ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

(ISO 3297: 2007 Certified Organization)

Vol. 2, Issue 8, August 2013

# THERMAL CONVECTION IN WALTERS'B′ VISCOELASTIC FLUID IN A DARCY – BRINKMAN POROUS MEDIUM WITH EFFECT OF DUSTY PARTICLES

K. Thirumurugan [1], R. Vasanthakumari [2]

Research – Scholar, Department of Mathematics, Bharathiar University, Coimbatore, India.

Principal, Kasthurba College for Women, Villianur, Puducherry, India.

**Abstract:** The problem of convection of compressible walters'B′ viscoelastic fluid in a Darcy – Brinkman porous medium with effect of dusty particles is considered . By applying normal mode analysis method, the dispersion relation has been derived and solved analytically. It has been found that the medium permeability, dusty particles, gravity field and viscoelasticity introduce oscillatory modes. For stationary convection, it is found that the Darcy number has stabilizing effect whereas the dusty particles and medium permeability has destabilizing effects on the system. The effect of dusty particles, Darcy number and medium permeability have also been shown graphically.

**Key Words:** Walters'B′ fluid, Compressibility, Brinkman porous medium, Dusty particles, viscoelasticity.

## I. INTRODUCTION

In recent years, considerable interest has been evinced in the study of thermal instability in a porous medium because it has various applications in geophysics, food processing and nuclear reactor Rana[6]. A detailed account of the thermal instability of a Newtonian fluid under varying assumptions of hydrodynamics and hydromagnetics has been given by Chandrasekhar [1]. Lapwood[4] has studied the connective flow in porous medium using linearized stability theory. The Rayleigh instability of a thermal boundary layer in flow through a porous medium has been considered by Wooding[15]. Scanlon and Segel[10] have considered the effect of suspended particles on the onset of Be 'nard and found that the critical Rayleigh number was reduced solely because the heat capacity of pure gas was supplemented by the particles. Sharma and Sunil [11] have studied the thermal instability of an Oldroydian viscoelastic fluid with suspended particles in hydromagnetics in a porous medium. There are many elastico – viscous fluids that cannot be characterized by Maxwell's constitutive relations or Oldroyd's constitutive relations. One such class of elastic – viscous fluid in Rivlin – Erickson fluid. Rivlin and Erickson [9] have proposed a theoretical model for such another elastico – viscous fluid.

The investigation in porous media has been started with the simple Darcy model and gradually was extended to Darcy – Brinkman model. A good account of convection problem in a porous medium is given by vafai and Hadim [14], Ingham and pop [2] and Nield and Bejan [5]. Kuzentsov and Nield [5] have studied the thermal instability of porous medium. Sharma et al.[12] have studied the instability of streaming Rivlin-Erickson fluids in porous medium. Recently, Rana and Thakur[8] studied the instability of couple – stress fluid permeated with suspended particles saturating a porous medium. Rana et al[7] have studied an effect of rotation on thermal instability of compressible Walters'B′ elastico – viscous fluid in porous medium. Shivakumara et al[13] has studied an effect of thermal modulation on the onset of thermal convection in Walters'B′ viscoelastic fluid in a porous medium. The interest of investigations of non – Newtonian fluids is also motivated

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

(ISO 3297: 2007 Certified Organization)

Vol. 2, Issue 8, August 2013

by a wide range of engineering application which includes ground pollutions by chemicals which were non – Newtonian like lubricants and polymers in the treatment of sewage sludge in drying beds. Recently, polymers are used in agriculture, communications applications and in bio medical applications. Examples of these application filtration processes, packed bed reactors, insulation system, ceramic processing, enhanced oil recovery, chromatography etc.

Keeping mind the importance in various applications mentioned above, the objective of the present paper is to study the effect of suspended particles on thermal convection in Walters'B′ elastico – viscous fluid in a brinkman porous medium. This necessitates of additional parameter namely Darcy number.

## II. MATHEMATICAL MODEL AND PERTURBATION EQUATIONS

Consider an infinite, horizontal, compressible Walters' B′ elastic – viscous fluid layer fluid layer of thickness d, heated and soluted from below so that the temperatures and densities at the bottom surface z = 0 are $T_0$ and $\rho_0$ and at the upper surface z = 0 are $T_d$ and $\rho_d$, respectively, and that a uniform temperature gradient $\beta(= |dT/dz|)$ is maintained. The gravity field $\mathbf{g}(0,0,-g)$, and a uniform vertical magnetic field $\mathbf{H}(0,0,H)$, act on the system.

