

The Zhang and Fu's Similarity Measure on Intuitionistic Fuzzy Multi Sets

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ABSTRACT: In this paper we introduce a novel Similarity measure for Intuitionistic fuzzy multi sets (IFMS) based on Zhang and Fu's measure, as Similarity Measure is an important topic in fuzzy set theory. The unique feature of this method is that it considers multi membership, non membership and hesitancy degree for the same element. Since the proposed measure of IFMS is mathematically valid and demonstrates the superiority of the existing methods, we apply this measure to medical diagnosis and, pattern recognition problems.

KEYWORDS: Intuitionistic fuzzy set, Fuzzy Multi sets, Intuitionistic Fuzzy Multi sets, Similarity measure.

I. INTRODUCTION

The Intuitionistic Fuzzy sets (*IFS*) proposed by **Krassimir T. Atanassov** [1], [2] was the generalisation of the Fuzzy set (*FS*) introduced by **Lofti A. Zadeh** [3]. The object, partially belong to a set with a membership degree (μ) between 0 and 1 are represented by the *FS* whereas the *IFS* represent the uncertainty with respect to both membership ($\mu \in [0,1]$) and non membership ($\vartheta \in [0,1]$) such that $\mu + \vartheta \leq 1$. Here, the number $\pi = 1 - \mu - \vartheta$ is called the hesitation degree or intuitionistic index.

The study of distance and similarity measure of *IFSs* gives lots of measures, each representing specific properties and behaviour in real-life decision making and pattern recognition works. For measuring the degree of similarity between vague sets, **Chen and Tan** [4] proposed two similarity measures. The Hamming, Euclidean distance and similarity measures were introduced by **Szmidt and Kacprzyk** [5], [6], [7], [8]. The Geometric distance and similarity measures were given by **Xu** [9]. Most of the similarity measures reflect the degree of membership and non membership; **Li et al** [10] made a comparative study for similarity measures between *IFSs* and found the inadequate conditions for similarity measures. Therefore the hesitation degree was introduced for similarity measure. **Zhang and Fu** [11] proposed a new similarity measure for *IFSs* by considering the hesitation degree also. Later some modifications were made by **Binyamin et al** [12] on Zhang and Fu's method for better results.

The Multi set [13] allows the repeated occurrences of any element and hence the Fuzzy Multi set (*FMS*) can occur more than once with the possibly of the same or the different membership values was introduced by **R. R. Yager** [14]. Recently, the new concept Intuitionistic Fuzzy Multi sets (*IFMS*) was proposed by **T.K Shinoj and Sunil Jacob John** [15] which allows the repeated occurrences of different membership and non membership functions.

As various distance and similarity methods of *IFS* are extended for *IFMS* distance and similarity measures [16], [17], [18], [19] and [20]. And in this paper we extend the Zhang and Fu's measure of *IFSs* to *IFMSs*. The numerical results of the examples show that the developed similarity measures are well suited to use any linguistic variables. The organization of this paper is as follows: In section 2, the Fuzzy Multi sets, Intuitionistic Fuzzy Multi sets and similarity measures of *IFMS* are presented. The section 3 deals with the proposed Zhang and Fu's measure for the *IFMS*. The

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significance of this new measure along with the numerical evaluation is in the section 4. The section 5 and 6 are dedicated for the application of the similarity measure in medical diagnosis and pattern recognition.

II. PRELIMINARIES

Definition: 2.1

Let X be a nonempty set. A **fuzzy set** A in X is given by $A = \{ \langle x, \mu_A(x) \rangle / x \in X \}$ -- (2.1)

where $\mu_A : X \rightarrow [0, 1]$ is the membership function of the fuzzy set A (i.e.) $\mu_A(x) \in [0,1]$ is the membership of $x \in X$ in A . The generalizations of fuzzy sets are the Intuitionistic fuzzy (IFS) set proposed by **Atanassov [1], [2]** is with independent memberships and non memberships.

