

The Effect of Parameter Variation on Open and Closed Loop of Control System Sensitivity

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ABSTRACT: Usually control system does not remain constant throughout its life cycle, so there are always changes in system parameters because of environmental effects and other perturbations and disturbances. These changes and affections are called parameter variations where can be studied using mathematical and physical term expressions to illustrate entire control system change. The more keeping prosperities control system with entire changes the lesser sensitivity of control system. This paper discusses a method to minimize the control system sensitivity.

KEYWORDS: parameters variation, sensitivity, control system, open loop, closed loop, feedback.

I. INTRODUCTION

The prediction of control system probability is one of the most control system design problems arising, where the physical and mathematical models are used then the complete prosperities of control system must appear and reflected. In some special cases model parameters can involve some characteristics such as operation conditions, disturbances and sort of exogenous beside parameter variations. By mathematics parameter it meant any independent variable included in its mathematic model [1].

II. SENSITIVITY

The study of system sensitivity as initiated by Bode is concerned first with the definition of a measure of the change for some system characteristic (arising from a class of disturbances) ; and then second with the development of design procedures to minimize, with respect to this measure, fluctuations in the system characteristic. The sensitivity of a transfer function T(s) with respect to a parameter G(s) is:

$$S = \frac{\frac{dT}{T}}{\frac{dG}{G}} \quad (1)$$

Note that then

$$\frac{dT}{T} = S \frac{dG}{G} \quad (2)$$

Ratio of change in the overall TF is equal to Sensitivity multiplied by ratio of change in the Plant. Negative feedback usually reduces the error between control action and the response what lead to minimize the sensitivity of the control system to parameter variation and un desired internal and external disturbances. However it important to make the control system insensitive to such parameter variation to keep it stable as far as possible. So we are going to study the sensitivity affection in both open loop and close loop [2].

III. OPEN LOOP SENSITIVITY

Consider the open loop control system shown in figure 1, where u(t) represents the input control action of the system and y(t) point to system output response. Then the transfer function is given as:

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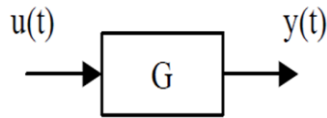


Fig 1 open loop control system

The overall transfer function of the system in figure 1 is given as follows:

$$T = \frac{Y}{U} = G \tag{3}$$

Therefore

$$\frac{T}{G} = 1 \quad \text{then} \quad \frac{dT}{dG} = 1 \tag{4}$$

The sensitivity with respect to G can be obtained as:

$$S = \frac{\frac{dT}{dG}}{\frac{T}{G}} = 1 \tag{5}$$

In other hand let ΔG represents the changes in G in figure 1 due to parameter variation, then the corresponding change in the output $Y(t)$

$$Y + \Delta Y = (G + \Delta G)R$$

In which obtain

$$\Delta Y = \Delta G \cdot R \tag{6}$$

Equation 5 gives the change of transfer function due to effect of variation on open loop output response. Now suppose that $G=10$, a constant parameter. If G changes by 10% to $G=11$ then

$$\frac{dG}{G} = \frac{11-10}{10} = 0.1 \quad \text{then according to equation 2, one has}$$

$$\frac{dT}{T} = S \frac{dG}{G} = 1 \frac{1}{10} = 0.1, \quad \text{so that } T \text{ also change by 10\%}$$

However in open loop systems, there is little protection from uncertainty or disturbances. A 10% change in parameter is reflected as 10% change in the overall transfer function.

IV. CLOSED LOOP SENSITIVITY

The closed loop control system is shown in figure 2, $r(t)$ represents the input control action, $y(t)$ is the output response, $G(s)$ gives the control system plant and $H(s)$ appears the control system negative feedback

Sensitivity in a Closed-Loop System

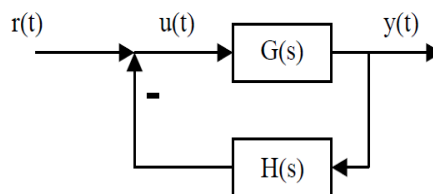


Fig 2 Closed loop control system

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The closed loop control system transfer function is given as:

$$T = \frac{Y}{R} = \frac{G}{1 + HG} \tag{7}$$

Therefore one has

$$\frac{T}{G} = \frac{1}{1+HG} \tag{8}$$

By using the quotient derivative rule one can write:

$$\frac{dT}{dG} = \frac{(1 + HG) - GH}{(1 + HG)^2} = \frac{1}{(1 + HG)^2} \tag{9}$$

Then the sensitivity is thus:

$$S = \frac{\frac{dT}{dG}}{\frac{T}{G}} = \frac{\frac{1}{(1 + HG)^2}}{\frac{1}{1 + HG}} = \frac{1}{1 + HG} \tag{10}$$

Let ΔG represents closed loop system change which is due to parameter variation in the system corresponding to change in the output ΔY after substitute in equation 6 we then get:

$$Y + \Delta Y = \frac{(G + \Delta G)}{1 + H(G + \Delta G)} \cdot R \tag{11}$$

In which at last can obtain the following considering that the term $\Delta G \cdot H$ in equation 10 is neglected, that

$$\Delta Y = \frac{\Delta G}{1 + HG} \cdot R \tag{12}$$

The equation 11 gives the change of output due to parameter variation in closed loop control system, in practice the system magnitude $1 + HG$ is greater than unity, then we can observe that from equation 11 that in close loop due to the feedback the change of output with respect of parameter variation in $G(s)$, is minimized by the factor $1 + GH$ [2]. Let the plant with the constant value of $G=1000$ and $H=0.099$, then:

$$T = \frac{G}{1 + HG} = \frac{1000}{1 + 99} = 10. \tag{13}$$

Then we can obtain the sensitivity by using equation (10) as follows:

$$S = \frac{1}{1 + HG} = \frac{1}{1 + 99} = 0.01. \tag{14}$$

Its observed that from equations (13) and (14) that a 10% increase in G amounts to increase from 1000 to 1100, but the preformed change in the transfer function T is given by the equation (2):

$$\frac{dT}{T} = S \frac{dG}{G} = 0.1 \frac{100}{1000} = 0.001$$

That means the change is only 0.1% and the transfer function increases to 10.01. However it is clear that feedback obtains valuable and significant insulation of transfer function in system parameters.

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V. RESULTS DISCUSSION

By referring to equations (5) and (10), feedback effect in the transfer function sensitivity then make it reduced by the ratio of $\frac{1}{1+HG}$ compared to open loop sensitivity that effect by the factor of unity. Table 1 bellow shows control system open and closed loop sensitivity and parameter variation. Consider the control system in fig (2), various plant values will be substituted in equations (3), (5), (7) and (10) respectively to obtain values of sensitivity and overall transfer function value for both open and closed loop control system. We can observe that in case of closed loop, the overall transfer function values T remains with in constant value when entire parameter variation is little bit, for open loop any change in the parameters leads to the overall transfer function value.

Table 1 open loop closed loop sensitivity and parameter variation

The plant G with several values	Open loop sensitivity S H=0	Open loop transfer function value T H=0	Closed loop sensitivity S H=1	Closed loop transfer function value T H=1
10	1	10	0.099	0.99
20	1	20	0.047	0.94
30	1	30	0.032	0.96
40	1	40	0.024	0.96

VI. CONCLUSION

For any control system feedback is essential mater to keep the output response without changes even if the entire parameters change, this leads for a stable control system design with less affection of the environmental changes. By the methods and equations discussed previously it easy to evaluate the sensitivity of control system and its ability to remain healthy despite of parameter variations.

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