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Study on Graphs of Kinematic Chains

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Abstract: In this paper the author used graph theory and combinatorial analysis for enumeration of graphs of kinematic chains. Contracted graphs with up to four loops and six vertices are provided in Table 4. Based on the concept of expansion from contracted graphs, conventional graphs are generated. Conventional graphs with up to three independent loops and eight vertices are tabulated in Tables 5,6,7, and 8.. Using this collection, a large number of mechanisms can be developed by labelling the edges of the graphs according to the available joint types and the choice of fixed links.

Keywords: Kinematic chain, kinematic graphs or graph, contracted graphs, mechanisms.

I. INTRODUCTION

We know that the topological structures of kinematic chains can be represented by graphs. Several useful structural characteristics of graphs of kinematic chains were derived. In this paper we show that graphs of kinematic chains can be enumerated systematically by using graph theory and combinatorial analysis. There are a number of graphs but all of them are not suitable for construction of kinematic chains. Only graphs satisfying the structural characteristics are said to be *feasible solutions.* The following guidelines are framed to reduce the complexity of enumeration:

1. We are interested in closed-loop kinematic chains so, all graphs should be connected with a minimal vertex degree of 2.
2. All graphs should have no articulation points or bridges.
3. Only planar graphs are used here for the purpose of study. It has been shown that for the graph of a planar one-dof linkage to be non-planar, it must have at least 10 links according to Freudenstein [6].

Although there is no general theory for enumeration of graphs at this time, various algorithms have been developed [1, 2, 3, 4, 5, 14]. Woo [13] presented an algorithm based on contraction of graphs. Sohn and Freudenstein [10] employed the concept of dual graphs to reduce the complexity of enumeration (See also [9]). Tuttle et al. [11, 12] applied group theory. In this paper several methods for the enumeration of contracted graphs and graphs of kinematic chains have been introduced.

II. NOTATIONS USED

The following notations have been used to derive the dof equation.

ci : degrees of constraint on relative motion imposed by joint *i*.
F or dof: degrees of freedom of a mechanism.
fi : degrees of relative motion permitted by joint *i*.
j : number of joints in a mechanism, assuming that all joints are binary.
ji: number of joints with *i* dof; namely, *j*1 denotes the number of 1-dof joints, *j*2 denotes the number of 2-dof joints, and so on.
L: number of independent loops in a mechanism.
n: number of links in a mechanism, including the fixed link.
λ: degrees of freedom of the space in which a mechanism is intended to function.
S: spherical kinematic pair (dof =3).
E: plane kinematic pair (dof =3).
G: gear pair (dof =2)
Cp: cam pair (dof =2).
fp: number of passive dof in a mechanism
λ is called motion parameter.λ = 6 (for spatial mechanisms kinematic chain) and λ = 3 (for planar and spherical mechanism kinematic chain).

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III. ENUMERATION OF CONTRACTED GRAPHS

Structural analysis means the study of the connection among the members of a mechanism kinematic chains and its mobility. Mainly, it is related with the fundamental relationships among the dof, number of links, number and the type of joints used in a mechanism. The structural analysis does not deal with the physical dimensions of the links.

Mostly, graph theory is used as a helping tool in the study of the kinematic structure of mechanisms. The present study focused only on mechanisms whose corresponding graphs are planar and contain no articulation points or bridges. The topological structure of a mechanism kinematic chain is represented by a graph. The correspondence between the elements of a mechanism kinematic chain and that of a graph has been shown in Table 1 and Table 2.

Table 1 Correspondence between Mechanisms and Graphs.

Graphs	Symbol	Mechanism	Symbol
Number of vertices	v	Number of links	n
Number of edges	e	Number of joints	j
Number of vertices of degree *i*	v_i	Number of links having *i* joints	n_i
Degree of vertex *i*	d_i	Number of joints on link *i*	d_i
Number of independent loops	L	Number of independent loops	L
Total number of loops (L + 1)	L'	Total number of loops (L + 1)	L'
Number of loops with *i* edges	L_i	Number of loops with *i* joints	L_i

Table 2: Structural Characteristics of Mechanisms and Graphs	
Graphs	Mechanisms
$L = e - v + 1$	$L = j - n + 1$
$e - v + 2 \geq di \geq 2$	$j\text{-}n+2 \geq d_i \geq 2$
$\sum_i di = 2e$	$\sum_i d_i = 2j$
$\sum_i v_i = v$	$\sum_i n_i = n$
$\sum_i iv_i = 2e$	$\sum_i in_i = 2j$
$V_2 \geq 3v\text{-}2e$	$n_2 \geq 3n - 2j$
$\sum_i L_i = L' = L+1$	$\sum_i L_i = L' = L+1$
$\sum_i iL_i = 2e$	$\sum_i iLi = 2j$
Isomorphic graphs	Isomorphic mechanisms

As we know that a conventional graph can be transformed into a contracted graph by a process known as *contraction.* In a contracted graph, the number of vertices is equal to that of the conventional graph diminished by the number of binary vertices, $vc = v - v2$; the number of edges is also equal to that of the conventional graph diminished by the number of binary vertices, $ec = e - v2$; whereas the total number of loops remains unchanged, $Lc = L$. Since there are fewer vertices and edges in a contracted graph, enumeration of conventional graphs can be greatly simplified by enumerating an atlas of contracted graphs followed by an expansion of the graphs.