Definition: 2.2

An **Intuitionistic fuzzy set (IFS)**, A in X is given by $A = \{ \langle x, \mu_A(x), \vartheta_A(x) \rangle / x \in X \}$ -- (2.2)

where $\mu_A : X \rightarrow [0,1]$ and $\vartheta_A : X \rightarrow [0,1]$ with the condition $0 \leq \mu_A(x) + \vartheta_A(x) \leq 1, \forall x \in X$. Here $\mu_A(x)$ and $\vartheta_A(x) \in [0,1]$ denote the membership and the non membership functions of the fuzzy set A ; For each Intuitionistic fuzzy set in X , $\pi_A(x) = 1 - \mu_A(x) - \vartheta_A(x) = 0$ for all $x \in X$ that is

$\pi_A(x) = 1 - \mu_A(x) - \vartheta_A(x)$ is the hesitancy degree of $x \in X$ in A . Always $0 \leq \pi_A(x) \leq 1, \forall x \in X$.

The **complementary set** A^c of A is defined as $A^c = \{ \langle x, \vartheta_A(x), \mu_A(x) \rangle / x \in X \}$ -- (2.3)

Definition: 2.3

Let X be a nonempty set. A **Fuzzy Multi set (FMS)** A in X is characterized by the count membership function Mc such that $Mc : X \rightarrow Q$ where Q is the set of all crisp multi sets in $[0,1]$. Hence, for any $x \in X$, $Mc(x)$ is the crisp multi set from $[0, 1]$. The membership sequence is defined as

$(\mu_A^1(x), \mu_A^2(x), \dots, \dots, \mu_A^p(x))$ where $\mu_A^1(x) \geq \mu_A^2(x) \geq \dots \geq \mu_A^p(x)$.

Therefore, A FMS A is given by $A = \{ \langle x, (\mu_A^1(x), \mu_A^2(x), \dots, \dots, \mu_A^p(x)) \rangle / x \in X \}$ -- (2.4)

Definition: 2.4

Let X be a nonempty set. A **Intuitionistic Fuzzy Multi set (IFMS)** A in X is characterized by two functions namely count membership function Mc and count non membership function NMc such that $Mc : X \rightarrow Q$ and $NMc : X \rightarrow Q$ where Q is the set of all crisp multi sets in $[0,1]$. Hence, for any $x \in X$, $Mc(x)$ is the crisp multi set from $[0, 1]$ whose membership sequence is defined as $(\mu_A^1(x), \mu_A^2(x), \dots, \dots, \mu_A^p(x))$ where $\mu_A^1(x) \geq \mu_A^2(x) \geq \dots \geq \mu_A^p(x)$ and the corresponding non membership sequence $NMc(x)$ is defined as $(\vartheta_A^1(x), \vartheta_A^2(x), \dots, \dots, \vartheta_A^p(x))$ where the non membership can be either decreasing or increasing function. such that $0 \leq \mu_A^i(x) + \vartheta_A^i(x) \leq 1, \forall x \in X$ and $i = 1, 2, \dots, p$. Therefore,

An **IFMS** A is given by

$A = \{ \langle x, (\mu_A^1(x), \mu_A^2(x), \dots, \dots, \mu_A^p(x)), (\vartheta_A^1(x), \vartheta_A^2(x), \dots, \dots, \vartheta_A^p(x)) \rangle / x \in X \}$ -- (2.5)

where $\mu_A^1(x) \geq \mu_A^2(x) \geq \dots \geq \mu_A^p(x)$. The **complementary set** A^c of A is defined as

$A^c = \{ \langle x, (\vartheta_A^1(x), \vartheta_A^2(x), \dots, \dots, \vartheta_A^p(x)), (\mu_A^1(x), \mu_A^2(x), \dots, \dots, \mu_A^p(x)) \rangle / x \in X \}$ -- (2.6)

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where $\vartheta_A^1(x) \geq \vartheta_A^2(x) \geq \dots \geq \vartheta_A^p(x)$

Definition: 2.5

The **Cardinality** of the membership function $\text{Mc}(x)$ and the non membership function $\text{NMc}(x)$ is the length of an element x in an *IFMS* A denoted as η , defined as $\eta = |\text{Mc}(x)| = |\text{NMc}(x)|$

If A, B, C are the *IFMS* defined on X , then their cardinality $\eta = \text{Max} \{ \eta(A), \eta(B), \eta(C) \}$.