Let $\mathbf{q}(u,v,w)$, p, ρ, T, υ,υ′$, \eta, \mu_e, N, and\ e$ denote the velocity, pressure, density, temperature, kinematic viscosity, and kinematic viscoelasticity, resistivity, magnetic permeability, electron number, density and charge of an electron respectively, and $\mathbf{r}(x,y,z)$.

The equations of motion, continuity, heat conduction, and Maxwell's equations governing the flow of Walters'B′ viscoelastic fluid and the equation of the state are,

$$\frac{1}{\varepsilon}\left[\frac{\partial v}{\partial t}+\frac{1}{\varepsilon}(\boldsymbol{v}.\nabla)\boldsymbol{v}\right] = -\left(\frac{1}{\rho_0}\right)\nabla p + \boldsymbol{g}\left(1+\frac{\delta_\rho}{\rho_0}\right)-\frac{1}{k_1}\left(v-v'\frac{\partial}{\partial t}\right)\boldsymbol{v}+\frac{\mu_e}{\rho_0}\nabla^2\boldsymbol{v}+\frac{\boldsymbol{K'N}}{\boldsymbol{\rho_0\varepsilon}}(\boldsymbol{v_d}-\boldsymbol{v}) \quad (1)$$

$$\frac{\partial q}{\partial t}+\nabla(\rho.q)=0, \quad (2)$$

$$E\frac{\partial T}{\partial t}+(\mathbf{v}.\nabla)T+\frac{mNC_{pt}}{\rho_0 c_f}\left[\varepsilon\frac{\partial}{\partial t}+v_d.\boldsymbol{\nabla}\right]T=K\nabla^2 T \quad (3)$$

$$\rho = \rho_0[1-\alpha(T-T_0) \quad (4)$$

Where the suffix zero refers to values at the references level z = 0.

Here $v_d(\bar{x},t)$ and $N(\bar{x},t)$ denote the velocity and number density of the particles respectively, $K' = 6\pi\eta\rho v$, where $\eta$ is the particle radius, is the strokes drag coefficient, $v_d = (l,r,s)$ and $\bar{x}=(x,y,z)$.

$$E=\varepsilon+(1-\varepsilon)\left(\frac{\rho_s C_s}{\rho_0 C_f}\right) \quad (5)$$

Which is constant, k is thermal diffusivity $\rho_s, C_s$; $\rho_0, C_f$ denote the density and heat capacity of solid(porous) matrix and fluid, respectively. If $mN$ is the mass of the particles per unit volume. Then equations of motion continuity for the particles are expressed Eq.(5) and Eq.(6) respectively.

$$mN\left[\frac{\partial v_d}{\partial t}+\frac{1}{\varepsilon}(\boldsymbol{v_d}.\boldsymbol{\nabla})\boldsymbol{v_d}\right]=\mathrm{K}'\mathrm{N}(\boldsymbol{v}-\boldsymbol{v_d}) \quad (6)$$

The presence of particles adds an extra force term proportional to the velocity difference between particles and fluid and appears in the equation of motion(Eq.(1)). Since the force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid, there must be an extra force term, equal in magnitude but opposite sign, in the equation of the motion for the particles (Eq.(6)). The buoyancy force on the particle is neglected. Inter particles reactions are not considered either since the distance between the particles is quite large compared with their diameters. These assumptions have been used in writing the equation of motion (Eq.(6)) for the particles. The initial state of the system is taken to be quiescent layer(no settling) with a uniform particle distribution number. The initial state is defined as Eq.(7).

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

(ISO 3297: 2007 Certified Organization)

Vol. 2, Issue 8, August 2013

The basic state of the fluid is

$\boldsymbol{v} = (0,0,0), \qquad T = -\beta z + T_0$

$\rho = \rho_0(1 + \alpha\beta z)$ (7)

This is an exact solution to the governing equations. Let $\boldsymbol{v}(u, v, w), \theta, \delta p\ and \delta p$ denote the perturbations in fluid velocity $\boldsymbol{v}(0,0,0)$, Temperature $T$, pressure p and density $\rho$ respectively. The change in density $\delta p$ caused by the perturbation $\theta$ in temperature is given by Eq. (8).