IV. SYSTEMATIC PROCEDURE FOR ENUMERATION OF CONTRACTED GRAPHS

To facilitate the study, the vertices of a graph are labeled sequentially from 1 to *v*. A vertex-to-vertex *adjacency matrix,* A^c, is defined as follows:

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$$A^C = \{ a_{ij} \}$$

where

$$a_{ij} = \left\{ \begin{array}{l} 1 \text{ if vertex i is adjacent to vertex j} \\ 0 \text{ otherwise (including i = j)} \end{array} \right\}$$

Here $a_{i,j}$ denotes the (i, j) element of $[A^c]$. It means that $[A^c]$ is a $v \times v$ square matrix having zero diagonal elements. Each row (or column) sum of $[A^c]$ corresponds to the degree of a vertex. Given a graph, the adjacency matrix is uniquely determined. On the other hand, given an adjacency matrix, one can construct the corresponding graph. In other words,The *adjacency matrix*, A^c, of a contracted graph is a $v^c \times v^c$ symmetric matrix with all the diagonal elements equal to zero and the off-diagonal elements equal to the number of parallel edges connecting the two corresponding vertices. From this definition, we observe that the sum of all elements in each row of A^c is equal to the degree of the vertex. Let $a_{i,j}$ denote the (i, j) element of Ac. It follows in equation (1).

$$\begin{array}{l} 0 + a_{1,2} + a_{1,3} + \cdots + a_{1,l} = d_1 , \\ a_{2,1} + 0 + a_{2,3} + \cdots + a_{2,l} = d_2 , \\ a_{3,1} + a_{3,2} + 0 + \cdots + a_{3,l} = d_3 , \\ \ldots \\ a_{l,1} + a_{l,2} + \cdots + a_{l,l-1} + 0 = d_l , \end{array} \quad \text{------------------------------(1)}$$

Where,

$l = v^c$ denotes the number of vertices in a contracted graph and d_i represents the degree of vertex i. Since the adjacency matrix is symmetric, $a_{i,j} = a_{j,i}$. To eliminate articulation points, every vertex must be connected to at least two other vertices. Hence, there must be at least two nonzero elements in each equation (1).Hence,

$d_i - 1 \geq a_{i,j} \geq 0$ --(2)

For a contracted graph, each edge joins two vertices, so,

$d_1 + d_2 + \cdots + d_l = 2e^c$ ---(3)

Since there are no binary vertices, so,

$L' \geq d_i \geq 3$ --(4)

Given v^c and e^c, the adjacency matrix of a contracted graph can be derived by solving Equations (1) and (3) subject to the constraints imposed by Equations (2) and (4).

First, we solve Equation (3) for the d_is. Without loss of generality, we assume that $d_1 \leq d_2 \leq \cdots \leq d_l$. The solution to Equation (3) can be regarded as the number of partitions of $2e^c$ objects into l places with repetition allowed. The solutions can be obtained by the nested-do loops computer program that can be written easily.

V. ENUMERATION OF CONVENTIONAL GRAPHS

Here we use a systematic enumeration methodology developed by Woo [13]. The method is based on the concept of *expansion* from contracted graphs. The procedure involves the following steps:

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(a). Given the number of links and the number of joints, solve Equations (5) and (6) for all possible link assortments and each link assortment is known as a family.

$n2 + n3 + n4 +\cdot\cdot\cdot+nr = n$---(5)

where$r$ = largest number of joints on a link.

$2n2 + 3n3 + 4n4 +\cdot\cdot\cdot+rnr= 2j$ -------------------------------------- (6)

(b). For each family, identify the corresponding contracted graphs from Table 3.

(c). Solve Equations (7) and (8) for all possible combinations of binary link chains. We call each combination of binary link chains a branch.

b1 + 2b2 + 3b3 +· · ·+qbq= v2, --(7)

Where bi denotes the number of binary strings of length i, and q denotes the longest binary string in a conventional graph. We may consider a binary string of zero length as a special case in which two vertices of degree greater than two are connected directly by an edge. From the definition of a contracted graph, it follows that

b0 + b1 + b2 + b3 +· · ·+bq= e^c .---(8)

(d). Permute the edges of each contracted graph with each combination of binary link chains obtained in (c) in as many arrangements as possible. This is equivalent to the problem of coloring the edges of a graph. Here, the concept of similar edges can be employed to reduce the number of permutations.

(e). Eliminate those graphs that contain either parallel edges or partially locked sub-chains.

(f). Check for graph isomorphism. Note that only those graphs that belong to the same family and same branch can possibly be isomorphic to one another.

VI. ILLUSTRATIVE EXAMPLES

Example- 1 : Enumeration of (2, 4) Contracted Graphs

We wish to enumerate all feasible contracted graphs with 2 vertices and 4 edges.