Definition: 2.6

Sim (A, B) is said to be the **similarity measure** between A and B , where $A, B \in X$ and X is an *IFMS*, as $\text{Sim}(A, B)$ satisfies the following properties

1. $\text{Sim}(A, B) \in [0,1]$
2. $\text{Sim}(A, B) = 1$ if and only if $A = B$
3. $\text{Sim}(A, B) = \text{Sim}(B, A)$

ZHANG AND FU’S SIMILARITY MEASURE OF IFSs

The similarity measure of *IFSs* proposed by **Zhang and Fu’s** was as follows

$$\text{Sim}_{ZF}(A, B) = 1 - \frac{1}{2n} \sum_{i=1}^n \{ |\mu_A(x_i) - \mu_B(x_i)| + |(1 - \vartheta_A(x_i)) - (1 - \vartheta_B(x_i))| \}$$

consisting of the membership and non membership functions.

Later Zhang and Fu’s new similarity measure was

$$\text{Sim}_{ZF}(A, B) = 1 - \frac{1}{2n} \sum_{i=1}^n \{ |\delta_A - \delta_B| + |\alpha_A - \alpha_B| \}$$

where $\delta_A(x_i) = \mu_A(x_i) + (1 - \mu_A(x_i) - \vartheta_A(x_i)) \mu_A(x_i)$ and

$$\alpha_A(x_i) = \vartheta_A(x_i) + (1 - \mu_A(x_i) - \vartheta_A(x_i)) \vartheta_A(x_i)$$

And if there are three parameters like membership, non membership and hesitation function then the modified Zhang and Fu’s similarity measure of **Binyamin et al [12]** for *IFSs* becomes

$$\text{Sim}_{modified\ ZF}(A, B) = 1 - \frac{1}{2n} \sum_{i=1}^n \{ |\delta_A - \delta_B| + |\alpha_A - \alpha_B| + |\beta_A - \beta_B| \}$$

where $\delta_A(x_i) = \mu_A(x_i) + (1 - \mu_A(x_i) - \vartheta_A(x_i)) \mu_A(x_i)$

$$\alpha_A(x_i) = \vartheta_A(x_i) + (1 - \mu_A(x_i) - \vartheta_A(x_i)) \vartheta_A(x_i) \text{ and } \beta_A(x_i) = (1 - \delta_A(x_i) - \alpha_A(x_i))$$

**III. PROPOSED ZHANG AND FU'S SIMILRITY MEASURES
FOR INTUITIONISTIC MULTI FUZZY SETS**

In *IFS*, the Similarity measures are considered for the membership and non membership functions only once. But in *IFMS*, it should be considered more than once; because of their multi membership and non membership functions. And, their considerations are combined together by means of Summation concept based on their cardinality.

Definition: 3.1

$$IFMS_{ZF1}(A, B) = \frac{1}{\eta} \sum_{j=1}^{\eta} \left[1 - \frac{1}{2n} \sum_{i=1}^n \left\{ |\mu_A^j(x_i) - \mu_B^j(x_i)| + |(1 - \vartheta_A^j(x_i)) - (1 - \vartheta_B^j(x_i))| \right\} \right]$$

of the membership and non membership functions. Also the new similarity measure becomes

$$IFMS_{ZF2}(A, B) = \frac{1}{\eta} \sum_{j=1}^{\eta} \left[1 - \frac{1}{2n} \sum_{i=1}^n \left\{ |\delta_A^j - \delta_B^j| + |\alpha_A^j - \alpha_B^j| \right\} \right]$$