$\delta_\rho = -\alpha\rho_0\theta$ (8)

Then the linearized perturbation equations are

$$\frac{1}{\varepsilon}\frac{\partial \boldsymbol{v}}{\partial t} = -\frac{\mathbf{1}}{\boldsymbol{\rho_0}}(\nabla\delta p) - \boldsymbol{g}\frac{\delta p}{\rho_0} - \frac{1}{k_1}\left(v - v'\frac{\boldsymbol{\partial}}{\boldsymbol{\partial t}}\right)\boldsymbol{v} + \frac{\bar{\boldsymbol{\mu}}}{\boldsymbol{\rho_0}}\nabla^2\boldsymbol{v} + \frac{\boldsymbol{K'N}}{\boldsymbol{\rho_0\varepsilon}}(\boldsymbol{v_d} - \boldsymbol{v}) \tag{9}$$

$$\nabla.\boldsymbol{v} = 0 \tag{10}$$

$$(1 + b\varepsilon)\frac{\partial\theta}{\partial t} = \left(\beta - \frac{g}{C_p}\right)(w + bs) + k\nabla^2\theta \tag{11}$$

Where $b = \frac{mNC_pt}{\rho_0 c_f}$ and $w, s$ are the vertical fluid and particles velocity.

In Cartesian form, Eqs. (9) – (13) with the help of Eq. (8) can be expressed as Eq. (12) and Eq. (13).

$$\frac{1}{\varepsilon}\frac{\partial \boldsymbol{u}}{\partial t} = -\frac{\mathbf{1}}{\boldsymbol{\rho_0}}\frac{\partial}{\partial x}(\delta p) - \frac{1}{k_1}\left(v - v'\frac{\partial}{\partial t}\right)\boldsymbol{u} + \frac{\bar{\boldsymbol{\mu}}}{\boldsymbol{\rho_0}}\nabla^2\boldsymbol{u} - \frac{mN}{\in\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\boldsymbol{\rho_0}}\frac{\partial \boldsymbol{u}}{\partial t}, \tag{12}$$

$$\frac{1}{\varepsilon}\frac{\partial \boldsymbol{v}}{\partial t} = -\frac{\mathbf{1}}{\boldsymbol{\rho_0}}\frac{\partial}{\partial y}(\delta p) - \frac{1}{k_1}\left(v - v'\frac{\partial}{\partial t}\right)\boldsymbol{v} + \frac{\bar{\boldsymbol{\mu}}}{\boldsymbol{\rho_0}}\nabla^2 v - \frac{mN}{\in\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\boldsymbol{\rho_0}}\frac{\partial v}{\partial t}, \tag{13}$$

$$\frac{1}{\varepsilon}\frac{\partial \boldsymbol{w}}{\partial t} = -\frac{\mathbf{1}}{\boldsymbol{\rho_0}}\frac{\partial}{\partial z}(\delta p) + g\alpha\theta - \frac{1}{k_1}\left(v - v'\frac{\partial}{\partial t}\right)\boldsymbol{w} + \frac{\bar{\boldsymbol{\mu}}}{\boldsymbol{\rho_0}}\nabla^2\boldsymbol{w} - \frac{mN}{\in\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\boldsymbol{\rho_0}}\frac{\partial \boldsymbol{w}}{\partial t}, \tag{14}$$

$$\frac{\partial \boldsymbol{u}}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \boldsymbol{w}}{\partial z} \tag{15}$$

$$(1 + b\varepsilon)\frac{\partial\theta}{\partial t} = \left(\beta - \frac{g}{C_p}\right)(w + bs) + k\nabla^2\theta \tag{16}$$