For v^c= 2 and e^c= 4, L’ = $e^c- v^c+ 2$ = 4. Equation (3) reduces to d1 + d2 = 8 ,

where 4 ≥ di ≥ 3. There is only one feasible solution: d1 = d2 = 4. Since there are only 2 vertices, all edges emanating from 1 vertex must terminate at the other. The resulting contracted graph is shown in Figure 1.

Example-2: Enumeration of (3, 5) Contracted Graphs

We wish to enumerate all feasible contracted graphs having 3 vertices and 5 edges.

For v^c= 3 and e^c= 5, L = $e^c- v^c+ 2$ = 4. Equation (3) reduces to

d1 + d2 + d3 = 10 ,

where 4 ≥ di ≥ 3. Solving the this equation by a computer programme which yields d1 = d2 = 3 and d3 = 4 as the only solution.

Substituting d1 = d2 = 3 and d3 = 4 into Equation (1) results in a system of 3 equations in 3 unknowns:

a1,2+ a1,3 = 3 ,

a1,2+ a2,3 = 3 ,

a1,3+ a2,3 = 4 , ---(9)

where 2 ≥ ai,j≥ 0. Solving Equation (9) yields a1,2= 1 and a1,3 = a2,3 = 2.

Hence, the adjacency matrix of the contracted graph is

$$A^c = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 2 \\ 2 & 2 & 0 \end{bmatrix}$$

The corresponding graph is shown in Figure 2.

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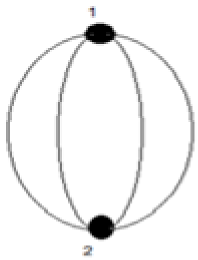

Figure 1 : A (2,4) contacted graph

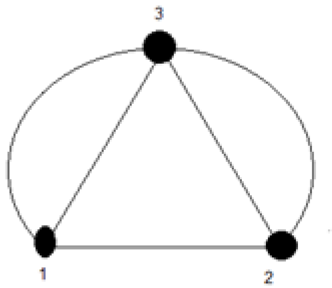

FIGURE 2:–A (3,5) contracted graph.

Example -3 Enumeration of (4, 6) Contracted Graphs
We wish to enumerate all feasible contracted graphs with 4 vertices and 6 edges.
For v^c= 4 and e^c= 6, L' = ec− vc+ 2 = 4. Hence, Equation (3) reduces to
d1 + d2 + d3 + d4 = 12 ,---(10)
where 4 ≥ di ≥ 3. Solving Equation (10) by the computer programme,
d1 = d2 = d3 = d4 = 3 as the only solution.
Substituting d1 = d2 = d3 = d4 = 3 into Equation (1) leads to a system of 4 equations in 6 unknowns:
a1,2+ a1,3 + a1,4 = 3 , ------------------------------ (11)
a1,2+ a2,3 + a2,4 = 3 ,------------------------------- (12)
a1,3+ a2,3 + a3,4 = 3 , --------------------------------(13)
a1,4+ a2,4 + a3,4 = 3 , ----------------------------------(14)
where 2 ≥ ai,j≥ 0. Equation (11) contains 3 unknowns: a1,2, a1,3, and a1,4. At this point, no distinction can be made among the 4 vertices. Without loss of generality, we assume that a1,2≤ a1,3 ≤ a1,4. Note that solutions obtained in any other order will simply lead to isomorphic graphs. Solving Equation (11) for a1,2, a1,3, and a1,4 yields the following two solutions:

Solution	a1,2	a1,3	a1,4
1	0	1	2
2	1	1	1

Solution 1: Substituting a1,2= 0, a1,3 = 1, and a1,4 = 2 into Equations (12),(13), and (14) yields
a2,3+ a2,4 = 3 ,--- (15)
a2,3+ a3,4 = 2 , --(16)
a2,4+ a3,4 = 1 .--- (17)
Solving Equations (15), (16), and (17), we obtain *a*2,3= 2, *a*2,4 = 1, and *a*3,4 = 0. Thus, the adjacency matrix of the contracted graph is

$$A^c = \begin{bmatrix} 0 & 0 & 1 & 2 \\ 0 & 0 & 2 & 1 \\ 1 & 2 & 0 & 0 \\ 2 & 1 & 0 & 0 \end{bmatrix}$$

The corresponding contracted graph is shown in Figure 3(a).

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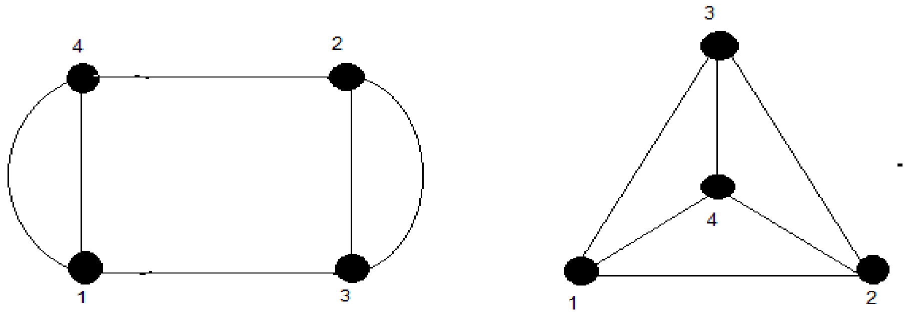

(a) (4,6) contracted graph-1 (b) (4,6) contracted graph-2

FIGURE:--3 Two (4,6) contracted graphs.