Where $\delta_A^j(x_i) = \mu_A^j(x_i) + (1 - \mu_A^j(x_i) - \vartheta_A^j(x_i)) \mu_A^j(x_i)$ and

$$\alpha_A^j(x_i) = \vartheta_A^j(x_i) + (1 - \mu_A^j(x_i) - \vartheta_A^j(x_i)) \vartheta_A^j(x_i)$$

And if there are three parameters like membership, non membership and hesitation function then the IFMS new similarity measure becomes

$$IFMS_{ZF}(A, B) = \frac{1}{\eta} \sum_{j=1}^{\eta} \left[1 - \frac{1}{2n} \sum_{i=1}^n \left\{ |\delta_A^j - \delta_B^j| + |\alpha_A^j - \alpha_B^j| + |\beta_A^j - \beta_B^j| \right\} \right]$$

Where $\delta_A^j(x_i) = \mu_A^j(x_i) + (1 - \mu_A^j(x_i) - \vartheta_A^j(x_i)) \mu_A^j(x_i)$

$$\alpha_A^j(x_i) = \vartheta_A^j(x_i) + (1 - \mu_A^j(x_i) - \vartheta_A^j(x_i)) \vartheta_A^j(x_i) \text{ and}$$

$$\beta_A^j(x_i) = (1 - \delta_A^j(x_i) - \alpha_A^j(x_i))$$

PROPOSITION: 3.2

The defined similarity measure $IFMS(A, B)$ between *IFMS* A and B satisfies the following properties

- D1.** $0 \leq IFMS_{ZF}(A, B) \leq 1$
- D2.** $A = B$ if and only if $IFMS_{ZF}(A, B) = 1$
- D3.** $IFMS_{ZF}(A, B) = IFMS_{ZF}(B, A)$

Proof

- D1.** $0 \leq IFMS_{ZF}(A, B) \leq 1$

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As the membership and the non membership functions of the *IFMSs* lies between 0 and 1, the similarity measure based on Zhang and Fu's function also lies between 0 and 1.

D2. $A = B$ if and only if $IFMS_{ZF}(A, B) = 1$

(i) Let the two *IFMS* A and B be equal (i.e.) $A = B$. This implies for any $\mu_A^j(x_i) = \mu_B^j(x_i)$ and $\vartheta_A^j(x_i) = \vartheta_B^j(x_i)$ which states that $|\mu_A^j(x_i) - \mu_B^j(x_i)|$ and $|\vartheta_A^j(x_i) - \vartheta_B^j(x_i)| = 0$. Hence $IFMS_{ZF}(A, B) = 1$

(ii) Let the $IFMS_{ZF}(A, B) = 1$

The unit measure is possible only if both $|\mu_A^j(x_i) - \mu_B^j(x_i)|$ and $|\vartheta_A^j(x_i) - \vartheta_B^j(x_i)| = 0$,

This refers that $\mu_A^j(x_i) = \mu_B^j(x_i)$ and $\vartheta_A^j(x_i) = \vartheta_B^j(x_i)$ for all i, j values. Hence $A = B$.

D3. $IFMS_{ZF}(A, B) = IFMS_{ZF}(B, A)$

It is obvious that $\mu_A^j(x_i) - \mu_B^j(x_i) \neq \mu_B^j(x_i) - \mu_A^j(x_i)$ and $\vartheta_A^j(x_i) - \vartheta_B^j(x_i) \neq \vartheta_B^j(x_i) - \vartheta_A^j(x_i)$