Operating Eqs. (12) and (13) by $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$ respectively, adding using Eq. (8), we get

$$\frac{1}{\varepsilon}\frac{\partial}{\partial t}\left(\frac{\partial \boldsymbol{w}}{\partial z}\right) = -\frac{\mathbf{1}}{\boldsymbol{\rho_0}}\frac{\partial}{\partial x}\left(\nabla^2 - \frac{\partial^2}{\partial z^2}\right) - \frac{1}{k_1}\left(v - v'\frac{\partial}{\partial t}\right)\left(\frac{\partial \boldsymbol{w}}{\partial z}\right) + \frac{\bar{\boldsymbol{\mu}}}{\boldsymbol{\rho_0}}\nabla^2\left(\frac{\partial \boldsymbol{w}}{\partial z}\right) - \frac{mN}{\in\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\boldsymbol{\rho_0}}\frac{\partial}{\partial t}\left(\frac{\partial \boldsymbol{w}}{\partial z}\right) \tag{17}$$

Operating Eqs. (14) and (17) by $\left(\nabla^2 - \frac{\partial^2}{\partial z^2}\right)$ and $\frac{\partial}{\partial z}$ respectively and adding to eliminate $\delta p$ between Eqs. (14) and (17), we get

$$\frac{1}{\varepsilon}\frac{\partial}{\partial t}(\nabla^2 w) = -\frac{1}{k_1}\left(v - v'\frac{\partial}{\partial t}\right)(\nabla^2 w) + \frac{\bar{\boldsymbol{\mu}}}{\boldsymbol{\rho_0}}(\nabla^4 w) + \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\alpha\theta - \frac{mN}{\in\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\boldsymbol{\rho_0}}\frac{\partial}{\partial t}(\nabla^2 \mathrm{w}) \tag{18}$$

Where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

## III. THE DISPERSION RELATION

Following the normal mode analyses, we assume that the perturbation quantities have x and y and t dependence of the form

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

(ISO 3297: 2007 Certified Organization)

Vol. 2, Issue 8, August 2013

$$[w,\theta] = [W(z), \Theta(z)] \exp(ilx + imy + nt) \quad (19)$$

Using Eq. (19) in Eqs. (18) and (16) become

$$\frac{n}{\varepsilon}\left[\frac{d^2}{dz^2} - k^2\right] W = -gk^2\alpha\theta - \frac{1}{k_1}(v - v'n)\left(\frac{d^2}{dz^2} - k^2\right) W + \frac{\bar{\mu}}{\rho_0}\left(\frac{d^2}{dz^2} - k^2\right) W - \frac{mN}{\in\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\rho_0}\left(\frac{d^2}{dz^2} - k^2\right) W \quad (20)$$

Eq. (20) and (21) in non – dimensional form, become

$$\left[1 + \left(\frac{p_l}{\varepsilon} + \frac{Mp_l}{\varepsilon(1+\tau_1\sigma)} + F\right)\sigma - D_A(D^2 - a^2)\right](D^2 - a^2)W + \frac{ga^2d^2p_l\alpha\Theta}{v} = 0 \quad (21)$$

$$(1 + b\varepsilon)\frac{\partial\theta}{\partial t} = \beta(w + bs) + k\left(\frac{d^2}{dz^2} - k^2\right)\theta \quad (22)$$

Where $a = kd, \sigma = \frac{nd^2}{v}, P_1 = \frac{V}{\chi}, F = \frac{v'}{d^2}, and\ p_l = \frac{k_1}{d^2}$, is the dimensionless medium permeability, $p_1 = \frac{v}{k}$, is the themal Prandtl number and $D_A = \frac{\tilde{\mu}k_1}{\mu d^2}$, is the Darcy number modified by the viscosity ratio.

Eliminating Θ between Eqs. (22) and (23), we obtain

$$\left[1 + \left(\frac{p_l}{\varepsilon} + \frac{Mp_l}{\varepsilon(1+\tau_1\sigma)} + F\right)\sigma - D_A(D^2 - a^2)\right](D^2 - a^2)W + \frac{ga^2d^2p_l\alpha\Theta}{v} = 0 \quad (23)$$

$$[D^2 - a^2 - E_1p_1\sigma]\Theta = -\frac{\beta d^2}{k}\left(\frac{B+\tau_1\sigma}{1+\tau_1\sigma}\right) W \quad (24)$$

Where $R = \left(\frac{g\alpha\beta\, d^4}{v\chi}\right)$ is the Thermal Rayleigh Number