Solution 2: Substituting $a1,2= a1,3 = a1,4 = 1$ into Equations (12), (13), and (14) yields
a2,3+ a2,4 = 2 , ---(18)
a2,3+ a3,4 = 2 ,-- (19)
a2,4+ a3,4 = 2 . ---(20)
Solving Equations (18), (19), and (20), yields $a2,3= a2,4 = a3,4 = 1$. Hence, the adjacency matrix of the contracted graph is

$$A^c = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

The corresponding contracted graph is shown in Figure 3(b)
The above procedure can be employed to enumerate contracted graphs having several vertices and edges. A collection of contracted graphs up to four independent loops and six vertices is given in Table 3.
Example-4 : Enumeration of (6,8) Graphs
We wish to enumerate all possible (6, 8) graphs of planar one-dof kinematic chains.
For n = 6 and j = 8, we have L' = 8 − 6 + 2 = 4. then
n2 + n3 + n4 = 6 ---, (21)
2n2 + 3n3 + 4n4 = 16 ---.(22)
The minimal number of binary links is given by $n_2 \geq 2$.
Solving Equations (21) and (22) for nonnegative integers of niby using Crossley's operator yields three families of link assortments:

Family	n2	n3	n4
2400	2	4	0
3210	3	2	1
4020	4	0	2

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Next, we find the corresponding contracted graphs and solve Equations (7) and (8) for all possible partitions of binary links. Since we are interested in F = 1 kinematic chains, the length of any binary link chain is bounded by $q \leq 2$ as the upper boundon the length of a binary link chain

2400 family: For the 2400 family, the corresponding contracted graphs have $v^c = 6 - 2 = 4$ vertices and $e^c = 8 - 2 = 6$ edges. There are two contracted graphs with 4 vertices and 6 edges as shown in Figure 3.

With n2 = 2, ec= 6, and q = 2,

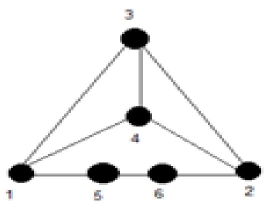
b1 + 2b2 = 2, ---(23)

b0 + b1 + b2 = 6. ---(24)

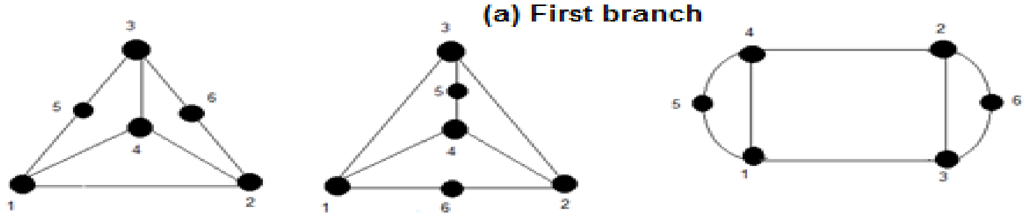
Solving Equation (23) for nonnegative integersb1 and b2, and then Equation (24) for b0 yields two branches of binary link chains:

Branch	*b0*	*b1*	*b2*
1	5	0	1
2	4	2	0

For the first branch, we replace one of the six edges in each contracted graph shown in Figure 3 with a binary link chain of length two. This is equivalent to a problem of labeling one edge in one color and the remaining edges in another color. There are six possible ways of labeling each contracted graph. Due to the existence of similar edges, only two labelings of the graph shown in Figure 3(a) are nonisomorphic. However, both labelings lead to a graph with parallel edges and, therefore, are not feasible. Since all edges in Figure 3(b) are similar, there is only one nonisomorphiclabeling of the graph as shown in Figure 4(a). Similarly, for the second branch, we replace two of the six edges with a binary link chain of length one. This is equivalent to a problem of labeling two edges in one color and the remaining edges in another color. There are 15 possible ways of labeling each contracted graph. After eliminating those graphs that are isomorphic or contain parallel edges, we obtain three labelednonisomorphic graphs as shown in Figure 4(b).



(a) First branch



(b) Secondary branch

Figure 4: Four nonisomorphic graphs derived from 2400 family

3210 family: For the 3210 family, the corresponding contracted graphs have $v^c = 6 - 3 = 3$ vertices and ec= 8 − 3 = 5 edges. There is only one contracted graph with 3 vertices and 5 edges as shown in Figure 2.