But $|\mu_A^j(x_i) - \mu_B^j(x_i)| = |\mu_B^j(x_i) - \mu_A^j(x_i)|$ and $|\vartheta_A^j(x_i) - \vartheta_B^j(x_i)| = |\vartheta_B^j(x_i) - \vartheta_A^j(x_i)|$

$$\begin{aligned} \text{Hence } IFMS_{ZF}(A, B) &= \frac{1}{\eta} \sum_{j=1}^{\eta} \left[1 - \frac{1}{2n} \sum_{i=1}^n \{ |\delta_A^j - \delta_B^j| + |\alpha_A^j - \alpha_B^j| + |\beta_A^j - \beta_B^j| \} \right] \\ &= \frac{1}{\eta} \sum_{j=1}^{\eta} \left[1 - \frac{1}{2n} \sum_{i=1}^n \{ |\delta_B^j - \delta_A^j| + |\alpha_B^j - \alpha_A^j| + |\beta_B^j - \beta_A^j| \} \right] = IFMS_{ZF}(B, A) \end{aligned}$$

IV. SIGNIFICANCE OF THE PROPOSED SIMILARITY MEASURE

EXAMPLE : 4.1

Let $X = \{A_1, A_2, A_3, A_4, \dots, A_n\}$ with $Y = \{A_1, A_{10}\}$ and $Z = \{A_1, A_{10}\}$ are the *IFMS* defined as

$Y = \{ \langle A_1 : (0.1, 0.2) \rangle, \langle A_{10} : (0.2, 0.3) \rangle \}$ and the Pattern $Z = \{ \langle A_1 : (0.1, 0.2) \rangle, \langle A_{10} : (0.2, 0.3) \rangle \}$

Here, the cardinality $\eta = 2$ as $|\text{Mc}(Y)| = |\text{NM}(Y)| = 2$ and $|\text{Mc}(Z)| = |\text{NM}(Z)| = 2$, and the new modified Similarity measure between the

$$\text{Patten}(Y, Z) = \frac{1}{2} \sum_{j=1}^2 \left[1 - \frac{1}{2(1)} \sum_{i=1}^1 \{ |\delta_A^j - \delta_B^j| + |\alpha_A^j - \alpha_B^j| + |\beta_A^j - \beta_B^j| \} \right] = 1$$

Thus the similarity measure of any two *IFMSs* equals to one if and only if the two *IFMSs* are the same.

EXAMPLE : 4.2

Let $X = \{A_1, A_2, A_3, A_4, \dots, A_n\}$ with $A = \{A_1, A_2\}$, $B = \{A_1, A_{10}\}$ and $C = \{A_1, A_4\}$ are the *IFMS* defined as
 $A = \{ \langle A_1 : (0.1, 0.2) \rangle, \langle A_2 : (0.3, 0.3) \rangle \}$, $B = \{ \langle A_9 : (0.1, 0.2) \rangle, \langle A_{10} : (0.2, 0.3) \rangle \}$ and
 $C = \{ \langle A_9 : (0.1, 0.2) \rangle, \langle A_{10} : (0.2, 0.2) \rangle \}$

Here, the cardinality $\eta = 2$ as $|\text{Mc}(A)| = |\text{NM}(A)| = 2$ and $|\text{Mc}(B)| = |\text{NM}(B)| = 2$, Also $|\text{Mc}(C)| = |\text{NM}(C)| = 2$, then

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$$IFMS_{ZF1}(A, B) = \frac{1}{2} \sum_{j=1}^2 \left[1 - \frac{1}{2(1)} \sum_{i=1}^1 \left\{ |\mu_A^j(x_i) - \mu_B^j(x_i)| + \left| (1 - \vartheta_A^j(x_i)) - (1 - \vartheta_B^j(x_i)) \right| \right\} \right] = 0.975 \text{ and}$$

$$IFMS_{ZF1}(A, C) = \frac{1}{2} \sum_{j=1}^2 \left[1 - \frac{1}{2(1)} \sum_{i=1}^1 \left\{ |\mu_A^j(x_i) - \mu_C^j(x_i)| + \left| (1 - \vartheta_A^j(x_i)) - (1 - \vartheta_C^j(x_i)) \right| \right\} \right] = 0.975$$