Assume that the temperature at the boundaries is kept fixe, the fluid layer is confirmed between two boundaries and adjoining medium is electrically non – conducting. The boundary conditions appropriate to the problem are (Chandrasekhar, 1981)

$W = D^2W = 0$, $DZ = 0$, $\Theta = 0$ at $z = 0$, $z=l$, (25)

The case of two boundaries, though a little artificial is the most appropriate for steller atmospheres. Using the boundary conditions(Eq. (25)), all the even order derivatives of $W$ characterizing the lowest mode is

$W = W_0 \sin \pi z$, where $W_0$ is a constant. (26)

Substituting Eq. (27) in Eq. (25) we get

$$R_1xP = \left(\frac{G}{G-1}\right)\left[1 + \left(\frac{p_l}{\varepsilon} + \frac{MP}{\varepsilon(1+\tau_1 i\sigma_1)} + \pi^2F\right) i\sigma_1 + D_{A1}(1 + x)\right](1 + x)(1 + x - E_1p_1 i\sigma_1) + \left(\frac{1+i\tau_1\pi^2 i\sigma_1}{B+i\tau_1\pi^2 i\sigma_1}\right) \quad (27)$$

Where $R_1 = \frac{R}{\pi^4}, T_{A1} = \frac{T_A}{\pi^4}, D_{A1} = \frac{D_A}{\pi^2}, x = \frac{a^2}{\pi^2}, i\sigma_1 = \frac{\sigma}{\pi^2}, P = \pi^2 p_l, \tau = \frac{m}{K'}, \tau_1 = \frac{\tau v}{d^2}, M = \frac{mN}{\rho_0}, E_1 = 1 + b\varepsilon, B = b + 1$.

Eq. (27) is required dispersion relation accounting for the onset of thermal convection in Walters'B′ elastico – viscous fluid permeated with suspended particles in a Brinkman porous medium.

## IV. STABILITY OF THE SYSTEM AND OSCILLATORY MODES

The possibility of oscillatory modes in Walters' B′ elastic – viscous fluid due to the presence of suspended particles, viscoelasticity, medium permeability and gravity field will be examined. Multiply Eq. (22) by $W^*$ the complex conjugate of W, integrating over the range of z and making use of Eq. (23) with the help of boundary conditions (Eq. (25)), we obtain

$$\left[1 + \left(\frac{p_l}{\varepsilon} + \frac{MP}{\varepsilon(1+\tau_1 i\sigma_1)} + F\right)\sigma\right] I_1 - D_A I_2 - \frac{ga^2d^2\alpha k p_l}{v\beta} + \left(\frac{G}{G-1}\right)\left(\frac{B+\tau_1\sigma^*}{1+\tau_1\sigma^*}\right)(I_3 + Ep_1\sigma^* I_4) = 0 \quad (28)$$

Where

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

*(ISO 3297: 2007 Certified Organization)*

**Vol. 2, Issue 8, August 2013**

$I_1 = \int_0^1 (|DW|^2 + a^2|W|^2)dz,$
$I_2 = \int_0^1 (|DW|^2 + 2a^2|W|^2 + a^4|W|^2)dz,$
$I_3 = \int_0^1 (|D\Theta|^2 + a^2|\Theta|^2)dz,$
$I_4 = \int_0^1 |\Theta|^2 dz,$

The integral part $I_1 - I_4$ are all positive definite, putting $\sigma = i\sigma_1$in Eq. (28), where $\sigma_1$ is real and equating the imaginary parts, we obtain

$$\sigma_i\left[\left(\frac{p_l}{\varepsilon} + \frac{MP}{\varepsilon(1+\tau_1 i\sigma_1)} + F\right)I_1 - \frac{ga^2d^2\alpha k p_l}{\nu\beta}\left\{\left(\frac{\tau_1(B-1)}{B^2+\tau_1^2\sigma_i^2}\right)I_3 + \left(\frac{G}{G-1}\right)\left(\frac{\tau_1(B-1)}{B^2+\tau_1^2\sigma_i^2}\right)Ep_1\sigma^* I_4\right\}\right] = 0 \tag{29}$$

Eq. (29) implies that $\sigma_i = 0$ or $\sigma_i \neq 0$ which means that modes may be non – oscillatory or oscillatory modes introduced due to the presence of viscosity, viscoelasticity, suspended particles and medium permeability which were non – existent in their absence.