With n2 = 3, $e^c = 5$, and q = 2,

b1 + 2b2 = 3 ,--(25)

b0 + b1 + b2 = 5 .--(26)

Solving Equation (5 for nonnegative integers b1 and b2, and then Equation (26) forb0 yields two families of binary link chains:

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Branch	*b*0	*b*1	*b*2
1	3	1	1
2	2	3	0

For the first branch, we replace one of the edges in the contracted graph shown in Figure 2 by a binary link chain of length one and another by a binary link chain of length two. Note that there are two sets of two parallel edges. To avoid parallel edges in the conventional graph, one in each set of two parallel edges must be replaced by a binary link chain. In addition, the two sets of parallel edges are similar. Hence,we obtain only one labelednonisomorphic graph as shown in Figure 5(a). For the second branch, we replace three edges of the contracted graph each with a binary link chain of length one. Again, at least one in each set of two parallel edges must be replaced by a binary link chain. Thus, there are three possible ways of labeling the edges. Due to similar edges, only two are nonisomorphic as shown in Figure 5(b).

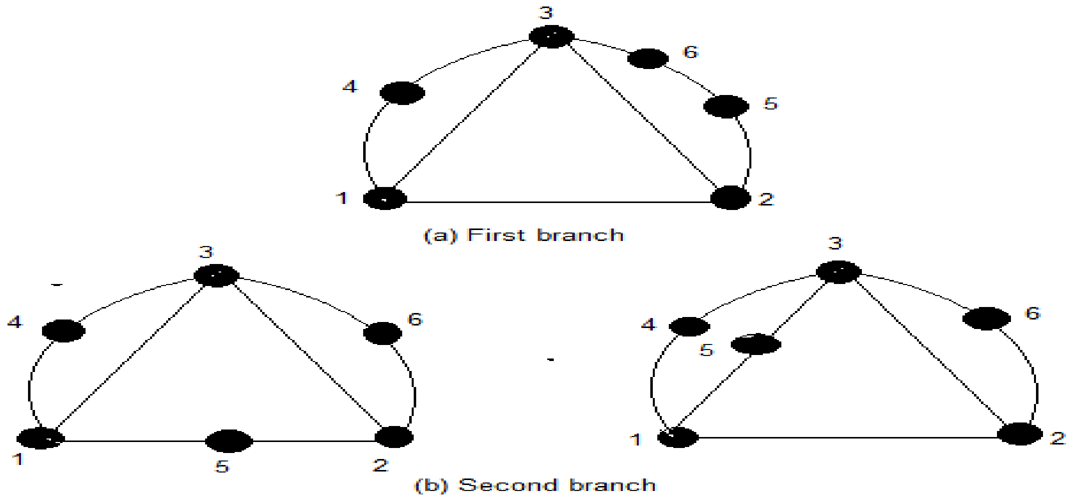

FIGURE 5:--Three nonisomorphic graphs derived from the 3210 family.

4020 family: For the 4020 family, the corresponding contracted graphs have $v^c = 6 - 4 = 2$ vertices and $e^c = 8 - 4 = 4$ edges. There is only one contracted graph with 2 vertices and 4 edges as shown in Figure 1.

With n2 = 4, ec= 4, and q = 2, then

b1 + 2b2 = 4 , --(27)

b0 + b1 + b2 = 4 .---------------------------------------(28)

Solving Equation (27) for nonnegative integersb1 and b2, and then Equation (28) for b0 yields three families of binary link chains:

Branch	*b*0	*b*1	*b*2
1	2	0	2
2	1	2	1
3	0	4	0

For the first branch, we replace two of the four parallel edges of the contracted graph shown in Figure 1 each with a binary link chain of length two. This produces no feasible solutions because the resulting graphs always contain two parallel edges. For the second branch, we replace two of the four edges of the contracted graph each with a binary link chain of length one and one with a binary link chain of length two. For the third branch, we replace all four edges of the

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contracted graph each with a binary link chain of length one. As a result, we obtain two non-isomorphic graphs as shown in Figure 6.

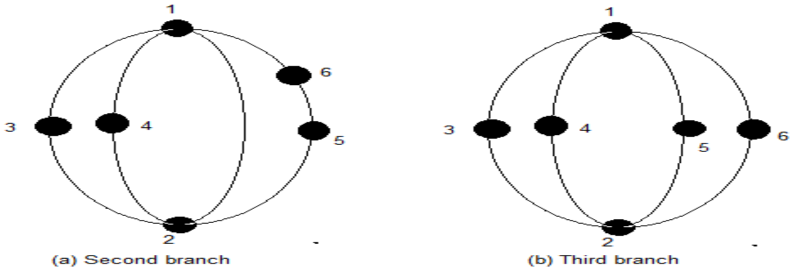

FIGURE 6:--Two nonisomorphic graphs derived from the 4020 family.

VII. CONCLUSIONS

We have shown that systematic algorithms using graph theory and combinatorial analysis can be developed for enumeration of graphs of kinematic chains. Contracted graphs with up to four loops and six vertices are provided in TABLE 4. Based on the concept of expansion from contracted graphs, an algorithm for enumeration of conventional graphs is explained. Conventional graphs with up to three independent loops and eight vertices are tabulated in Tables 4,5,6,7, and 8. Using these collection, an enormous number of mechanisms can be developed by labeling the edges of the graphs according to the available joint types and the choice of fixed links. All graphs given in Tables 4,5,6,7, and 8 are sketched in such a way that the longest circuit forms the peripheral loop and the vertex of highest degree appears on the top (or upper-left corner), provided that it does not cause crossing of the edges.These graphs are arranged in the order of complexity according to the number of loops, number of vertices, and length of peripheral loop.