Using the **First Similarity Measure**, we cannot differentiate whether (A, B) or (A, C) is more similar

$$IFMS_{ZF2}(A, B) = \frac{1}{2} \sum_{j=1}^2 \left[1 - \frac{1}{2(1)} \sum_{i=1}^1 \left\{ |\delta_A^j - \delta_B^j| + |\alpha_A^j - \alpha_B^j| \right\} \right] = 0.9625 \text{ and}$$

$$IFMS_{ZF2}(A, C) = \frac{1}{2} \sum_{j=1}^2 \left[1 - \frac{1}{2(1)} \sum_{i=1}^1 \left\{ |\delta_A^j - \delta_C^j| + |\alpha_A^j - \alpha_C^j| \right\} \right] = 0.9625$$

where $\delta_A^j(x_i) = \mu_A^j(x_i) + (1 - \mu_A^j(x_i) - \vartheta_A^j(x_i)) \mu_A^j(x_i)$ and

$$\alpha_A^j(x_i) = \vartheta_A^j(x_i) + (1 - \mu_A^j(x_i) - \vartheta_A^j(x_i)) \vartheta_A^j(x_i)$$

Also from the **Second Similarity Measure**, we cannot differentiate the similarity

But, using the modified Similarity Measure consisting of the membership, non membership and hesitation degrees, we can clearly differentiate the similarity measure correctly.

$$IFMS_{ZF}(A, B) = \frac{1}{2} \sum_{j=1}^2 \left[1 - \frac{1}{2(1)} \sum_{i=1}^1 \left\{ |\delta_A^j - \delta_B^j| + |\alpha_A^j - \alpha_B^j| + |\beta_A^j - \beta_B^j| \right\} \right] = 0.935 \text{ and}$$

$$IFMS_{ZF}(A, C) = \frac{1}{2} \sum_{j=1}^2 \left[1 - \frac{1}{2(1)} \sum_{i=1}^1 \left\{ |\delta_A^j - \delta_C^j| + |\alpha_A^j - \alpha_C^j| + |\beta_A^j - \beta_C^j| \right\} \right] = 0.940$$

where $\delta_A^j(x_i) = \mu_A^j(x_i) + (1 - \mu_A^j(x_i) - \vartheta_A^j(x_i)) \mu_A^j(x_i)$

$$\alpha_A^j(x_i) = \vartheta_A^j(x_i) + (1 - \mu_A^j(x_i) - \vartheta_A^j(x_i)) \vartheta_A^j(x_i) \text{ and } \beta_A^j(x_i) = (1 - \delta_A^j(x_i) - \alpha_B^j(x_i))$$

Hence, for pattern recognition or decision making situations the new similarity measure of IFMS using the Zhang and Fu's method of the first and second measures cannot obtain the correct recognition results. While using the modified Zhang and Fu's similarity measure with three parameters distinguish clearly and correctly the recognition patterns.

Therefore, **the modified and enhanced similarity measure is capable of classification and makes a decision correctly.**

V. MEDICAL DIAGNOSIS USING IFMS - ZHANG AND FU'S MEASURE

As Medical diagnosis contains lots of uncertainties, they are the most interesting and fruitful areas of application for Intuitionistic fuzzy set theory. Due to the increased volume of information available to physicians from new medical technologies, the process of classifying different set of symptoms under a single name of disease becomes difficult. In some practical situations, there is the possibility of each element having different membership and non membership functions. The proposed similarity measure among the Patients Vs Symptoms and Symptoms Vs diseases gives the proper medical diagnosis. The unique feature of this proposed method is that it considers multi membership and non membership. By taking one time inspection, there may be error in diagnosis. Hence, this multi time inspection, by taking the samples of the same patient at different times gives best diagnosis.

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Let $P = \{ P_1, P_2, P_3, P_4 \}$ be a set of Patients. $D = \{ \text{Fever, Tuberculosis, Typhoid, Throat disease} \}$ be the set of diseases and $S = \{ \text{Temperature, Cough, Throat pain, Headache, Body pain} \}$ be the set of symptoms.

