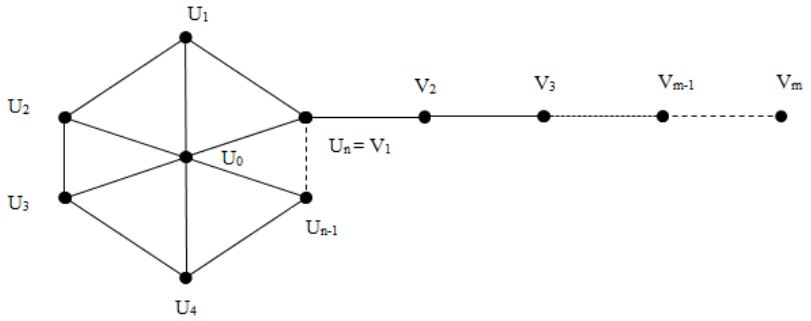
$u_i = 1+\mathrm{i}\ ;\ \ 1 \le i \le \mathrm{n}$

Assume $u_n = v_1$

$v_{i+1} = (n+1) + \mathrm{i}\ ;\ \ 1 \le i \le (\mathrm{m}-1)$

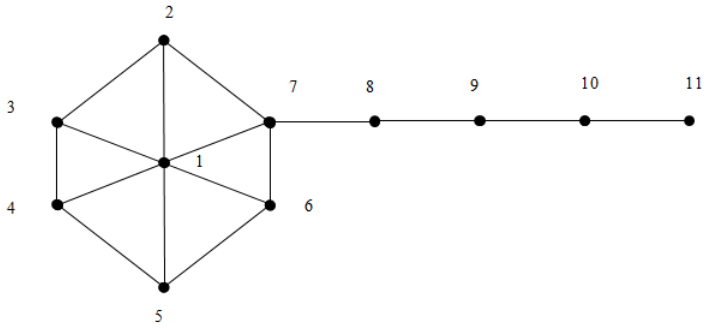
Clearly vertex labels are distinct. Then f admits prime labeling. Thus, W_n P_m is a prime graph.

Figure 9. Prime labeling of graph W_n P_m.



Prime labeling of graph W_6 P_5.
The prime labeling of the graph is presented in the following graph.

Figure 10. Prime labeling of wheel graph W_6 P_5.



Theorem

The Gear and path graph are a prime graph. Then the graph G_n P_m prime labeling of the graph

Proof

Let G be the graph obtained by joining Gear G_n by a path P_m.

Let $u_1, u_2, \ldots$ and $v_1, v_2, \ldots v_n$ be the vertices of Gear G_n and $w_1, w_2, \ldots w_n$ be the vertices path P_m.

Assume $v_0 = 1$

$v_i = 2i, \qquad 1 \le \mathrm{i} \le \mathrm{n}$

$u_i = (2i+1), \quad 1 \le \mathrm{i} \le \mathrm{n}$

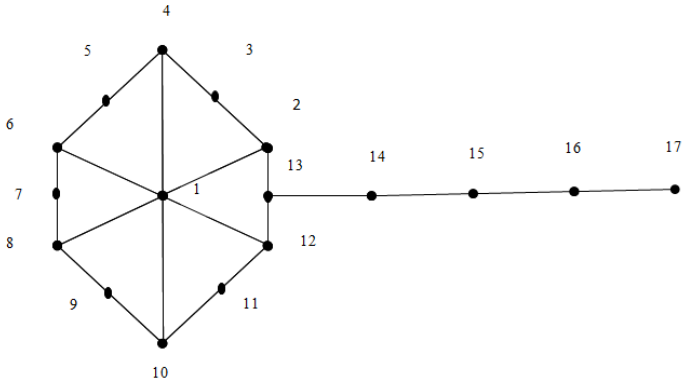
Assume $u_n = w_1$

$w_{i+1} = (2n+i); \quad 2 \le i \le (\mathrm{m}-1)$

Clearly vertex labels are distinct.

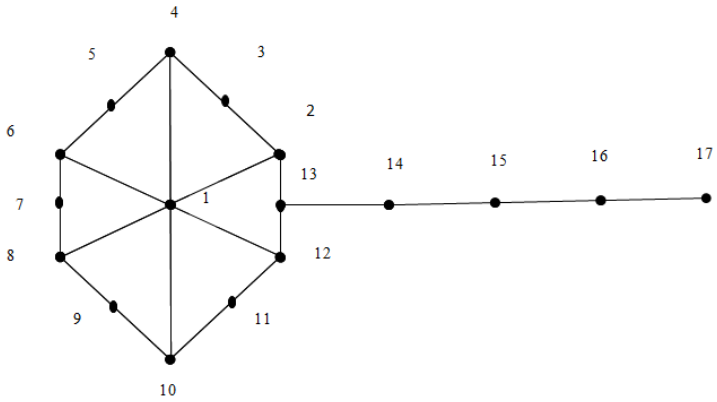
Then f admits prime labeling. Thus, G_n P_m is a prime graph.

Figure 11. Prime labeling of graph $G_n P_m$.



Prime labeling of graph $G_6 P_5$.
The prime labeling of the graph is presented in the following graph.

Figure 12. Prime labeling of graph $G_5 P_6$.



CONCLUSION

Prime labeling has been studied for then five decades. A huge number of research articles published in the area of graph theory and discrete mathematics. In this paper, we studied the prime labeling of some graph for, $C_n P_m$, $F_n P_m$, crown $C_n P_m$, $S_n P_m$ and $W_n P_m$, $G_n P_m$ in necessary conditions, In future work for some connected graphs.

REFERENCES

1. Gallian JA, et al. A Dynamic Survey of Graph Labeling. Electron J Comb. 2017.
2. Tout A, et al. Prime labeling of Graphs, Nati Acada Sci Lett. 1982;11:365-368.
3. Vaidya K, et al. Prime Labeling for some cycle related graphs. J Mat Res. 2010;23:98-104.
4. Lee SM et al. On the amalgamation of prime graphs. Bull Malays Math Sci Soc. 1988;11:59-67.
5. Bondy JA, et al. Graph theory with applications. Elsevier North Holland. New York. (1976).
6. Vaithilingam K. et al. Prime Labeling For Some Crown Related Graphs. Int J Sci. 2013;2.
7. Dretsky et al. Vertex prime labeling of graphs in graph theory. Combinatories and applications 1991;1:299-359.
8. Meena S et al. Prime labeling for some fan related graphs. Int J Eng Res. Technol 2012;1.
9. Parmar YM. Edge Vertex Prime Labeling for Wheel, Fan and Friendship Graph. Int J Math Stat Invent. 2017;5:23-

29.
10. Fu HC, et al. On prime labeling discrete math. 1994;127:181-186.
11. Ramya N, et al. on prime labeling of some classes of graphs. Int J Comput Appl. 2012;44:975-8887.