## V. THE STATIONARY CONVECTION

For stationary convection putting $\sigma = 0$ in Eq. (27), we obtain

$$R_1 = \left(\frac{G}{G-1}\right)\frac{(1+x)^2}{xPB}[1 + (1+x)D_{A1}] \tag{30}$$

Eq. (30) expresses the modified Rayleigh number $R_1$ as a function of the dimensionless wave number x and the parameters $B, D_{A1}, P$ and Walters'B′ viscous fluid behaves like an ordinary Newtonian fluid since elastico – viscous parameter F vanishes with $\sigma$. To study the effects of suspended particles, Darcy number, medium permeability, the behavior of $\frac{dR_1}{dB}, \frac{dR_1}{dD_{A1}}$ and $\frac{dR_1}{dP}$ has been examined analytically, from Eq. (30) we get

$$\frac{dR_1}{dB} = -\left(\frac{G}{G-1}\right)\left(\frac{(1+x)}{xPB^2}\right)[1 + (1+x)D_{A1}] \tag{31}$$

Which is negative. Hence suspended particles have destabilizing effect on the thermal convection Walters'B′ elastico-viscous fluid in the Brinkman porous medium From Eq. (30), we get

$$\frac{dR_1}{dD_{A1}} = \left(\frac{G}{G-1}\right)\frac{(1+x)^3}{xPB} \tag{32}$$

Which is positive implying thereby the stabilizing effect of Darcy medium on the thermal convection in Walters'B′ elastico-viscous fluid permeated with the suspended particles in a Brinkman porous medium?

It is evident from eq. (31) that

$$\frac{dR_1}{dD_{A1}} = \left(\frac{G}{G-1}\right)\frac{(1+x)^2}{xPB^2}[1 + (1+x)D_{A1}] \tag{33}$$

From Eq. (33), it is observed that the medium permeability has destabilizing effect on thermal convection in Walters'B′ elastico-viscous fluid permeated with the suspended particles in a Brinkman porous medium.

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

(ISO 3297: 2007 Certified Organization)

Vol. 2, Issue 8, August 2013

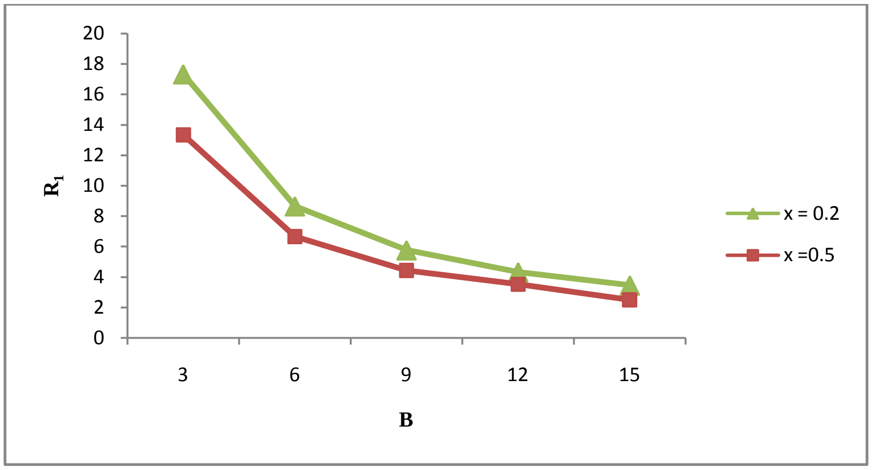


**Fig. 1.** Variation of Rayleigh number $R_1$ with the dust particles B for P = 2, $DA_1$=10 and G=10 for fixed wave numbers x = 0.2, 0.5.