Our solution is to examine the patient at different time intervals (three times a day), which in turn give arise to different membership and non membership function for each patient.

TABLE : 5.1 – IFMs Q : The Relation between Patient and Symptoms

Q	Temperature	Cough	Throat Pain	Head Ache	Body Pain
P ₁	(0.6, 0.2)	(0.4, 0.3)	(0.1, 0.7)	(0.5, 0.4)	(0.2, 0.6)
	(0.7, 0.1)	(0.3, 0.6)	(0.2, 0.7)	(0.6, 0.3)	(0.3, 0.4)
	(0.5, 0.4)	(0.4, 0.4)	(0, 0.8)	(0.7, 0.2)	(0.4, 0.4)
P ₂	(0.4, 0.5)	(0.7, 0.2)	(0.6, 0.3)	(0.3, 0.7)	(0.8, 0.1)
	(0.3, 0.4)	(0.6, 0.2)	(0.5, 0.3)	(0.6, 0.3)	(0.7, 0.2)
	(0.5, 0.4)	(0.8, 0.1)	(0.4, 0.4)	(0.2, 0.7)	(0.5, 0.3)
P ₃	(0.1, 0.7)	(0.3, 0.6)	(0.8, 0)	(0.3, 0.6)	(0.4, 0.4)
	(0.2, 0.6)	(0.2, 0)	(0.7, 0.1)	(0.2, 0.7)	(0.3, 0.7)
	(0.1, 0.9)	(0.1, 0.7)	(0.8, 0.1)	(0.2, 0.6)	(0.2, 0.7)

Let the samples be taken at **three different timings** in a day (morning, noon and night)

TABLE : 5.2 – IFMs R : The Relation among Symptoms and Diseases

R	Viral Fever	Tuberculosis	Typhoid	Throat disease
Temperature	(0.8, 0.1)	(0.2, 0.7)	(0.5, 0.3)	(0.1, 0.7)
Cough	(0.2, 0.7)	(0.9, 0)	(0.3, 0.5)	(0.3, 0.6)
Throat Pain	(0.3, 0.5)	(0.7, 0.2)	(0.2, 0.7)	(0.8, 0.1)
Head ache	(0.5, 0.3)	(0.6, 0.3)	(0.2, 0.6)	(0.1, 0.8)
Body ache	(0.5, 0.4)	(0.7, 0.2)	(0.4, 0.4)	(0.1, 0.8)

TABLE : 5.3 – The Similarity Measure between IFMs Q and R :

Zhang And Fu's Similarity measure	Viral Fever	Tuberculosis	Typhoid	Throat disease
P ₁	0.7473	0.5650	0.8157	0.5287
P ₂	0.6580	0.7977	0.7167	0.5880
P ₃	0.5827	0.6413	0.6553	0.8407

The **highest similarity measure** from the table 4.3 gives the proper medical diagnosis.

The diagnosis refers that Patient **P1** suffers from **Typhoid**, Patient **P2** suffers from **Tuberculosis** and Patient **P3** suffers from **Throat disease**.

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VI. PATTERN RECOGNITION OF THE PROPOSED SIMILARITY MEASURE

PATTERN RECOGNITION: 6.1

Let $X = \{A_1, A_2, A_3, A_4, \dots, A_n\}$ with $A = \{A_1, A_2, A_3, A_4, A_5\}$ and $B = \{A_2, A_5, A_7, A_8, A_9\}$ are the *IFMS* defined as

$$\begin{aligned} \text{Pattern I} &= \{ \langle A_1 : (0.6, 0.4), (0.5, 0.5) \rangle, \langle A_2 : (0.5, 0.3), (0.4, 0.5) \rangle, \langle A_3 : (0.5, 0.2), (0.4, 0.4) \rangle, \\ &\quad \langle A_4 : (0.3, 0.2), (0.3, 0.2) \rangle, \langle A_5 : (0.2, 0.1), (0.2, 0.2) \rangle \} \\ \text{Pattern II} &= \{ \langle A_2 : (0.5, 0.3), (0.4, 0.5) \rangle, \langle A_5 : (0.2, 0.1), (0.2, 0.2) \rangle, \langle A_7 : (0.7, 0.3), (0.4, 0.2) \rangle, \\ &\quad \langle A_8 : (0.4, 0.5), (0.3, 0.3) \rangle, \langle A_9 : (0.2, 0.7), (0.1, 0.8) \rangle \} \end{aligned}$$