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Table 3 - Contracted Graphs Having Two to four Independent Loops

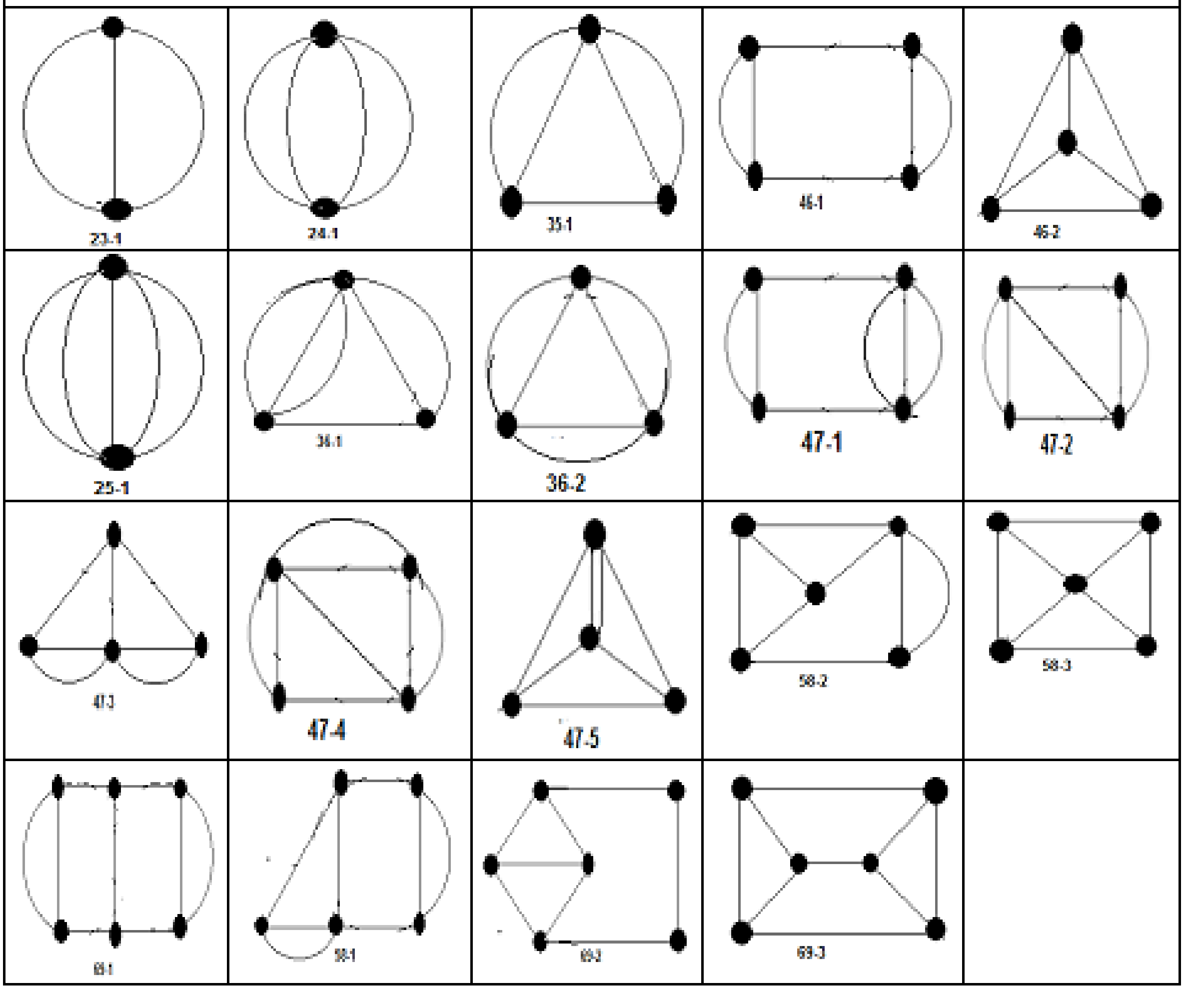

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Table 4- Graphs having one independent loop: three to eight vertices