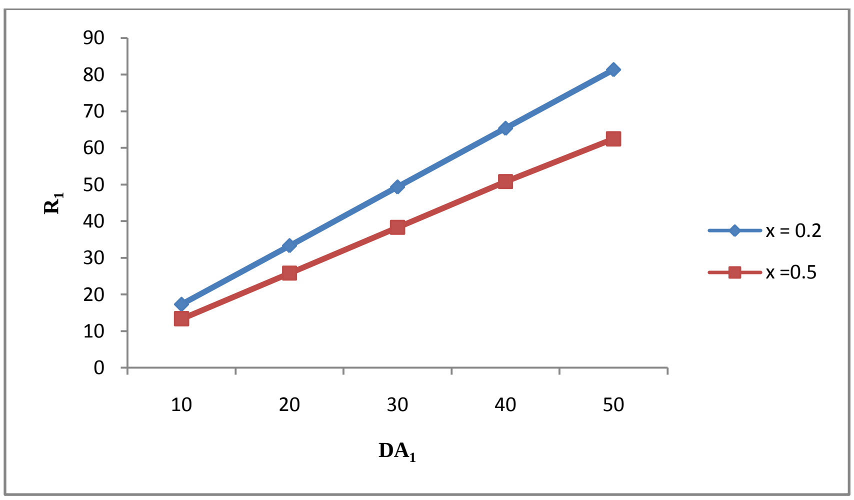


**Fig. 2.** Variation of Rayleigh number $R_1$ with Darcy number $DA_1$ for P = 2, B = 3 and G=10 for fixed wave numbers x = 0.2, 0.5.

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

(ISO 3297: 2007 Certified Organization)

Vol. 2, Issue 8, August 2013

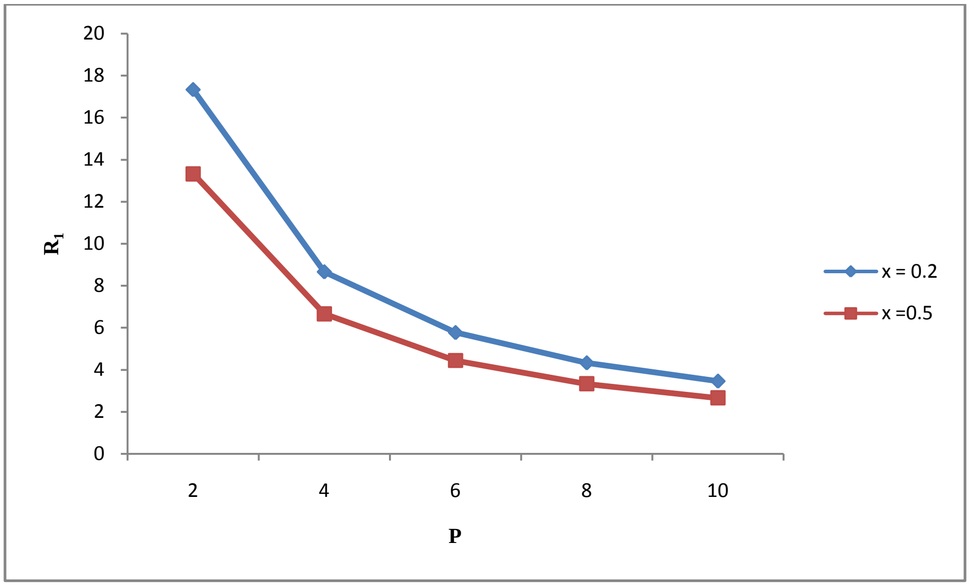


**Fig. 3.** Variation of Rayleigh number $R_1$ with medium permeability P for B = 3, $DA_1$=10 and G=10 for fixed wave numbers x = 0.2, 0.5.

The dispersion relation in Eq. (30) is analysed numerically to depict the stability characteristics. In Fig. (1), Rayleigh Number $R_1$ is plotted against suspended particles B for P = 2 and $D_{A1} = 10$ for fixed wave numbers $x = 0.2$, $x = 0.5$. This shows that suspended particles has destabilizing effect on thermal instability of Walters'B′ fluid in the Brinkman porous medium for fixed wave numbers $x = 0.2$, $x = 0.5$. Which clearly verifies the result numerically as derived in Eq. (31). In Fig. (2), Rayleigh Number $R_1$ is plotted against with Darcy Number has a stabilizing effect on thermal convection in Walters'B′ elastico viscous fluid permeated with dusty particles in a Brinkman porous medium which is clearly verifies the result numerically as derived in Equation (32). In Fig (3) Rayleigh number $R_1$ is plotted against medium permeability P for $D_{A1} = 10$ and B = 3 for fixed wave numbers $x = 0.2$, $x = 0.5$ . This slows that medium permeability has a destabilizing effect on the thermal convection in Walters'B′ elastico-viscous fluid permeated with suspended particles in a Brinkman porous medium which is clearly verifies the result numerically as derived in Eq. (33).