Then the testing *IFMS* Pattern III be $\{A_6, A_7, A_8, A_9, A_{10}\}$ such that $\{ \langle A_6 : (0.8, 0.1), (0.4, 0.6) \rangle, \langle A_7 : (0.7, 0.3), (0.4, 0.2) \rangle, \langle A_8 : (0.4, 0.5), (0.3, 0.3) \rangle, \langle A_9 : (0.2, 0.7), (0.1, 0.8) \rangle, \langle A_{10} : (0.2, 0.6), (0, 0.6) \rangle \}$

Here, the cardinality $\eta = 5$ as $|Mc(A)| = |NMc(A)| = 5$ and $|Mc(B)| = |NMc(B)| = 5$, then the Similarity measure between Pattern (I, III) is 0.681, Pattern (II, III) is **0.770**

The testing Pattern III belongs to Pattern II type

PATTERN RECOGNITION: 6.2

Let $X = \{A_1, A_2, A_3, A_4, \dots, A_n\}$ with $X_1 = \{A_1, A_2\}$; $X_2 = \{A_3, A_4\}$; $X_3 = \{A_1, A_4\}$ are the *IFMS* defined as

$$\begin{aligned} A &= \{ \langle A_1 : (0.4, 0.2), (0.3, 0.1), (0.2, 0.1), (0.1, 0.4) \rangle, \\ &\quad \langle A_2 : (0.6, 0.3), (0.4, 0.5), (0.4, 0.3), (0.2, 0.6) \rangle \} \\ B &= \{ \langle A_3 : (0.5, 0.2), (0.4, 0.2), (0.4, 0.1), (0.1, 0.1) \rangle, \\ &\quad \langle A_4 : (0.4, 0.6), (0.4, 0.5), (0.3, 0.4), (0.2, 0.4) \rangle \} \\ C &= \{ \langle A_1 : (0.4, 0.2), (0.3, 0.1), (0.2, 0.1), (0.1, 0.4) \rangle, \\ &\quad \langle A_4 : (0.4, 0.6), (0.4, 0.5), (0.3, 0.4), (0.2, 0.4) \rangle \} \end{aligned}$$

then the Pattern D of *IFMS* referred as $\{ \langle A_5 : (0.4, 0.6), (0.4, 0.5), (0.3, 0.4), (0.2, 0.4) \rangle, \langle A_6 : (0.4, 0.2), (0.5, 0.5), (0.2, 0.4), (0.2, 0.5) \rangle \}$

The cardinality $\eta = 2$ as $|Mc(A)| = |NMc(A)| = |Hc(A)| = 2$ and $|Mc(B)| = |NMc(B)| = |Hc(B)| = 2$, then the Proposed Similarity measure between the Pattern (A, D) is **0.7857**; the Pattern (B, D) is 0.7425 and the Pattern (C, D) is 0.775.

Hence, the testing Pattern D belongs to Pattern A type

VII. CONCLUSION

A new similarity measure of *IFMS* from *IFS* theory is derived. The prominent characteristic of this method is that it considers multi membership, non membership, hesitation functions and this similarity measure guarantee that the similarity measure of any two *IFMSs* equals to one if and only if the two *IFMSs* are the same referred in **example 4.1**. Also, the **example 4.2** shows that the new modified measure perform well in the case of three representatives of *IFMS* - membership, non membership and hesitation functions than the proposed measure with two representatives of *IFMS* - membership and non membership functions Finally, this novel similarity method is applied to medical diagnosis and pattern recognitions problems.

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