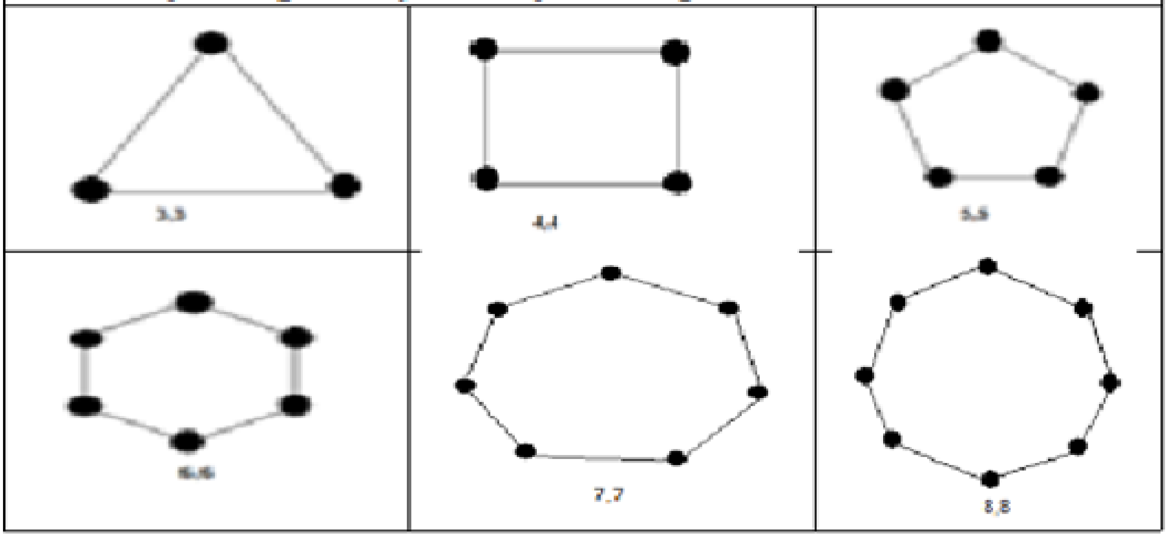

Table 5- Graphs Having Two Independent Loops: Four To Eight Vertices

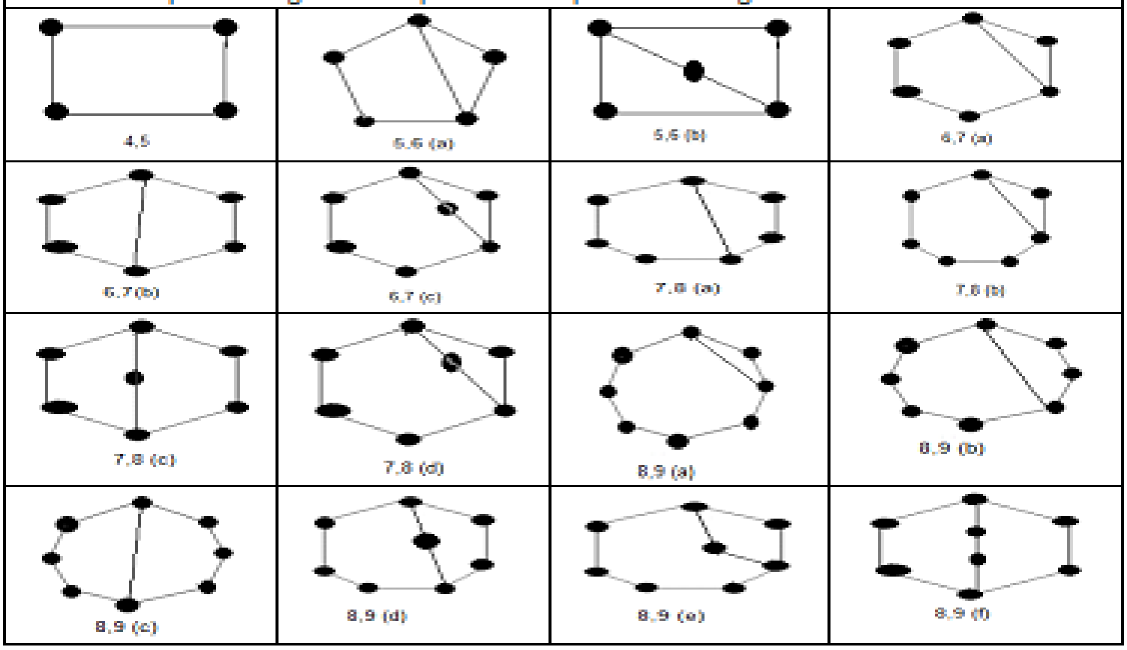

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Table 6 - Graphs Having Three Independent Loops: five To Six Vertices