## VI. CONCLUSION

The effect of suspended particles on thermal convection in Walters'B′ elastico-viscous fluid heated from below in the Brinkman porous medium has been investigated. The dispersion relation, including the effects of dusty particles, Darcy number, medium permeability and viscoelasticity on the thermal convection in Walters'B′ fluid in porous medium is derived. From the analysis, the main conclusions are as follows:

(i) For the case of stationary convection, Walters'B′ elastico-viscous fluid behaves like an ordinary Newtonian fluid as elastico-viscous parameter F vanishes with $\sigma$.

(ii) The expression for $\frac{dR_1}{dB}, \frac{dR_1}{dD_{A1}}$ and $\frac{dR_1}{dP}$ has been examined analytically and it has been found that the Darcy number has stabilizing effect whereas the dusty particles and medium permeability has a destabilizing effect on the system

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

(ISO 3297: 2007 Certified Organization)

Vol. 2, Issue 8, August 2013

(iii) The effects of dusty particles, Darcy Number and medium permeability on thermal convection in Walters'B′ elastico – viscous fluid permeated with dusty particles, in a Brinkman porous medium have been shown analyzed.

(iv) The oscillatory modes introduced due to the presence of viscoelasticity, dusty particles, gravity field and medium permeability, which were non – existent in their absence.

## REFERENCES

[1] Chandrasekhar, S. "Hydrodynamics and Hydromagnetics stability". New York : Dover publication, 1981.

[2] Ingham, D.D. and Pop L. "Transport phenomena in porous media". New York: Elseiver, 1981.

[3] Kuzentsov, A.V and Nield, D.A. "Thermal instability in a porous medium layer saturated by a nano fluid. Brinkman model. Transport in Porous Media", 81(3): 409-422. . 2010

[4] Lapwood, E.R. "Convection of a fluid in porous medium. Mathematical Proceedings of the Cambridge Philosophical Society", 44:508-519. 1948

[5] Nield, D.A. and Bejan, A. Convection in Porous medium". New York: Springer 2006.

[6] Rana G.C, "Effect of suspended particles on thermal instability of Walters' (model B′) fluid in a Brinckman porous medium", IJMA, 3, 399 – 406, 2012.

[7] Rana G. C, Kango S.K, "Effect of rotation on thermal instability of compressible Walters' (model B′) elastico – viscousfluid in porous medium", JARAM, 3, 44 – 57, 2011.

[8] Rana, G.C and Thakur, R.C. "A mathematical theorem on the onset of couple stress fluid permeated with suspended particles saturating a porous medium. International journal of Multiphysics" 2012.

[9] Rivlin R.S and ericksen, J.L.. "Stress-deformation relation for isotropic materials". Chemistry Biodiversity, 4(2):323 – 425, 1955.

[10] Scanlon, J.W. and Segel, L.A. "Effect of suspended particles on the onset of Be'nard convection". Physics fluids,16: 1537 -1578, 1973.

[11] Sharma, R.C. and Sunil. "Thermal instability of an Oldroydian fluid with suspended particles in hydromagnetics in porous medium". Polymer-plastics Technology and Engineering, 33(3): 323 – 339, 1994.

[12] Sharma, V., Kishore, K. and Rana, G.C. "The instability of streaming Rivlin – Erickson fluids in porous medium. Studia Geotechinica et Mechanica", 13: 83 – 93, 2001.

[13] Shivakumara. I.S, Lee. J, Malarshetty.M.S, Sureshkumara.S," Effect of thermal modulation on the onset of thermal convection in Walters'B′ viscoelastic fluid in a porous medium, transport in porous media", 87, 297 – 307, 2011.

[14] Vafai, K. and Hadim, H.A. "Handbook of porous media". New York: M.Decker, 2000.

[15] Wooding, R.A.. "Rayleigh instability of thermal boundary layer in flow through a porous medium. Journal of fluid mechanics", 9:183 – 192, 1960.