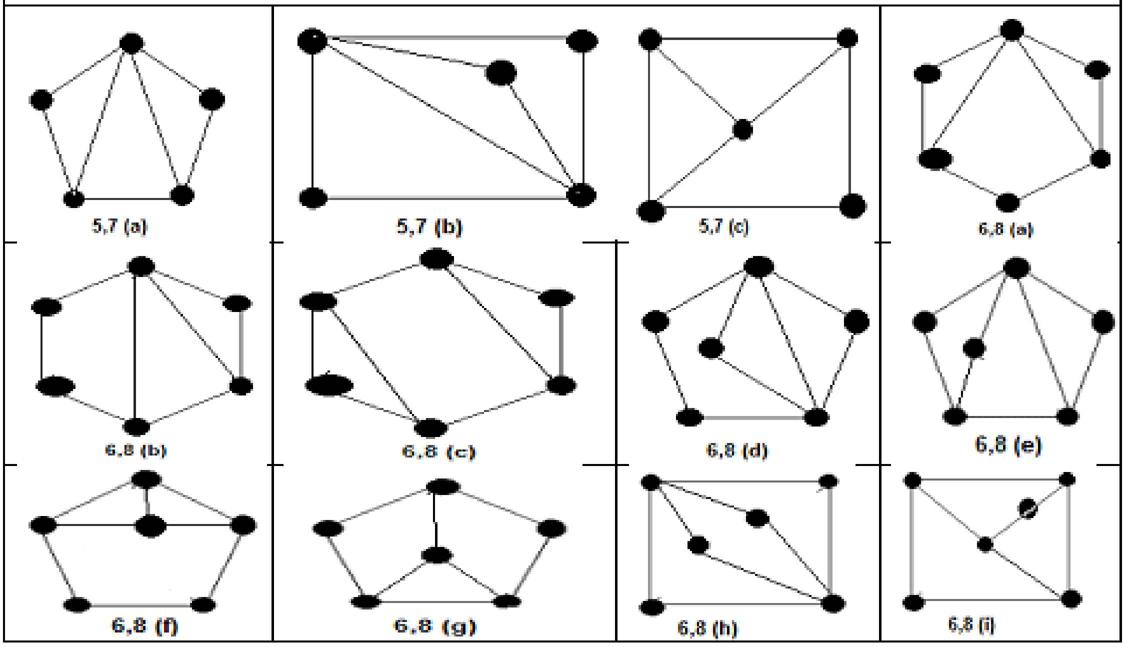

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Table 7 – Graphs Having Three Independent Loops: Seven Vertices

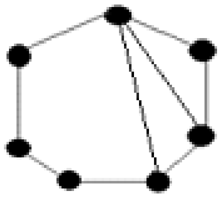
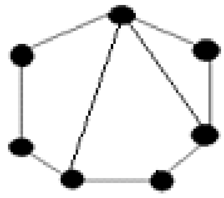
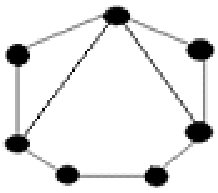
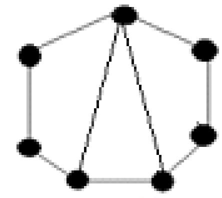
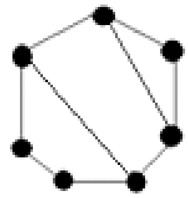
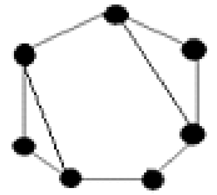
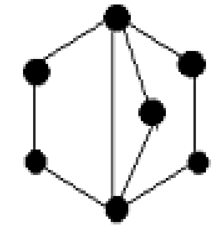
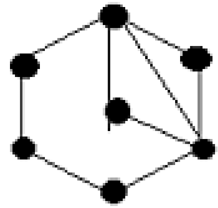
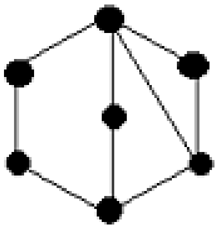
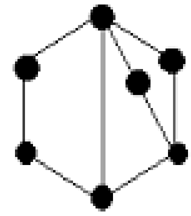
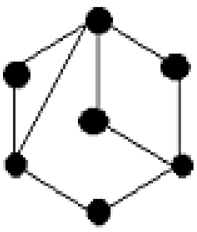
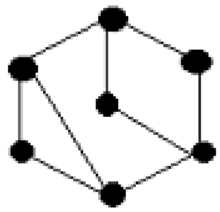
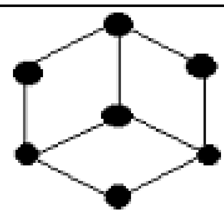
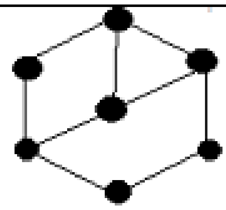
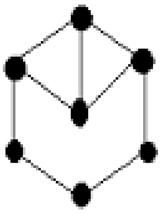
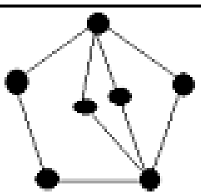
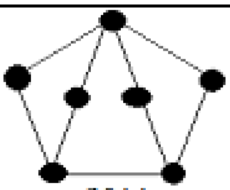
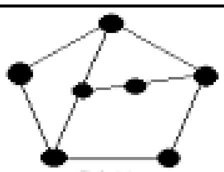
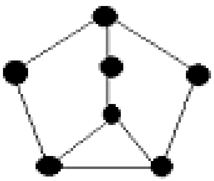
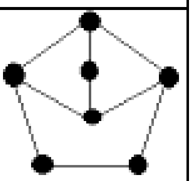
 7,9 (a)	 7,9 (b)	 7,9(c)	 7,9 (d)	 7,9 (e)
 7,9 (f)	 7,9 (g)	 7,9 (h)	 7,9 (i)	 7,9 (j)
 7,9 (k)	 7,9 (l)	 7,9 (m)	 7,9 (n)	 7,9 (o)
 7,9 (p)	 7,9 (q)	 7,9 (r)	 7,9 (s)	 7,9 (t)

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Table 8 - Graphs Having Three Independent Loops: 5 To Six Vertices.